# 1.11 Pure: Algebra and Functions Depth

EDEXCEL 9MA0 · Calculator allowed · about 230 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

**Answer ALL questions. Show all your working.**

**1** (5x + 7) / ((x + 2)(x - 1)), x is real, x not equal to -2 or 1.

**(a)** Express (5x + 7) / ((x + 2)(x - 1)) in partial fractions. **(4)**

**(b)** State the two values of x for which the partial fraction expression is not valid. **(2)**

**(c)** Find the exact value of (5x + 7) / ((x + 2)(x - 1)) when x = 3, using both the original expression and the partial fraction form, showing they agree. **(2)**

**(Total for Question 1 is 8 marks)**

**2** Fully factorise $x^2 - 5x + 6$.

**(Total for Question 2 is 1 mark)**

**3** $g(x) = x^3 + ax^2 + bx - 12$, where a and b are constants. $(x - 1)$ and $(x + 3)$ are both factors of $g(x)$.

**(a)** Show that $a + b = 11$, form a second equation connecting a and b, and hence find the values of a and b. **(5)**

**(b)** Fully factorise $g(x)$. **(3)**

**(c)** Hence write down all the roots of $g(x) = 0$. **(2)**

**(Total for Question 3 is 10 marks)**

**4** $h(x) = |2x - 5| - 3$.

**(a)** Solve $h(x) = 4$. **(4)**

**(b)** Solve $h(x) \leq 2$, giving your answer in set notation. **(4)**

**(c)** State the coordinates of the vertex of $y = h(x)$, and the coordinates of the points where the graph crosses the coordinate axes. **(3)**

**(Total for Question 4 is 11 marks)**

**5** $f(x) = x^3 - 3x^2 + 4$. Use the factor theorem to state whether $(x - 2)$ is a factor of $f(x)$.

**(Total for Question 5 is 1 mark)**

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**6** $f(x) = x^3 + x^2 - 10x + 8$. Show that $(x - 2)$ is a factor of $f(x)$.

**(Total for Question 6 is 2 marks)**

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**7** Find the remainder when $g(x) = 2x^3 - 5x^2 + 3$ is divided by $(x + 2)$.

**(Total for Question 7 is 2 marks)**

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**8** $f(x) = (x - 3)^2 + 5$, for $x \geq 3$. State the range of f.

**(Total for Question 8 is 2 marks)**

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**9** Find the remainder when $f(x) = x^3 + 2x^2 - 5x + 1$ is divided by $(x - 1)$.

**(Total for Question 9 is 1 mark)**

**10** State the values of x for which 7 / ((x-2)(x+5)) is not defined.

**(Total for Question 10 is 1 mark)**

**11** Solve $|x - 4| = 7$.

**(Total for Question 11 is 1 mark)**

**12** Simplify fully $(x^2 - 9) / (x - 3)$, for $x \neq 3$.

**(Total for Question 12 is 1 mark)**

**13** $f(x) = 2x - 5$, $g(x) = x + 3$. Find $fg(x)$.

**(Total for Question 13 is 1 mark)**

**14** $f(x) = 2x - 5$. Find $f^{-1}(x)$.

**(Total for Question 14 is 1 mark)**

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**15** $f(x) = x^2 - 6x + 5$, $x \geq 3$.

**(a)** Express $f(x)$ in the form $(x - a)^2 - b$, stating the values of a and b. **(3)**

**(b)** State the range of f, given the domain $x \geq 3$. **(2)**

**(c)** Find $f^{-1}(x)$ and state its domain. **(4)**

**(Total for Question 15 is 9 marks)**

**16** $f(x) = x^3 - 2x^2 - 5x + 6$.

**(a)** Show that $(x - 1)$ is a factor of $f(x)$. **(2)**

**(b)** Hence fully factorise $f(x)$. **(2)**

**(Total for Question 16 is 4 marks)**

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**17** $h(x) = x^3 + kx^2 - 7x - 2$, where k is a constant. Given that $(x + 2)$ is a factor of $h(x)$, find the value of k.

**(Total for Question 17 is 3 marks)**

**18** $f(x) = 2x^3 + ax^2 - 3x + b$, where a and b are constants. When f(x) is divided by (x - 2), the remainder is 15. When f(x) is divided by (x + 1), the remainder is -6. Find the values of a and b.

**(Total for Question 18 is 4 marks)**

**19** Solve the equation $|3x - 2| = x + 4$.

**(Total for Question 19 is 3 marks)**

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**20** $f(x) = 2x^2 - 12x + 7$, for $x \leq 1$.

**(a)** Express $f(x)$ in the form $a(x-p)^2 + q$, stating the values of a, p and q. **(2)**

**(b)** State the range of f, given the domain $x \leq 1$. **(2)**

**(Total for Question 20 is 4 marks)**

**21** $(x + 1)/(x - 1) - (x - 1)/(x + 1)$, x not equal to 1 or -1.

**(a)** Show that $(x + 1)/(x - 1) - (x - 1)/(x + 1)$ can be written as $4x/(x^2 - 1)$. **(4)**

**(b)** Hence solve $(x + 1)/(x - 1) - (x - 1)/(x + 1) = 3$, giving your answer(s) as exact simplified surds. **(4)**

**(Total for Question 21 is 8 marks)**

**22** Solve the following involving the modulus function.

**(a)** Solve $|4x - 3| = 2x - 2$. **(4)**

**(b)** Solve $|4x - 3| < 3$. **(2)**

**(Total for Question 22 is 6 marks)**

**23** The equation $x^2 + 2kx + (3k + 10) = 0$, where k is a constant, is to be considered.

**(a)** Find the discriminant of the equation, simplifying fully. **(3)**

**(b)** Find the set of values of k for which the equation has two distinct real roots. **(3)**

**(c)** Find the value(s) of k for which the equation has equal roots, and find the repeated root in each case. **(3)**

**(Total for Question 23 is 9 marks)**

**24** $f(x) = x^3 + px^2 + qx - 4$, where p and q are constants. When f(x) is divided by (x - 1) the remainder is -10, and when f(x) is divided by (x - 3) the remainder is -16.

**(a)** Show that p and q satisfy the simultaneous equations $p + q = -7$ and $9p + 3q = -39$. **(3)**

**(b)** Hence find the values of p and q. **(3)**

**(c)** Find the remainder when f(x) is divided by (x - 1). **(2)**

**(Total for Question 24 is 8 marks)**

**25** $f(x) = (8x + 26)/((x + 2)(x + 4))$, $x \neq -2$, $x \neq -4$.

**(a)** Express f(x) in partial fractions. **(4)**

**(b)** Hence find the exact value of f(x) when x = 0. **(2)**

**(Total for Question 25 is 6 marks)**

**26** The function f is defined by $f(x) = (3x + 8)/(x - 4)$, x in R, $x \neq 4$. The function g is defined by $g(x) = x^2 + 5$, x in R.

**(a)** Find $f^{-1}(x)$, stating its domain. **(4)**

**(b)** Write down the range of f. **(1)**

**(c)** Find fg(x) and hence solve $fg(x) = 71/17$. **(4)**

**(Total for Question 26 is 9 marks)**

**27** $g(x) = (3x^2 - 17x + 26)/((x + 2)(x - 4)^2)$, $x \neq -2$, $x \neq 4$.

**(a)** Express $g(x)$ in the form $A/(x + 2) + B/(x - 4) + C/(x - 4)^2$. **(4)**

**(b)** Find the exact value of $g(6)$. **(2)**

**(Total for Question 27 is 6 marks)**

**28** The equation $x^2 + 2kx + (-5k - 4) = 0$, where k is a constant, is to be considered.

**(a)** Find the discriminant of the equation, simplifying fully. **(3)**

**(b)** Find the set of values of k for which the equation has two distinct real roots. **(3)**

**(c)** Find the value(s) of k for which the equation has equal roots, and find the repeated root in each case. **(3)**

**(Total for Question 28 is 9 marks)**

**29** Solve the following involving the modulus function.

**(a)** Solve $|3x - 4| = 2x - 3$. **(4)**

**(b)** Solve $|3x - 4| < 3$. **(2)**

**(Total for Question 29 is 6 marks)**

**30** The function f is defined by $f(x) = (6x + 7)/(x - 3)$, x in R, $x \neq 3$. The function g is defined by $g(x) = x^2 + 3$, x in R.

**(a)** Find $f^{-1}(x)$, stating its domain. **(4)**

**(b)** Write down the range of f. **(1)**

**(c)** Find fg(x) and hence solve $fg(x) = 49/4$. **(4)**

**(Total for Question 30 is 9 marks)**

**31** $h(x) = (2x^2 + 14x + 18)/((x + 4)(x + 1))$, $x \neq -4$, $x \neq -1$.

**(a)** Show that h(x) can be written in the form $2 + A/(x + 4) + B/(x + 1)$, and find the values of A and B. **(4)**

**(b)** Hence find h(1). **(2)**

**(Total for Question 31 is 6 marks)**

**32** $(3x^2 - x + 4) / ((x + 1)(x - 1)^2) = A/(x + 1) + B/(x - 1) + C/(x - 1)^2$.

**(a)** Find the values of the constants A, B and C. **(6)**

**(b)** Explain why the partial fraction expression requires no additional polynomial term. **(2)**

**(c)** Find the exact value of $(3x^2 - x + 4) / ((x + 1)(x - 1)^2)$ when $x = 2$, using the partial fraction form, and verify using the original expression. **(3)**

**(Total for Question 32 is 11 marks)**

**33** Functions f and g are defined by $f(x) = 2x/(x - 3)$, x is real, x not equal to 3, and $g(x) = x + 1$, x is real.

**(a)** Find fg(x), simplifying your answer fully. **(3)**

**(b)** Find $f^{-1}(x)$, stating its domain. **(4)**

**(c)** Solve the equation $fg(x) = f^{-1}(x)$, showing that there is no solution. **(3)**

**(Total for Question 33 is 10 marks)**

**34** $f(x) = (5 + x) / ((1 + 2x)(1 - x))$, $|x| < 1/2$.

**(a)** Express $f(x)$ in the form $A/(1 + 2x) + B/(1 - x)$, stating the values of A and B. **(5)**

**(b)** Hence expand $f(x)$ in ascending powers of x, up to and including the term in $x^2$, giving each coefficient as an exact number. State the range of values of x for which the expansion is valid. **(4)**

**(Total for Question 34 is 9 marks)**

# Mark scheme · 1.11 Pure: Algebra and Functions Depth

## Question 1

(a) writes (5x+7)/((x+2)(x-1)) = A/(x+2) + B/(x-1) and forms the identity 5x + 7 = A(x - 1) + B(x + 2) **M1**

(a) substitutes x = 1 (or x = -2) to find one constant **M1**

(a) A = 1 **A1**

(a) B = 4 oe **A1**

**(a) Answer: 1/(x + 2) + 4/(x - 1)**

(b) x = -2 stated **B1**

(b) x = 1 stated **B1**

**(b) Answer: x = -2 and x = 1**

(c) substitutes x = 3 into both the original expression and the partial fraction form **M1**

(c) both give 11/5 (= 2.2) cao **A1**

**(c) Answer: 11/5 (= 2.2)**

## Question 2

(x-2)(x-3) cao **B1**

**Answer: (x-2)(x-3)**

## Question 3

(a) uses g(1) = 0 to form a + b = 11 oe (1 + a + b - 12 = 0) **M1**

(a) uses g(-3) = 0 to form 3a - b = 13 oe (-27 + 9a - 3b - 12 = 0) **M1**

(a) solves the simultaneous equations **M1**

(a) a = 6 **A1**

(a) b = 5 **A1**

**(a) Answer: a = 6, b = 5**

(b) divides g(x) = $x^3 + 6x^2 + 5x - 12$ by (x - 1)(x + 3) = $x^2 + 2x - 3$ **M1**

(b) third factor (x + 4) obtained **A1**

(b) g(x) = (x - 1)(x + 3)(x + 4) oe **A1**

**(b) Answer: g(x) = (x - 1)(x + 3)(x + 4)**

(c) x = 1 and x = -3 stated **B1**

(c) x = -4 stated, all three roots oe **B1**

**(c) Answer: x = 1, x = -3, x = -4**

## Question 4

(a) forms |2x - 5| = 7 **M1**

(a) sets up two linear equations 2x-5=7 and 2x-5=-7 **M1**

(a) x = 6 **A1**

(a) x = -1 oe **A1**

**(a) Answer: x = 6 or x = -1**

(b) forms |2x - 5| ≤ 5 **M1**

(b) forms the compound inequality -5 ≤ 2x-5 ≤ 5 **M1**

(b) 0 ≤ x ≤ 5 oe **A1**

(b) correct set notation {x : 0 ≤ x ≤ 5} oe **A1**

**(b) Answer: {x : 0 ≤ x ≤ 5}**

(c) vertex (2.5, -3) oe **B1**

(c) y-intercept (0, 2) **B1**

(c) x-intercepts (1, 0) and (4, 0) **B1**

**(c) Answer: Vertex (2.5, -3); y-intercept (0, 2); x-intercepts (1, 0) and (4, 0)**

## Question 5

f(2) = 0, so (x-2) is a factor **B1**

**Answer: Yes, (x-2) is a factor**

## Question 6

substitutes x = 2 into f(x) **M1**

f(2) = 0, so (x-2) is a factor, cso **A1**

**Answer: f(2) = 0, so (x-2) is a factor**

## Question 7

substitutes x = -2 into g(x) **M1**

remainder = -33 cao **A1**

**Answer: -33**

## Question 8

recognises the vertex of f is at (3, 5) **M1**

f(x) ≥ 5 cao **A1**

**Answer: f(x) ≥ 5**

## Question 9

remainder = -1 cao **B1**

**Answer: -1**

## Question 10

x = 2 and x = -5 **B1**

**Answer: x = 2, x = -5**

## Question 11

x = 11 or x = -3, both values **B1**

**Answer: x = 11 or x = -3**

## Question 12

x + 3 cao **B1**

**Answer: x + 3**

## Question 13

2x + 1 cao **B1**

**Answer: fg(x) = 2x + 1**

## Question 14

(x+5)/2 cao **B1**

**Answer: $f^{-1}(x) = (x+5)/2$**

## Question 15

(a) attempts to complete the square on $x^2 - 6x + 5$ **M1**

(a) a = 3 **A1**

(a) b = 4 oe, i.e. $f(x) = (x-3)^2 - 4$ **A1**

**(a) Answer: $f(x) = (x - 3)^2 - 4$, so a = 3, b = 4**

(b) recognises that for $x \geq 3$, $(x-3) \geq 0$ so $(x-3)^2 \geq 0$, and f is increasing on this domain **M1**

(b) $f(x) \geq -4$ oe **A1**

**(b) Answer: $f(x) \geq -4$**

(c) sets $y = (x-3)^2 - 4$ and rearranges to $(x-3)^2 = y+4$ **M1**

(c) takes the square root and selects the positive root, consistent with domain $x \geq 3$: $x-3 = \sqrt{y+4}$ **M1**

(c) $f^{-1}(x) = 3 + \sqrt{x+4}$ oe cao **A1**

(c) domain $x \geq -4$, ft from the range of f found in part (b) **B1ft**

**(c) Answer: $f^{-1}(x) = 3 + \sqrt{x + 4}$, domain: $x \geq -4$**

## Question 16

(a) substitutes x = 1 into f(x) **M1**

(a) f(1) = 0, so (x-1) is a factor, cso **A1**

**(a) Answer: f(1) = 0, so (x-1) is a factor**

(b) divides f(x) by (x-1) (or uses inspection) to find the quadratic factor $x^2 - x - 6$ **M1**

(b) $f(x) = (x-1)(x-3)(x+2)$ cao **A1**

**(b) Answer: $f(x) = (x-1)(x-3)(x+2)$**

## Question 17

substitutes x = -2 into h(x) and sets the result equal to 0 **M1**

forms the equation 4k + 4 = 0 oe **A1**

k = -1 cao **A1**

**Answer: k = -1**

## Question 18

forms the equation f(2) = 15, i.e. 4a + b = 5 oe **M1**

forms the equation f(-1) = -6, i.e. a + b = -7 oe **M1**

solves the two equations simultaneously **M1**

a = 4, b = -11 cao **A1**

**Answer: a = 4, b = -11**

## Question 19

solves 3x - 2 = x + 4 to get x = 3 **M1**

solves 3x - 2 = -(x + 4) to get x = -0.5 **M1**

both values checked as valid in the original equation, x = 3 or x = -0.5 cao **A1**

**Answer: x = 3 or x = -0.5**

## Question 20

(a) completes the square: $2(x^2-6x) + 7 = 2[(x-3)^2-9] + 7$ **M1**

(a) $2(x-3)^2 - 11$, so a=2, p=3, q=-11 **A1**

**(a) Answer: $f(x) = 2(x-3)^2 - 11$**

(b) evaluates f(1) = -3 (or uses the vertex form) and recognises f is decreasing throughout $x \leq 1$, since the vertex is at x=3 **M1**

(b) $f(x) \geq -3$ cao **A1**

**(b) Answer: $f(x) \geq -3$**

## Question 21

(a) writes over the common denominator (x-1)(x+1): $[(x+1)^2 - (x-1)^2] / [(x-1)(x+1)]$ **M1**

(a) expands both squares: $(x^2+2x+1) - (x^2-2x+1)$ **M1**

(a) numerator simplifies to 4x **A1**

(a) denominator $(x-1)(x+1) = x^2-1$, giving $4x/(x^2-1)$, cso **A1**

**(a) Answer: $4x / (x^2 - 1)$, as required**

(b) uses the result from part (a) to form $4x/(x^2-1) = 3$ **M1**

(b) rearranges to $3x^2 - 4x - 3 = 0$ oe **M1**

(b) applies the quadratic formula: $x = (4 \pm \sqrt{16+36}) / 6$ **M1**

(b) $x = (2 \pm \sqrt{13}) / 3$ oe cao, both values valid since neither equals 1 or -1 **A1**

**(b) Answer: $x = (2 + \sqrt{13})/3$ or $x = (2 - \sqrt{13})/3$**

## Question 22

(a) squares both sides, or considers 4x - 3 = 2x - 2 and 4x - 3 = -(2x - 2) **M1**

(a) obtains the candidate solutions 1/2, 5/6 **A1**

(a) checks each candidate in the original equation (the right side must be non-negative) **M1**

(a) no solutions **A1**

**(a) Answer: no solutions**

(b) -3 < 4x - 3 < 3 **M1**

(b) 0 < x < 3/2 **A1**

**(b) Answer: 0 < x < 3/2**

## Question 23

(a) uses $b^2 - 4ac$ with a = 1, b = 2k **M1**

(a) $4k^2 - 4(3k + 10)$ **A1**

(a) $= 4(k^2 - 3k - 10)$ **A1**

**(a) Answer: discriminant = $4(k + 2)(k - 5)$**

(b) discriminant > 0 and finds the critical values k = -2 and k = 5 **M1**

(b) chooses the outside region for a positive quadratic in k **M1**

(b) k < -2 or k > 5 **A1**

**(b) Answer: k < -2 or k > 5**

(c) sets the discriminant to zero **M1**

(c) k = -2 or k = 5 **A1**

(c) repeated root x = -k, so x = 2 or x = -5 **A1**

**(c) Answer: k = -2, x = 2; k = 5, x = -5**

## Question 24

(a) uses the remainder theorem: f(1) = -10 **M1**

(a) p + q = -7 **A1**

(a) f(3) = -16 gives 9p + 3q = -39 (cso) **A1**

**(a) Answer: equations as stated**

(b) solves the pair of simultaneous equations (elimination or substitution) **M1**

(b) p = -3 **A1**

(b) q = -4 **A1**

**(b) Answer: p = -3, q = -4**

(c) evaluates f(1) **M1**

(c) remainder = -10 **A1**

**(c) Answer: -10**

## Question 25

(a) writes $f(x) = A/(x + 2) + B/(x + 4)$ and equates numerators **M1**

(a) substitutes $x = -2$ (or equates coefficients) to find A **M1**

(a) $A = 5$ **A1**

(a) $B = 3$ **A1**

**(a) Answer: $f(x) = 5/(x + 2) + 3/(x + 4)$**

(b) substitutes into either form **M1**

(b) 13/4 **A1**

**(b) Answer: 13/4**

## Question 26

(a) sets $y = (3x + 8)/(x - 4)$ and rearranges: $y(x - 4) = 3x + 8$ **M1**

(a) collects x terms: $x(y - 3) = 4y + 8$ **M1**

(a) $f^{-1}(x) = (4x + 8)/(x - 3)$ **A1**

(a) domain x in R, $x \neq 3$ **B1**

**(a) Answer: $f^{-1}(x) = (4x + 8)/(x - 3)$, $x \neq 3$**

(b) $f(x) \neq 3$, i.e. y in R, $y \neq 3$ **B1**

**(b) Answer: $y \neq 3$**

(c) $fg(x) = f(x^2 + 5) = (3(x^2 + 5) + 8)/(x^2 + 5 - 4)$ **M1**

(c) sets equal to 71/17 and rearranges to a quadratic in $x^2$ (or x) **M1**

(c) $x^2 = 16$ **A1**

(c) $x = 4$ or $x = -4$ **A1**

**(c) Answer: $x = \pm 4$**

## Question 27

(a) writes the identity $A(x - q)^2 + B(x - p)(x - q) + C(x - p) =$ numerator **M1**

(a) substitutes $x = -2$ and $x = 4$ to find A and C **M1**

(a) $A = 2$, $C = 1$ **A1**

(a) compares coefficients of $x^2$ to find $B = 1$ **A1**

**(a) Answer: $A = 2$, $B = 1$, $C = 1$**

(b) substitutes into the partial fraction form or the original **M1**

(b) 1 **A1**

**(b) Answer: 1**

## Question 28

(a) uses $b^2 - 4ac$ with $a = 1$, $b = 2k$ **M1**

(a) $4k^2 - 4(-5k - 4)$ **A1**

(a) $= 4(k^2 + 5k + 4)$ **A1**

**(a) Answer: discriminant $= 4(k + 4)(k + 1)$**

(b) discriminant $> 0$ and finds the critical values $k = -4$ and $k = -1$ **M1**

(b) chooses the outside region for a positive quadratic in k **M1**

(b) $k < -4$ or $k > -1$ **A1**

**(b) Answer: $k < -4$ or $k > -1$**

(c) sets the discriminant to zero **M1**

(c) $k = -4$ or $k = -1$ **A1**

(c) repeated root $x = -k$, so $x = 4$ or $x = 1$ **A1**

**(c) Answer: $k = -4$, $x = 4$; $k = -1$, $x = 1$**

## Question 29

(a) squares both sides, or considers $3x - 4 = 2x - 3$ and $3x - 4 = -(2x - 3)$ **M1**

(a) obtains the candidate solutions 1, 7/5 **A1**

(a) checks each candidate in the original equation (the right side must be non-negative) **M1**

(a) no solutions **A1**

**(a) Answer: no solutions**

(b) $-3 < 3x - 4 < 3$ **M1**

(b) $1/3 < x < 7/3$ **A1**

**(b) Answer: $1/3 < x < 7/3$**

## Question 30

(a) sets $y = (6x + 7)/(x - 3)$ and rearranges: $y(x - 3) = 6x + 7$ **M1**

(a) collects x terms: $x(y - 6) = 3y + 7$ **M1**

(a) $f^{-1}(x) = (3x + 7)/(x - 6)$ **A1**

(a) domain x in R, $x \neq 6$ **B1**

**(a) Answer: $f^{-1}(x) = (3x + 7)/(x - 6)$, $x \neq 6$**

(b) $f(x) \neq 6$, i.e. y in R, $y \neq 6$ **B1**

**(b) Answer: $y \neq 6$**

(c) $fg(x) = f(x^2 + 3) = (6(x^2 + 3) + 7)/(x^2 + 3 - 3)$ **M1**

(c) sets equal to 49/4 and rearranges to a quadratic in $x^2$ (or x) **M1**

(c) $x^2 = 4$ **A1**

(c) $x = 2$ or $x = -2$ **A1**

**(c) Answer: $x = \pm 2$**

## Question 31

(a) recognises the numerator and denominator have the same degree and divides, quotient 2 **M1**

(a) forms the identity for the remainder over the two linear factors **M1**

(a) $A = 2$ **A1**

(a) $B = 2$ **A1**

**(a) Answer: $h(x) = 2 + 2/(x + 4) + 2/(x + 1)$**

(b) substitutes $x = 1$ **M1**

(b) 17/5 **A1**

**(b) Answer: 17/5**

## Question 32

(a) forms the identity $3x^2 - x + 4 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$ **M1**

(a) substitutes x = 1 to find C **M1**

(a) C = 3 **A1**

(a) substitutes x = -1 to find A **M1**

(a) A = 2 **A1**

(a) B = 1 oe, found by comparing coefficients of $x^2$ (or a further substitution) **A1**

**(a) Answer: A = 2, B = 1, C = 3**

(b) identifies that the numerator has degree 2 and the denominator has degree 3 **B1**

(b) states that, since 2 < 3, the fraction is already proper, so no polynomial division / extra term is needed oe **B1**

**(b) Answer: The numerator $3x^2 - x + 4$ has degree 2 and the denominator $(x+1)(x-1)^2$ has degree 3; since 2 < 3, the fraction is proper and no extra polynomial term is required**

(c) substitutes x = 2 into the partial fraction form $2/(x+1) + 1/(x-1) + 3/(x-1)^2$ **M1**

(c) value = 14/3 cao **A1**

(c) verifies using the original expression: (3(4)-2+4)/((3)(1)) = 14/3 **B1**

**(c) Answer: 14/3**

## Question 33

(a) attempts f(g(x)) = f(x+1) = 2(x+1)/((x+1)-3) **M1**

(a) simplifies the denominator to (x - 2) **A1**

(a) fg(x) = 2(x+1)/(x-2), x not equal to 2, oe cao **A1**

**(a) Answer: fg(x) = 2(x + 1)/(x - 2), x not equal to 2**

(b) sets y = 2x/(x-3) and rearranges: y(x-3) = 2x **M1**

(b) collects x terms: x(y-2) = 3y oe **M1**

(b) $f^{-1}(x) = 3x/(x-2)$ oe cao **A1**

(b) domain: x is real, x not equal to 2, ft from the denominator of $f^{-1}(x)$ **B1ft**

**(b) Answer: $f^{-1}(x) = 3x/(x - 2)$, domain: x is real, x not equal to 2**

(c) sets 2(x+1)/(x-2) = 3x/(x-2) **M1**

(c) multiplies both sides by (x-2), x not equal to 2, to obtain 2(x+1) = 3x, giving x = 2 **M1**

(c) rejects x = 2 as it is excluded from the domain of both functions, so there is no solution, cso **A1**

**(c) Answer: No solution (the only candidate, x = 2, is excluded from the domain)**

## Question 34

(a) forms the identity 5 + x = A(1 - x) + B(1 + 2x) **M1**

(a) substitutes x = 1 (or x = -1/2) to find one constant **M1**

(a) B = 2 **A1**

(a) A = 3 **A1**

(a) f(x) = 3/(1+2x) + 2/(1-x) oe cao **A1**

**(a) Answer: 3/(1 + 2x) + 2/(1 - x)**

(b) expands $3(1+2x)^{-1} = 3(1 - 2x + 4x^2 - ...)$ using the binomial series **M1**

(b) expands $2(1-x)^{-1} = 2(1 + x + x^2 + ...)$ using the binomial series **M1**

(b) $f(x) = 5 - 4x + 14x^2 + ...$ oe cao **A1**

(b) valid for |x| < 1/2 oe, identified as the more restrictive of the two ranges **B1**

**(b) Answer: $f(x) = 5 - 4x + 14x^2 + ...$, valid for |x| < 1/2**