1.12 Pure: Sequences, Series and Binomial Depth: 1-hour Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Consider the binomial expansion of $(2+3x)^7$ in ascending powers of x. Let T_k denote the term in x^k (so T_0 is the constant term).

(a) Find an expression for T_k in terms of k. **(2)**

(b) Show that $T_{k+1}/T_k = 3(7-k) / (2(k+1))$. **(3)**

(c) Hence find the value of k for which T_k is greatest, and find this greatest coefficient. **(3)**

(Total for Question 1 is 8 marks)

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2 Find the sum of the first 20 terms of the arithmetic series with first term 7 and common difference -2.

(Total for Question 2 is 2 marks)

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3 Find the coefficient of x^3 in the binomial expansion of $(1-x)^6$.

(Total for Question 3 is 2 marks)

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4 An arithmetic series has first term 6 and common difference 5.

(a) Find the 20th term. **(2)**

(b) Find the sum of the first 20 terms. **(2)**

(Total for Question 4 is 4 marks)

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5 Find the sum of the first 10 terms of an arithmetic series with first term 4 and common difference 6.

(Total for Question 5 is 1 mark)

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6 Find the sum of the first 4 terms of a geometric series with first term 2 and common ratio 3.

(Total for Question 6 is 1 mark)

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7 A geometric series has first term 8 and common ratio 0.5. State the sum to infinity.

(Total for Question 7 is 1 mark)

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8 Write down the first three terms, in ascending powers of x, of the binomial expansion of $(1+x)^4$.

(Total for Question 8 is 1 mark)

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9 An arithmetic sequence has first term 10 and 4th term 22. State the common difference.

(Total for Question 9 is 1 mark)

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10 The sequence u_r is defined by $u_r = 2r + 4$, r = 1, 2, 3, ...

(a) Write down u_1 and the common difference, and state the type of sequence. **(2)**

(b) Evaluate the sum from r = 1 to r = 13 of (2r + 4). **(3)**

(c) Find the least value of n for which the sum from r = 1 to r = n of (2r + 4) exceeds 610. **(3)**

(Total for Question 10 is 8 marks)

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11 $g(x) = (1+3x)^4 (1-x)^{-1}$, for $|x| < 1/3$.

(a) Find the binomial expansion of $(1+3x)^4$ in ascending powers of x, up to and including the term in x^2. **(2)**

(b) Given that $(1-x)^{-1} = 1 + x + x^2 + ...$ for $|x| < 1$, hence show that $g(x) = 1 + 13x + 67x^2 + ...$ for small values of x. **(2)**

(Total for Question 11 is 4 marks)

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12 $f(x) = 1/\sqrt{4-3x} = (4-3x)^{-1/2}$.

(a) Show that f(x) can be written as $(1/2)(1 - (3/4)x)^{-1/2}$. **(2)**

(b) Hence find the binomial expansion of f(x) in ascending powers of x, up to and including the term in x^2. Give each coefficient as a simplified fraction. **(5)**

(c) State the range of values of x for which the expansion in part (b) is valid. **(1)**

(d) By substituting x=0.2 into the expansion from part (b), find an estimate for $1/\sqrt{3.4}$, giving your unrounded answer to 5 decimal places. Given that $1/\sqrt{3.4} = 0.542326$ to 6 decimal places, use your unrounded estimate to calculate the percentage error, giving your answer to 2 significant figures. **(4)**

(Total for Question 12 is 12 marks)

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13 $f(x) = 1/\sqrt{1+4x} = (1+4x)^{-1/2}$, for $|x| < 1/4$.

(a) Find the binomial expansion of f(x) in ascending powers of x, up to and including the term in x^2, giving each coefficient as an exact number. **(3)**

(b) By substituting x = 0.01 into the expansion from part (a), find an estimate for $1/\sqrt{1.04}$, giving your answer to 4 decimal places. **(1)**

(Total for Question 13 is 4 marks)

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Mark scheme · 1.12 Pure: Sequences, Series and Binomial Depth: 1-hour Pack

Question 1

(a) correct structure using the general binomial term C(7,k) $(2)^{7-k}(3x)^k$ **M1**

(a) cao $T_k = C(7,k) * 2^{7-k} * 3^k$ **A1**

(a) Answer: $T_k = C(7,k) * 2^{7-k} * 3^k$

(b) uses C(7,k+1)/C(7,k) = (7-k)/(k+1) **M1**

(b) correctly finds the ratio of powers of 2 and 3 (a factor of 1/2 and a factor of 3) **M1**

(b) cso combines to 3(7-k)/(2(k+1)) **A1**

(b) Answer: $T_{k+1}/T_k = 3(7-k)/(2(k+1))$ (shown)

(c) sets $T_{k+1}/T_k > 1$ and solves the resulting inequality to k < 3.8 (oe) **M1**

(c) deduces that the coefficients increase up to k=4 and decrease after, so the greatest term is T_4 **M1**

(c) cao $T_4 = 22680$ **A1**

(c) Answer: k = 4; greatest coefficient = 22680

Question 2

substitutes into $S_n = (n/2)(2a + (n-1)d)$ **M1**

-240 cao **A1**

Answer: -240

Question 3

identifies the term as $C(6,3)x^3(-1)^3$ **M1**

-20 cao **A1**

Answer: -20

Question 4

(a) substitutes into a + 19d **M1**

(a) 101 cao **A1**

(a) Answer: 101

(b) substitutes into $S_n = (n/2)(2a+(n-1)d)$ **M1**

(b) 1070 cao **A1**

(b) Answer: 1070

Question 5

310 cao **B1**

Answer: 310

Question 6

80 cao **B1**

Answer: 80

Question 7

16 cao **B1**

Answer: 16

Question 8

$1 + 4x + 6x^2$, all three terms correct **B1**

Answer: $1 + 4x + 6x^2$

Question 9

d = 4 cao **B1**

Answer: 4

Question 10

(a) $u_1 = 6$ **B1**

(a) d = 2, arithmetic **B1**

(a) Answer: $u_1 = 6$, d = 2

(b) uses the arithmetic series formula with a = 6, d = 2, n = 13 **M1**

(b) (13/2)(2(6) + 12(2)) **A1**

(b) 234 **A1**

(b) Answer: 234

(c) forms (n/2)(2(6) + (n - 1)(2)) > 610 **M1**

(c) solves the quadratic inequality or tests consecutive values **A1**

(c) $S_{22} = 594$ and $S_{23} = 644$, so n = 23 **A1**

(c) Answer: n = 23

Question 11

(a) uses the binomial expansion with terms C(4,1)(3x) and $C(4,2)(3x)^2$ **M1**

(a) $1 + 12x + 54x^2$ **A1**

(a) Answer: $1 + 12x + 54x^2$

(b) multiplies $(1 + 12x + 54x^2)$ by $(1 + x + x^2)$, collecting terms up to x^2 **M1**

(b) cso: $1 + 13x + 67x^2$ as required **A1**

(b) Answer: $g(x) = 1 + 13x + 67x^2 + ...$

Question 12

(a) factorises 4-3x as 4(1-(3/4)x) **M1**

(a) cso correctly simplifies $4^{-1/2}=1/2$ to reach the given form **A1**

(a) Answer: $f(x) = (1/2)(1-(3/4)x)^{-1/2}$ (shown)

(b) correct general structure $(1+u)^n = 1+nu+n(n-1)/2\ u^2$ with n=-1/2 **M1**

(b) correct term1 in u: -(1/2)u **A1**

(b) correct term2 in u: $(3/8)u^2$ **A1**

(b) substitutes u=-(3/4)x throughout and multiplies by the (1/2) factor from part (a) **M1**

(b) cao $1/2 + (3/16)x + (27/256)x^2$ **A1**

(b) Answer: f(x) is approximately $1/2 + (3/16)x + (27/256)x^2$ (+...)

(c) |x| < 4/3 oe (e.g. -4/3 < x < 4/3) **B1**

(c) Answer: |x| < 4/3

(d) substitutes x=0.2 into the expansion from part (b) **M1**

(d) correct unrounded estimate awrt 0.54172 **M1**

(d) correct method for percentage error, |estimate - true|/true x 100 **M1**

(d) cao awrt 0.11% **A1**

(d) Answer: Estimate is approximately 0.54172; percentage error is approximately 0.11%

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Question 13

(a) uses the binomial series with n = -1/2 and u = 4x **M1**

(a) coefficient of x = -2 **A1**

(a) coefficient of $x^2 = 6$, giving $1 - 2x + 6x^2$ **A1**

(a) Answer: f(x) = $1 - 2x + 6x^2$ + ...

(b) 0.9806 (ft from (a)) **B1**

(b) Answer: 0.9806

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