1.12 Pure: Sequences, Series and Binomial Depth: 2-hour Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A sequence is defined by $u_1 = 5$ and $u_{n+1} = 1 - 1/u_n$ for $n \geq 1$.

(a) Find u_2, u_3 and u_4. **(3)**

(b) Explain why the sequence is periodic and state its order. **(2)**

(c) Find the value of u_{38} and the sum of the first 38 terms. **(4)**

(Total for Question 1 is 9 marks)

2 A company's weekly output of widgets is 29 in the first week and rises by 12 each week. The pattern continues for 17 weeks.

(a) Find the value in week 17. **(2)**

(b) Find the total over the first 17 weeks. **(3)**

(c) Find the smallest number of weeks for which the total exceeds 3900. **(3)**

(Total for Question 2 is 8 marks)

3 Consider the expansion of $(1 + 2x)^8$ in ascending powers of x.

(a) Find the first four terms in the expansion of $(1 + 2x)^8$, giving each coefficient as an integer. **(4)**

(b) Hence find the coefficient of x^2 in the expansion of $(3 + x)(1 + 2x)^8$. **(3)**

(Total for Question 3 is 7 marks)

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4 $f(x) = (5 + 4x)^{-2}$, $|x| < 5/4$.

(a) Find the binomial expansion of f(x) in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(4)**

(b) State the range of values of x for which the expansion is valid. **(1)**

(c) Use your expansion with $x = 0.01$ to estimate the value of $(5.04)^{-2}$, giving your answer to 5 significant figures. **(2)**

(Total for Question 4 is 7 marks)

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5 A sequence is defined by $u_1 = 2$ and $u_{n+1} = 2u_n + 1$ for $n \geq 1$.

(a) Find u_2, u_3 and u_4. **(3)**

(b) Find the value of the sum of $u_1 + u_2 + u_3 + u_4 + u_5$. **(3)**

(c) Given that the sequence has a fixed point L ($u_{n+1} = u_n = L$), find L. **(2)**

(Total for Question 5 is 8 marks)

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6 The sequence u_r is defined by $u_r = 5r + 5$, $r = 1, 2, 3, ...$

(a) Write down u_1 and the common difference, and state the type of sequence. **(2)**

(b) Evaluate the sum from $r = 1$ to $r = 19$ of $(5r + 5)$. **(3)**

(c) Find the least value of n for which the sum from $r = 1$ to $r = n$ of $(5r + 5)$ exceeds 520. **(3)**

(Total for Question 6 is 8 marks)

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7 $f(x) = (16 + x)^{1/2}$, $|x| < 16$.

(a) Find the binomial expansion of f(x) in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(4)**

(b) State the range of values of x for which the expansion is valid. **(1)**

(c) Use your expansion with $x = 0.02$ to estimate the value of $(16.02)^{1/2}$, giving your answer to 5 significant figures. **(2)**

(Total for Question 7 is 7 marks)

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8 A stack of tiles has 27 tiles in the first row, and each further row has 4 more tiles than the row before it. The pattern continues for 15 rows.

(a) Find the value in row 15. **(2)**

(b) Find the total over the first 15 rows. **(3)**

(c) Find the smallest number of rows for which the total exceeds 1500. **(3)**

(Total for Question 8 is 8 marks)

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9 Consider the expansion of $(3 + 2x)^5$ in ascending powers of x.

(a) Find the first four terms in the expansion of $(3 + 2x)^5$, giving each coefficient as an integer. **(4)**

(b) Hence find the coefficient of x^2 in the expansion of $(1 - 3x)(3 + 2x)^5$. **(3)**

(Total for Question 9 is 7 marks)

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10 A sequence is defined by $u_1 = 10$ and $u_{n+1} = 0.5\ u_n + 3$ for $n \geq 1$.

(a) Find the values of u2, u3 and u4. **(3)**

(b) Show that the sequence with terms $u_n - 6$ is a geometric sequence, and state its common ratio. **(4)**

(c) Hence show that $u_n = 6 + 4(0.5)^{n-1}$, and find the value that u_n approaches as n becomes very large. **(4)**

(Total for Question 10 is 11 marks)

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11 A geometric series has first term 10 and common ratio -0.5. Find the sum to infinity, giving your answer as an exact fraction.

(Total for Question 11 is 3 marks)

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12 The sum of the first n terms of an arithmetic series is given by $S_n = 2n^2 + 7n$. Find the first term and the common difference of the series.

(Total for Question 12 is 3 marks)

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13 Priya pays into a savings scheme. She pays £500 in year 1. Each subsequent year, she pays 4% more than the year before, so her annual payments form a geometric sequence with first term 500 and common ratio 1.04.

(a) Find the amount Priya pays in year 8, giving your answer to the nearest penny. **(3)**

(b) Find the total amount Priya has paid into the scheme by the end of year 15 (the sum of the first 15 payments), giving your answer to the nearest pound. **(3)**

(c) Priya wants the total amount paid into the scheme to first exceed £20000. Using logarithms, find the smallest number of years, n, for which this occurs. **(5)**

(d) State one assumption made in this model that may not hold in reality. **(1)**

(Total for Question 13 is 12 marks)

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Mark scheme · 1.12 Pure: Sequences, Series and Binomial Depth: 2-hour Pack

Question 1

(a) $u_2 = 1 - 1/5$ **M1**

(a) $u_2 = 4/5$, $u_3 = -1/4$ **A1**

(a) $u_4 = 5$ **A1**

(a) Answer: $u_2 = 4/5$, $u_3 = -1/4$, $u_4 = 5$

(b) $u_4 = u_1$ so the terms repeat **B1**

(b) period (order) 3 **B1**

(b) Answer: period 3

(c) $38 = 3 \times 12 + 2$, so $u_{38} = u_2$ **M1**

(c) $u_{38} = 4/5$ **A1**

(c) 12 complete cycles of sum 111/20 plus the remaining 2 terms **M1**

(c) sum = 362/5 **A1**

(c) Answer: $u_{38} = 4/5$, sum = 362/5

Question 2

(a) uses $u_n = a + (n - 1)d$ with a = 29, d = 12, n = 17 **M1**

(a) 221 **A1**

(a) Answer: 221

(b) uses $S_n = (n/2)(2a + (n - 1)d)$ or $(n/2)(a + l)$ **M1**

(b) $17/2 \times (29 + 221)$ **A1**

(b) 2125 **A1**

(b) Answer: 2125

(c) forms and solves $(m/2)(2(29) + (m - 1)(12)) > 3900$ **M1**

(c) critical value m = 23.65 (or tests m = 23 and m = 24) **A1**

(c) $S_{23} = 3703 < 3900$ and $S_{24} = 4008 > 3900$, so m = 24 **A1**

(c) Answer: 24

Question 3

(a) binomial expansion with the correct form for the x term: $8C1\ (1)^7(2)x$ **M1**

(a) first two terms $1 + 16x$ **A1**

(a) attempts the x^2 and x^3 terms using 8C2 and 8C3 **M1**

(a) $1 + 16x + 112x^2 + 448x^3$ (all four terms correct) **A1**

(a) Answer: $1 + 16x + 112x^2 + 448x^3$

(b) identifies the two contributions to x^2: 3 times the x^2 coefficient and 1 times the x coefficient **M1**

(b) 336 + 16 **A1**

(b) 352 **A1**

(b) Answer: 352

Question 4

(a) takes out 5 **M1**

(a) expands $(1 + (4/5)x)^{-2}$: $1 + n(kx) + n(n-1)(kx)^2/2! + ...$ **M1**

(a) first two terms $1/25 - 8/125x$ **A1**

(a) $1/25 - (8/125)x + (48/625)x^2 - (256/3125)x^3$ **A1**

(a) Answer: $1/25 - (8/125)x + (48/625)x^2 - (256/3125)x^3$

(b) $|x| < 5/4$ **B1**

(b) Answer: $|x| < 5/4$

(c) substitutes x = 0.01 into the series **M1**

(c) 0.039368 **A1**

(c) Answer: 0.039368

Question 5

(a) $u_2 = 2(2) + 1$ **M1**

(a) $u_2 = 5$ **A1**

(a) $u_3 = 11$, $u_4 = 23$ **A1**

(a) Answer: $u_2 = 5$, $u_3 = 11$, $u_4 = 23$

(b) calculates u_5 **M1**

(b) $u_5 = 47$ **A1**

(b) sum = 88 **A1**

(b) Answer: 88

(c) sets L = 2L + 1 **M1**

(c) L = -1 **A1**

(c) Answer: L = -1

Question 6

(a) $u_1 = 10$ **B1**

(a) d = 5, arithmetic **B1**

(a) Answer: $u_1 = 10$, d = 5

(b) uses the arithmetic series formula with a = 10, d = 5, n = 19 **M1**

(b) $(19/2)(2(10) + 18(5))$ **A1**

(b) 1045 **A1**

(b) Answer: 1045

(c) forms $(n/2)(2(10) + (n - 1)(5)) > 520$ **M1**

(c) solves the quadratic inequality or tests consecutive values **A1**

(c) $S_{13} = 520$ and $S_{14} = 595$, so n = 14 **A1**

(c) Answer: n = 14

Question 7

(a) takes out 16 **M1**

(a) expands $(1 + (1/16)x)^{1/2}$: $1 + n(kx) + n(n-1)(kx)^2/2! + ...$ **M1**

(a) first two terms $4 + 1/8x$ **A1**

(a) $4 + (1/8)x - (1/512)x^2 + (1/16384)x^3$ **A1**

(a) Answer: $4 + (1/8)x - (1/512)x^2 + (1/16384)x^3$

(b) $|x| < 16$ **B1**

(b) Answer: $|x| < 16$

(c) substitutes x = 0.02 into the series **M1**

(c) 4.0025 **A1**

(c) Answer: 4.0025

Question 8

(a) uses $u_n = a + (n - 1)d$ with a = 27, d = 4, n = 15 **M1**

(a) 83 **A1**

(a) Answer: 83

(b) uses $S_n = (n/2)(2a + (n - 1)d)$ or $(n/2)(a + l)$ **M1**

(b) $15/2 \times (27 + 83)$ **A1**

(b) 825 **A1**

(b) Answer: 825

(c) forms and solves $(m/2)(2(27) + (m - 1)(4)) > 1500$ **M1**

(c) critical value m = 21.84 (or tests m = 21 and m = 22) **A1**

(c) $S_{21} = 1407 < 1500$ and $S_{22} = 1518 > 1500$, so m = 22 **A1**

(c) Answer: 22

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Question 9

(a) binomial expansion with the correct form for the x term: $5C1\ (3)^4(2)x$ **M1**

(a) first two terms $243 + 810x$ **A1**

(a) attempts the x^2 and x^3 terms using 5C2 and 5C3 **M1**

(a) $243 + 810x + 1080x^2 + 720x^3$ (all four terms correct) **A1**

(a) Answer: $243 + 810x + 1080x^2 + 720x^3$

(b) identifies the two contributions to x^2: 1 times the x^2 coefficient and -3 times the x coefficient **M1**

(b) 1080 - 2430 **A1**

(b) -1350 **A1**

(b) Answer: -1350

Question 10

(a) correct method substituting into the recurrence at least once **M1**

(a) u2=8 and u3=7 correct **A1**

(a) cao u4=6.5 **A1**

(a) Answer: u2 = 8, u3 = 7, u4 = 6.5

(b) lets $v_n = u_n - 6$ and substitutes $u_{n+1}=0.5u_n+3$ into $v_{n+1}=u_{n+1}-6$ **M1**

(b) correctly simplifies to $v_{n+1} = 0.5(u_n - 6)$ **M1**

(b) cso identifies $v_{n+1} = 0.5\ v_n$, confirming a geometric sequence **A1**

(b) states common ratio 0.5 **A1**

(b) Answer: $u_n - 6$ is geometric with common ratio 0.5 (shown)

(c) writes the general term of the geometric sequence $v_n = v_1\ r^{n-1} = 4(0.5)^{n-1}$ **M1**

(c) cso $u_n = v_n + 6 = 6 + 4(0.5)^{n-1}$, matching the given result **A1**

(c) recognises that since $|0.5| < 1$, $(0.5)^{n-1}$ tends to 0 as n tends to infinity **M1**

(c) cao limit = 6 **A1**

(c) Answer: $u_n = 6 + 4(0.5)^{n-1}$; u_n tends to 6 as n tends to infinity

Question 11

substitutes a = 10, r = -0.5 into a/(1-r) **M1**

= 10/1.5 **A1**

= 20/3 cao (exact) **A1**

Answer: 20/3

Question 12

finds S1 = 9 (the first term) and S2 = 22 **M1**

second term = S2 - S1 = 13, so d = 13 - 9 = 4 **A1**

a = 9 cao **A1**

Answer: a = 9, d = 4

Question 13

(a) uses $u_n = a\ r^{n-1}$ with a=500, r=1.04, n=8 **M1**

(a) correct value of 1.04^7 (awrt 1.3159) **M1**

(a) awrt 657.97 (nearest penny) **A1**

(a) Answer: £657.97 (nearest penny)

(b) correct use of $S_n = a(r^n-1)/(r-1)$ with n=15 **M1**

(b) correct value of 1.04^{15} (awrt 1.8009) **M1**

(b) awrt 10012 (nearest pound) **A1**

(b) Answer: £10012 (nearest pound)

(c) forms the inequality $500(1.04^n-1)/0.04 > 20000$, i.e. $12500(1.04^n-1) > 20000$ **M1**

(c) rearranges to $1.04^n > 2.6$ **M1**

(c) dM1 takes logs of both sides, $n > \ln(2.6)/\ln(1.04)$ (oe base) **M1**

(c) correct unrounded value awrt 24.36 (or better) **A1**

(c) cao smallest integer n=25, with supporting check **A1**

(c) Answer: n = 25

(d) valid assumption, e.g. that Priya's payments increase by exactly 4% every year without fail, or that she is able and willing to keep increasing her payments indefinitely, which may not be realistic **B1**

(d) Answer: e.g. it assumes the 4% annual increase in payments continues exactly every year, which may not be realistic since Priya's circumstances or the scheme's terms could change.

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