1.12 Pure: Sequences, Series and Binomial Depth: Test Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 An arithmetic sequence has 4th term equal to 17 and 11th term equal to 45.

(a) Show that the common difference of the sequence is 4. **(2)**

(b) Find the first term, a, of the sequence. **(1)**

(c) Find the sum of the first 30 terms of the sequence. **(2)**

(Total for Question 1 is 5 marks)

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2 State the coefficient of x in the binomial expansion of $(1+2x)^3$.

(Total for Question 2 is 1 mark)

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3 Find the coefficient of x^2 in the binomial expansion of $(1+x)^5$.

(Total for Question 3 is 1 mark)

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4 State, with a reason, whether the geometric series with first term 5 and common ratio 1.2 has a sum to infinity.

(Total for Question 4 is 1 mark)

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5 A town's population is 17000. It is predicted to grow by 3% each year.

(a) Show that the population after n years is 17000×1.03^n. **(2)**

(b) Find the value after 11 years, to the nearest whole number. **(2)**

(c) Find the least whole number of years after which the value exceeds 36600. **(4)**

(Total for Question 5 is 8 marks)

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6 A geometric series has first term 20 and common ratio r = -0.4.

(a) Explain why this series has a sum to infinity. **(1)**

(b) Find the sum to infinity of the series, giving your answer as an exact fraction. **(3)**

(c) Find the smallest value of n such that the sum of the first n terms differs from the sum to infinity by less than 0.01. **(4)**

(Total for Question 6 is 8 marks)

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7 $g(x) = (1+2x)^5 (1-x)^{-2}$, for $|x| < 1/2$.

(a) Find the binomial expansion of $(1+2x)^5$ in ascending powers of x, up to and including the term in x^2. **(3)**

(b) Find the binomial expansion of $(1-x)^{-2}$ in ascending powers of x, up to and including the term in x^2. **(3)**

(c) Hence show that $g(x) = 1 + 12x + 63x^2 + ...$ for small values of x. **(4)**

(Total for Question 7 is 10 marks)

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8 A sequence is defined by $u1 = 20$ and $u_{n+1} = 0.5\, u_n + 3$, for $n \geq 1$.

(a) Find the values of u2 and u3. **(2)**

(b) Show that the sequence with terms $u_n - 6$ is a geometric sequence, and state its common ratio. **(2)**

(Total for Question 8 is 4 marks)

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9 This question concerns arithmetic sequences and series.

(a) Prove, using the method of pairing terms, that the sum of the first n terms of an arithmetic sequence with first term a and common difference d is given by $S_n = n/2\,(2a+(n-1)d)$. **(4)**

(b) Two arithmetic sequences, A and B, have the same common difference d. The first term of sequence A is p and the first term of sequence B is q, where $p \neq q$. Given that the sum of the first 10 terms of sequence A equals the sum of the first 6 terms of sequence B, show that $5p - 3q = -15d$. **(4)**

(c) Given additionally that d=2 and q=3p, find the value of p. **(3)**

(Total for Question 9 is 11 marks)

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Mark scheme · 1.12 Pure: Sequences, Series and Binomial Depth: Test Pack

Question 1

(a) forms two equations a+3d=17 and a+10d=45 (or equivalent) and eliminates a — **M1**

(a) d=4 cso, with full working shown — **A1**

(a) Answer: d = 4 (shown)

(b) a=5 ft (using d=4 found in part (a)) — **B1**

(b) Answer: a = 5

(c) correct substitution into $S_n = n/2(2a+(n-1)d)$ with n=30, a=5, d=4 — **M1**

(c) 1890 cao — **A1**

(c) Answer: 1890

Question 2

6 cao — **B1**

Answer: 6

Question 3

10 cao — **B1**

Answer: 10

Question 4

no, since |r| = 1.2 > 1 — **B1**

Answer: No, since |r| > 1

Question 5

(a) multiplies by 1.03 each year, a geometric sequence with common ratio 1.03 — **M1**

(a) after n years: 17000×1.03^n (cso) — **A1**

(a) Answer: 17000×1.03^n

(b) 17000×1.03^{11} — **M1**

(b) 23532 — **A1**

(b) Answer: 23532

(c) forms $1.03^n > 36600/17000 = 2.1529$ — **M1**

(c) takes logs: n > log(2.1529)/log(1.03) — **M1**

(c) n > 25.943 — **A1**

(c) n = 26 — **A1**

(c) Answer: n = 26

Question 6

(a) states |r| = 0.4 < 1 (or -1 < r < 1), so the series converges — **B1**

(a) Answer: Because |r| = 0.4 < 1, the series converges (has a sum to infinity).

(b) uses $S_\infty = a/(1-r)$ — **M1**

(b) ft correct substitution 20/(1-(-0.4)) — **A1**

(b) cao 100/7 — **A1**

(b) Answer: 100/7 (= 14 2/7)

(c) forms $S_\infty - S_n = a r^n/(1-r)$ (oe) — **M1**

(c) sets up the inequality $(100/7)(0.4)^n < 0.01$ (oe) — **M1**

(c) dM1 takes logs and solves for n, correctly reversing the inequality when dividing by a negative log value — **M1**

(c) cao n=8, with a supporting check — **A1**

(c) Answer: n = 8

Question 7

(a) correct structure using binomial coefficients C(5,1) and C(5,2) — **M1**

(a) 10x term correct — **A1**

(a) $40x^2$ term correct — **A1**

(a) Answer: $1 + 10x + 40x^2$ (+...)

(b) correct general expansion structure $(1+u)^n = 1+nu+n(n-1)/2\ u^2$ with n=-2, u=-x — **M1**

(b) 2x term correct — **A1**

(b) $3x^2$ term correct — **A1**

(b) Answer: $1 + 2x + 3x^2$ (+...)

(c) attempts to multiply the two expansions, considering only terms up to x^2 — **M1**

(c) correct x coefficient 12 (from 1x2 + 10x1) — **A1**

(c) dM1 correct method for the x^2 coefficient, combining all three contributing products (1x3, 10x2, 40x1) — **M1**

(c) cso 63, matching the given result — **A1**

(c) Answer: $g(x) = 1 + 12x + 63x^2 + ...$ (shown)

Question 8

(a) substitutes u1 = 20 into the recurrence — **M1**

(a) u2 = 13, u3 = 9.5 — **A1**

(a) Answer: u2 = 13, u3 = 9.5

(b) $u_{n+1} - 6 = 0.5u_n + 3 - 6 = 0.5(u_n - 6)$ — **M1**

(b) cso: common ratio = 0.5, confirming the sequence is geometric — **A1**

(b) Answer: Common ratio 0.5

Question 9

(a) writes $S_n = a+(a+d)+(a+2d)+...+(a+(n-1)d)$ — **B1**

(a) writes S_n again in reverse order, $(a+(n-1)d)+(a+(n-2)d)+...+a$ — **M1**

(a) adds the two expressions term by term, showing each pair sums to 2a+(n-1)d, giving $2S_n = n(2a+(n-1)d)$ — **M1**

(a) cso correctly divides by 2 to reach $S_n = n/2(2a+(n-1)d)$ — **A1**

(a) Answer: $S_n = n/2(2a+(n-1)d)$ (proved)

(b) correct expression for the sum of the first 10 terms of A, 10p+45d (or equivalent unsimplified form) — **M1**

(b) correct expression for the sum of the first 6 terms of B, 6q+15d — **M1**

(b) dM1 equates the two sums and rearranges to isolate terms in d — **M1**

(b) cso simplifies (dividing by 2) to reach 5p-3q=-15d exactly — **A1**

(b) Answer: 5p - 3q = -15d (shown)

(c) substitutes q=3p and d=2 into 5p-3q=-15d — **M1**

(c) solves the resulting linear equation in p — **M1**

(c) cao p=7.5 — **A1**

(c) Answer: p = 7.5

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