1.12 Pure: Sequences, Series and Binomial Depth

EDEXCEL 9MA0 · Calculator allowed · about 230 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A geometric sequence has first term 6 and third term 54.

(a) Find the possible value(s) of the common ratio, r. **(3)**

(b) Given that all terms of the sequence are positive, find the 7th term. **(2)**

(Total for Question 1 is 5 marks)

2 A factory records the number of defective items produced each month. In month 1 there are 84 defective items, and this decreases by 3 each month compared with the previous month, forming an arithmetic sequence, while the process remains uncorrected.

(a) Find the number of defective items modelled in month 12. **(2)**

(b) Find the total number of defective items modelled over months 6 to 15 inclusive. **(4)**

(c) The process is considered corrected once the modelled number of defective items first reaches zero or fewer. Find the first month in which this occurs according to the model, and comment on whether the arithmetic model remains appropriate after this point. **(3)**

(Total for Question 2 is 9 marks)

3 The sum of the first n terms of an arithmetic series is given by $S_n = 3n^2 + 5n$.

(a) Find the first term and the common difference of the series. **(4)**

(b) Hence find the 25th term of the series. **(2)**

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(Total for Question 3 is 6 marks)

4 An arithmetic sequence has first term 5 and common difference 3. Find the 6th term.

(Total for Question 4 is 1 mark)

5 A geometric sequence has first term 3 and common ratio 2. Find the 5th term.

(Total for Question 5 is 1 mark)

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6 Consider the binomial expansion of $(2+3x)^7$ in ascending powers of x. Let T_k denote the term in x^k (so T_0 is the constant term).

(a) Find an expression for T_k in terms of k. **(2)**

(b) Show that $T_{k+1}/T_k = 3(7-k) / (2(k+1))$. **(3)**

(c) Hence find the value of k for which T_k is greatest, and find this greatest coefficient. **(3)**

(Total for Question 6 is 8 marks)

7 The first three terms of a geometric series are 4, 4r and $4r^2$ respectively, where r is real and $r \neq 1$. The sum of the first 6 terms of the series is 5 times the sum of the first 3 terms.

(a) Show that $r^3 = 4$. **(3)**

(b) Find the value of r, giving your answer to 3 significant figures. **(2)**

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(c) Find the sum of the first 6 terms of the series, giving your answer to the nearest whole number. **(2)**

.....

(Total for Question 7 is 7 marks)

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8 An arithmetic sequence has 3rd term 11 and 8th term 31. Find the common difference and the first term.

(Total for Question 8 is 2 marks)

9 Find the sum of the first 20 terms of the arithmetic series with first term 7 and common difference -2.

(Total for Question 9 is 2 marks)

10 A geometric sequence has first term 4 and 4th term 32. Find the common ratio, given that it is positive.

(Total for Question 10 is 2 marks)

11 Find the coefficient of x^3 in the binomial expansion of $(1-x)^6$.

(Total for Question 11 is 2 marks)

12 An arithmetic series has first term 6 and common difference 5.

(a) Find the 20th term. **(2)**

(b) Find the sum of the first 20 terms. **(2)**

(Total for Question 12 is 4 marks)

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13 Find the sum of the first 10 terms of an arithmetic series with first term 4 and common difference 6.

(Total for Question 13 is 1 mark)

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14 Find the sum of the first 4 terms of a geometric series with first term 2 and common ratio 3.

(Total for Question 14 is 1 mark)

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15 A geometric series has first term 8 and common ratio 0.5. State the sum to infinity.

(Total for Question 15 is 1 mark)

16 Write down the first three terms, in ascending powers of x, of the binomial expansion of $(1+x)^4$.

(Total for Question 16 is 1 mark)

17 An arithmetic sequence has first term 10 and 4th term 22. State the common difference.

(Total for Question 17 is 1 mark)

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18 The sequence u_r is defined by $u_r = 2r + 4$, r = 1, 2, 3, ...

(a) Write down u_1 and the common difference, and state the type of sequence. **(2)**

(b) Evaluate the sum from r = 1 to r = 13 of (2r + 4). **(3)**

(c) Find the least value of n for which the sum from r = 1 to r = n of (2r + 4) exceeds 610. **(3)**

(Total for Question 18 is 8 marks)

19 A sequence is defined by $u_1 = 5$ and $u_{n+1} = 1 - 1/u_n$ for $n \geq 1$.

(a) Find u_2, u_3 and u_4. **(3)**

(b) Explain why the sequence is periodic and state its order. **(2)**

(c) Find the value of u_{38} and the sum of the first 38 terms. **(4)**

(Total for Question 19 is 9 marks)

20 A company's weekly output of widgets is 29 in the first week and rises by 12 each week. The pattern continues for 17 weeks.

(a) Find the value in week 17. **(2)**

(b) Find the total over the first 17 weeks. **(3)**

(c) Find the smallest number of weeks for which the total exceeds 3900. **(3)**

(Total for Question 20 is 8 marks)

21 Consider the expansion of $(1 + 2x)^8$ in ascending powers of x.

(a) Find the first four terms in the expansion of $(1 + 2x)^8$, giving each coefficient as an integer. **(4)**

(b) Hence find the coefficient of x^2 in the expansion of $(3 + x)(1 + 2x)^8$. **(3)**

(Total for Question 21 is 7 marks)

22 $f(x) = (5 + 4x)^{-2}$, $|x| < 5/4$.

(a) Find the binomial expansion of f(x) in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(4)**

(b) State the range of values of x for which the expansion is valid. **(1)**

(c) Use your expansion with $x = 0.01$ to estimate the value of $(5.04)^{-2}$, giving your answer to 5 significant figures. **(2)**

(Total for Question 22 is 7 marks)

23 A sequence is defined by $u_1 = 2$ and $u_{n+1} = 2u_n + 1$ for $n \geq 1$.

(a) Find u_2, u_3 and u_4. **(3)**

(b) Find the value of the sum of $u_1 + u_2 + u_3 + u_4 + u_5$. **(3)**

(c) Given that the sequence has a fixed point L ($u_{n+1} = u_n = L$), find L. **(2)**

(Total for Question 23 is 8 marks)

24 The sequence u_r is defined by $u_r = 5r + 5$, $r = 1, 2, 3, ...$

(a) Write down u_1 and the common difference, and state the type of sequence. **(2)**

(b) Evaluate the sum from $r = 1$ to $r = 19$ of $(5r + 5)$. **(3)**

(c) Find the least value of n for which the sum from $r = 1$ to $r = n$ of $(5r + 5)$ exceeds 520. **(3)**

(Total for Question 24 is 8 marks)

25 $f(x) = (16 + x)^{1/2}$, $|x| < 16$.

(a) Find the binomial expansion of f(x) in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(4)**

(b) State the range of values of x for which the expansion is valid. **(1)**

(c) Use your expansion with $x = 0.02$ to estimate the value of $(16.02)^{1/2}$, giving your answer to 5 significant figures. **(2)**

(Total for Question 25 is 7 marks)

26 A stack of tiles has 27 tiles in the first row, and each further row has 4 more tiles than the row before it. The pattern continues for 15 rows.

(a) Find the value in row 15. **(2)**

(b) Find the total over the first 15 rows. **(3)**

(c) Find the smallest number of rows for which the total exceeds 1500. **(3)**

(Total for Question 26 is 8 marks)

27 Consider the expansion of $(3 + 2x)^5$ in ascending powers of x.

(a) Find the first four terms in the expansion of $(3 + 2x)^5$, giving each coefficient as an integer. **(4)**

(b) Hence find the coefficient of x^2 in the expansion of $(1 - 3x)(3 + 2x)^5$. **(3)**

(Total for Question 27 is 7 marks)

28 A sequence is defined by $u_1 = 10$ and $u_{n+1} = 0.5\,u_n + 3$ for $n \geq 1$.

(a) Find the values of u2, u3 and u4. **(3)**

(b) Show that the sequence with terms $u_n - 6$ is a geometric sequence, and state its common ratio. **(4)**

(c) Hence show that $u_n = 6 + 4(0.5)^{n-1}$, and find the value that u_n approaches as n becomes very large. **(4)**

(Total for Question 28 is 11 marks)

29 A geometric series has first term 10 and common ratio -0.5. Find the sum to infinity, giving your answer as an exact fraction.

(Total for Question 29 is 3 marks)

30 $g(x) = (1+3x)^4 (1-x)^{-1}$, for $|x| < 1/3$.

(a) Find the binomial expansion of $(1+3x)^4$ in ascending powers of x, up to and including the term in x^2. **(2)**

(b) Given that $(1-x)^{-1} = 1 + x + x^2 + ...$ for $|x| < 1$, hence show that $g(x) = 1 + 13x + 67x^2 + ...$ for small values of x. **(2)**

(Total for Question 30 is 4 marks)

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31 The sum of the first n terms of an arithmetic series is given by $S_n = 2n^2 + 7n$. Find the first term and the common difference of the series.

(Total for Question 31 is 3 marks)

32 $f(x) = 1/\sqrt{4-3x} = (4-3x)^{-1/2}$.

(a) Show that f(x) can be written as $(1/2)(1 - (3/4)x)^{-1/2}$. **(2)**

(b) Hence find the binomial expansion of f(x) in ascending powers of x, up to and including the term in x^2. Give each coefficient as a simplified fraction. **(5)**

(c) State the range of values of x for which the expansion in part (b) is valid. **(1)**

(d) By substituting x=0.2 into the expansion from part (b), find an estimate for $1/\sqrt{3.4}$, giving your unrounded answer to 5 decimal places. Given that $1/\sqrt{3.4} = 0.542326$ to 6 decimal places, use your unrounded estimate to calculate the percentage error, giving your answer to 2 significant figures. **(4)**

(Total for Question 32 is 12 marks)

33 $f(x) = 1/\sqrt{1+4x} = (1+4x)^{-1/2}$, for $|x| < 1/4$.

(a) Find the binomial expansion of f(x) in ascending powers of x, up to and including the term in x^2, giving each coefficient as an exact number. **(3)**

(b) By substituting x = 0.01 into the expansion from part (a), find an estimate for $1/\sqrt{1.04}$, giving your answer to 4 decimal places. **(1)**

(Total for Question 33 is 4 marks)

34 Priya pays into a savings scheme. She pays £500 in year 1. Each subsequent year, she pays 4% more than the year before, so her annual payments form a geometric sequence with first term 500 and common ratio 1.04.

(a) Find the amount Priya pays in year 8, giving your answer to the nearest penny. **(3)**

(b) Find the total amount Priya has paid into the scheme by the end of year 15 (the sum of the first 15 payments), giving your answer to the nearest pound. **(3)**

(c) Priya wants the total amount paid into the scheme to first exceed £20000. Using logarithms, find the smallest number of years, n, for which this occurs. **(5)**

(d) State one assumption made in this model that may not hold in reality. **(1)**

(Total for Question 34 is 12 marks)

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Question 1

(a) forms $6r^2=54$ **M1**

(a) solves $r^2=9$ **M1**

(a) both r=3 and r=-3 stated (oe) **A1**

(a) Answer: r = 3 or r = -3

(b) correct substitution $u7=6(3)^6$ **M1**

(b) 4374 cao **A1**

(b) Answer: 4374

Question 2

(a) uses $u_n=a+(n-1)d$ with a=84, d=-3, n=12 **M1**

(a) 51 cao **A1**

(a) Answer: 51

(b) identifies that months 6 to 15 inclusive is 10 terms **B1**

(b) correct method, e.g. n/2(u6+u15) or S15-S5 **M1**

(b) correct intermediate values u6=69 and u15=42 (or S15=945, S5=390) **A1**

(b) 555 cao **A1**

(b) Answer: 555

(c) forms $84-3(n-1)\leq 0$ (oe) and solves to $n\geq 29$ **M1**

(c) cao month 29, with u28=3>0 and u29=0 checked **A1**

(c) valid comment that the model breaks down beyond this point since a count of defective items cannot be negative in reality **B1**

(c) Answer: Month 29; the arithmetic model is no longer appropriate beyond this point since a count cannot go negative.

Question 3

(a) finds the first term $a=S_1=8$ **M1**

(a) finds $u2=S_2-S_1$ (or equivalent method comparing coefficients) leading to d **M1**

(a) a=8 cao **A1**

(a) d=6 cao **A1**

(a) Answer: a = 8, d = 6

(b) uses $u_n=a+(n-1)d$ with n=25 **M1**

(b) 152 cao **A1**

(b) Answer: 152

Question 4

20 cao **B1**

Answer: 20

Question 5

48 cao **B1**

Answer: 48

Question 6

(a) correct structure using the general binomial term C(7,k) $(2)^{7-k}(3x)^k$ **M1**

(a) cao $T_k = C(7,k) * 2^{7-k} * 3^k$ **A1**

(a) Answer: $T_k = C(7,k) * 2^{7-k} * 3^k$

(b) uses C(7,k+1)/C(7,k) = (7-k)/(k+1) **M1**

(b) correctly finds the ratio of powers of 2 and 3 (a factor of 1/2 and a factor of 3) **M1**

(b) cso combines to 3(7-k)/(2(k+1)) **A1**

(b) Answer: $T_{k+1}/T_k = 3(7-k)/(2(k+1))$ (shown)

(c) sets $T_{k+1}/T_k > 1$ and solves the resulting inequality to k < 3.8 (oe) **M1**

(c) deduces that the coefficients increase up to k=4 and decrease after, so the greatest term is T_4 **M1**

(c) cao $T_4 = 22680$ **A1**

(c) Answer: k = 4; greatest coefficient = 22680

Question 7

(a) uses the sum formulae for S6 and S3 with the given condition S6=5S3 (or recognises terms 4 to 6 equal r^3 times terms 1 to 3) **M1**

(a) reaches $r^6-5r^3+4=0$ and factorises to $(r^3-1)(r^3-4)=0$ **M1**

(a) cso rejects $r^3=1$ (since $r \neq 1$ is given) leaving $r^3=4$ **A1**

(a) Answer: $r^3 = 4$ (shown)

(b) takes the cube root of both sides **M1**

(b) awrt 1.59 **A1**

(b) Answer: r = 1.59 (awrt, 3sf)

(c) correct method, e.g. $S6 = 4(1-r^6)/(1-r)$ using $r^6=(r^3)^2=16$, or S6=5*S3 **M1**

(c) awrt 102 **A1**

(c) Answer: 102

Question 8

forms 5d = 31 - 11 = 20 (or equivalent simultaneous equations) **M1**

d = 4 and a = 3 **A1**

Answer: d = 4, a = 3

Question 9

substitutes into $S_n = (n/2)(2a + (n-1)d)$ **M1**

-240 cao **A1**

Answer: -240

Question 10

forms $r^3 = 32/4 = 8$ **M1**

r = 2 cao **A1**

Answer: r = 2

Question 11

identifies the term as $C(6,3)x^3(-1)^3$ **M1**

-20 cao **A1**

Answer: -20

Question 12

(a) substitutes into a + 19d **M1**

(a) 101 cao **A1**

(a) Answer: 101

(b) substitutes into $S_n = (n/2)(2a+(n-1)d)$ **M1**

(b) 1070 cao **A1**

(b) Answer: 1070

Question 13

310 cao **B1**

Answer: 310

Question 14

80 cao **B1**

Answer: 80

Question 15

16 cao **B1**

Answer: 16

Question 16

$1 + 4x + 6x^2$, all three terms correct **B1**

Answer: $1 + 4x + 6x^2$

Question 17

d = 4 cao **B1**

Answer: 4

Question 18

(a) $u_1 = 6$ **B1**

(a) d = 2, arithmetic **B1**

(a) Answer: $u_1 = 6$, d = 2

(b) uses the arithmetic series formula with a = 6, d = 2, n = 13 **M1**

(b) (13/2)(2(6) + 12(2)) **A1**

(b) 234 **A1**

(b) Answer: 234

(c) forms (n/2)(2(6) + (n - 1)(2)) > 610 **M1**

(c) solves the quadratic inequality or tests consecutive values **A1**

(c) $S_{22} = 594$ and $S_{23} = 644$, so n = 23 **A1**

(c) Answer: n = 23

Question 19

(a) $u_2 = 1 - 1/5$ **M1**

(a) $u_2 = 4/5$, $u_3 = -1/4$ **A1**

(a) $u_4 = 5$ **A1**

(a) Answer: $u_2 = 4/5$, $u_3 = -1/4$, $u_4 = 5$

(b) $u_4 = u_1$ so the terms repeat **B1**

(b) period (order) 3 **B1**

(b) Answer: period 3

(c) $38 = 3 \times 12 + 2$, so $u_{38} = u_2$ **M1**

(c) $u_{38} = 4/5$ **A1**

(c) 12 complete cycles of sum 111/20 plus the remaining 2 terms **M1**

(c) sum = 362/5 **A1**

(c) Answer: $u_{38} = 4/5$, sum = 362/5

Question 20

(a) uses $u_n = a + (n - 1)d$ with a = 29, d = 12, n = 17 **M1**

(a) 221 **A1**

(a) Answer: 221

(b) uses $S_n = (n/2)(2a + (n - 1)d)$ or $(n/2)(a + l)$ **M1**

(b) 17/2 x (29 + 221) **A1**

(b) 2125 **A1**

(b) Answer: 2125

(c) forms and solves $(m/2)(2(29) + (m - 1)(12)) > 3900$ **M1**

(c) critical value m = 23.65 (or tests m = 23 and m = 24) **A1**

(c) $S_{23} = 3703 < 3900$ and $S_{24} = 4008 > 3900$, so m = 24 **A1**

(c) Answer: 24

Question 21

(a) binomial expansion with the correct form for the x term: $8C1\ (1)^7(2)x$ **M1**

(a) first two terms 1 + 16x **A1**

(a) attempts the x^2 and x^3 terms using 8C2 and 8C3 **M1**

(a) $1 + 16x + 112x^2 + 448x^3$ (all four terms correct) **A1**

(a) Answer: $1 + 16x + 112x^2 + 448x^3$

(b) identifies the two contributions to x^2: 3 times the x^2 coefficient and 1 times the x coefficient **M1**

(b) 336 + 16 **A1**

(b) 352 **A1**

(b) Answer: 352

Question 22

(a) takes out 5 **M1**

(a) expands $(1 + (4/5)x)^{-2}$: $1 + n(kx) + n(n-1)(kx)^2/2! + ...$ **M1**

(a) first two terms 1/25 - 8/125x **A1**

(a) $1/25 - (8/125)x + (48/625)x^2 - (256/3125)x^3$ **A1**

(a) Answer: $1/25 - (8/125)x + (48/625)x^2 - (256/3125)x^3$

(b) $|x| < 5/4$ **B1**

(b) Answer: $|x| < 5/4$

(c) substitutes x = 0.01 into the series **M1**

(c) 0.039368 **A1**

(c) Answer: 0.039368

Question 23

(a) $u_2 = 2(2) + 1$ **M1**

(a) $u_2 = 5$ **A1**

(a) $u_3 = 11$, $u_4 = 23$ **A1**

(a) Answer: $u_2 = 5$, $u_3 = 11$, $u_4 = 23$

(b) calculates u_5 **M1**

(b) $u_5 = 47$ **A1**

(b) sum = 88 **A1**

(b) Answer: 88

(c) sets L = 2L + 1 **M1**

(c) L = -1 **A1**

(c) Answer: L = -1

Question 24

(a) $u_1 = 10$ **B1**

(a) d = 5, arithmetic **B1**

(a) Answer: $u_1 = 10$, d = 5

(b) uses the arithmetic series formula with a = 10, d = 5, n = 19 **M1**

(b) (19/2)(2(10) + 18(5)) **A1**

(b) 1045 **A1**

(b) Answer: 1045

(c) forms (n/2)(2(10) + (n - 1)(5)) > 520 **M1**

(c) solves the quadratic inequality or tests consecutive values **A1**

(c) $S_{13} = 520$ and $S_{14} = 595$, so n = 14 **A1**

(c) Answer: n = 14

Question 25

(a) takes out 16 **M1**

(a) expands $(1 + (1/16)x)^{1/2}$: $1 + n(kx) + n(n-1)(kx)^2/2! + ...$ **M1**

(a) first two terms 4 + 1/8x **A1**

(a) $4 + (1/8)x - (1/512)x^2 + (1/16384)x^3$ **A1**

(a) Answer: $4 + (1/8)x - (1/512)x^2 + (1/16384)x^3$

(b) $|x| < 16$ **B1**

(b) Answer: $|x| < 16$

(c) substitutes x = 0.02 into the series **M1**

(c) 4.0025 **A1**

(c) Answer: 4.0025

Question 26

(a) uses $u_n = a + (n - 1)d$ with a = 27, d = 4, n = 15 **M1**

(a) 83 **A1**

(a) Answer: 83

(b) uses $S_n = (n/2)(2a + (n - 1)d)$ or $(n/2)(a + l)$ **M1**

(b) 15/2 x (27 + 83) **A1**

(b) 825 **A1**

(b) Answer: 825

(c) forms and solves (m/2)(2(27) + (m - 1)(4)) > 1500 **M1**

(c) critical value m = 21.84 (or tests m = 21 and m = 22) **A1**

(c) $S_{21} = 1407 < 1500$ and $S_{22} = 1518 > 1500$, so m = 22 **A1**

(c) Answer: 22

Question 27

(a) binomial expansion with the correct form for the x term: $5C1\ (3)^4(2)x$ **M1**

(a) first two terms 243 + 810x **A1**

(a) attempts the x^2 and x^3 terms using 5C2 and 5C3 **M1**

(a) $243 + 810x + 1080x^2 + 720x^3$ (all four terms correct) **A1**

(a) Answer: $243 + 810x + 1080x^2 + 720x^3$

(b) identifies the two contributions to x^2: 1 times the x^2 coefficient and -3 times the x coefficient **M1**

(b) 1080 - 2430 **A1**

(b) -1350 **A1**

(b) Answer: -1350

Question 28

(a) correct method substituting into the recurrence at least once **M1**

(a) u2=8 and u3=7 correct **A1**

(a) cao u4=6.5 **A1**

(a) Answer: u2 = 8, u3 = 7, u4 = 6.5

(b) lets $v_n = u_n - 6$ and substitutes $u_{n+1}=0.5u_n+3$ into $v_{n+1}=u_{n+1}-6$ **M1**

(b) correctly simplifies to $v_{n+1} = 0.5(u_n - 6)$ **M1**

(b) cso identifies $v_{n+1} = 0.5\ v_n$, confirming a geometric sequence **A1**

(b) states common ratio 0.5 **A1**

(b) Answer: $u_n - 6$ is geometric with common ratio 0.5 (shown)

(c) writes the general term of the geometric sequence $v_n = v_1\ r^{n-1} = 4(0.5)^{n-1}$ **M1**

(c) cso $u_n = v_n + 6 = 6 + 4(0.5)^{n-1}$, matching the given result **A1**

(c) recognises that since $|0.5| < 1$, $(0.5)^{n-1}$ tends to 0 as n tends to infinity **M1**

(c) cao limit = 6 **A1**

(c) Answer: $u_n = 6 + 4(0.5)^{n-1}$; u_n tends to 6 as n tends to infinity

Question 29

substitutes a = 10, r = -0.5 into a/(1-r) **M1**

= 10/1.5 **A1**

= 20/3 cao (exact) **A1**

Answer: 20/3

Question 30

(a) uses the binomial expansion with terms C(4,1)(3x) and $C(4,2)(3x)^2$ **M1**

(a) $1 + 12x + 54x^2$ **A1**

(a) Answer: $1 + 12x + 54x^2$

(b) multiplies $(1 + 12x + 54x^2)$ by $(1 + x + x^2)$, collecting terms up to x^2 **M1**

(b) cso: $1 + 13x + 67x^2$ as required **A1**

(b) Answer: $g(x) = 1 + 13x + 67x^2 + ...$

Question 31

finds S1 = 9 (the first term) and S2 = 22 **M1**

second term = S2 - S1 = 13, so d = 13 - 9 = 4 **A1**

a = 9 cao **A1**

Answer: a = 9, d = 4

Question 32

(a) factorises 4-3x as 4(1-(3/4)x) **M1**

(a) cso correctly simplifies $4^{-1/2}=1/2$ to reach the given form **A1**

(a) Answer: f(x) = (1/2)(1-(3/4)x)$^{-1/2}$ (shown)

(b) correct general structure $(1+u)^n = 1+nu+n(n-1)/2\ u^2$ with n=-1/2 **M1**

(b) correct term1 in u: -(1/2)u **A1**

(b) correct term2 in u: $(3/8)u^2$ **A1**

(b) substitutes u=-(3/4)x throughout and multiplies by the (1/2) factor from part (a) **M1**

(b) cao $1/2 + (3/16)x + (27/256)x^2$ **A1**

(b) Answer: f(x) is approximately $1/2 + (3/16)x + (27/256)x^2$ (+...)

(c) |x| < 4/3 oe (e.g. -4/3 < x < 4/3) **B1**

(c) Answer: |x| < 4/3

(d) substitutes x=0.2 into the expansion from part (b) **M1**

(d) correct unrounded estimate awrt 0.54172 **M1**

(d) correct method for percentage error, |estimate - true|/true x 100 **M1**

(d) cao awrt 0.11% **A1**

(d) Answer: Estimate is approximately 0.54172; percentage error is approximately 0.11%

Question 33

(a) uses the binomial series with n = -1/2 and u = 4x **M1**

(a) coefficient of x = -2 **A1**

(a) coefficient of $x^2 = 6$, giving $1 - 2x + 6x^2$ **A1**

(a) Answer: f(x) = $1 - 2x + 6x^2$ + ...

(b) 0.9806 (ft from (a)) **B1**

(b) Answer: 0.9806

Question 34

(a) uses $u_n = a\ r^{n-1}$ with a=500, r=1.04, n=8 **M1**

(a) correct value of 1.04^7 (awrt 1.3159) **M1**

(a) awrt 657.97 (nearest penny) **A1**

(a) Answer: £657.97 (nearest penny)

(b) correct use of $S_n = a(r^n-1)/(r-1)$ with n=15 **M1**

(b) correct value of 1.04^{15} (awrt 1.8009) **M1**

(b) awrt 10012 (nearest pound) **A1**

(b) Answer: £10012 (nearest pound)

(c) forms the inequality $500(1.04^n-1)/0.04 > 20000$, i.e. $12500(1.04^n-1) > 20000$ **M1**

(c) rearranges to $1.04^n > 2.6$ **M1**

(c) dM1 takes logs of both sides, n > ln(2.6)/ln(1.04) (oe base) **M1**

(c) correct unrounded value awrt 24.36 (or better) **A1**

(c) cao smallest integer n=25, with supporting check **A1**

(c) Answer: n = 25

(d) valid assumption, e.g. that Priya's payments increase by exactly 4% every year without fail, or that she is able and willing to keep increasing her payments indefinitely, which may not be realistic **B1**

(d) Answer: e.g. it assumes the 4% annual increase in payments continues exactly every year, which may not be realistic since Priya's circumstances or the scheme's terms could change.