1.13 Pure: Trigonometry Depth: 1-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The acute angles A and B satisfy sin A = 24/25 and cos B = 15/17.

(a) Find the exact values of cos A, sin B and tan A. **(3)**

(b) Find the exact value of cos(A + B). **(3)**

(c) Find the exact value of tan(A - B). **(3)**

(d) Find the exact value of sin 2A and of cos 2A. **(3)**

(Total for Question 1 is 12 marks)

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2 Solve, for 0 degrees ≤ x < 360 degrees, the equation

$2\cos^2 x - 3 \sin x = 3$

giving all solutions as exact values.

(Total for Question 2 is 5 marks)

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3 State the exact value of cos(60 degrees).

(Total for Question 3 is 1 mark)

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4 Solve, for 0 degrees ≤ θ < 360 degrees, the equation

sin 2theta = cos θ

giving all four solutions as exact values. Take care not to lose any solutions.

(Total for Question 4 is 5 marks)

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5 Solve, for 0 degrees ≤ x < 360 degrees, the equation

sin(x + 45 degrees) = 2 sin(x - 45 degrees)

giving your answers to 1 decimal place.

(Total for Question 5 is 5 marks)

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6 Starting from the compound angle formula $\cos(\theta + \theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$:

(a) Prove that $\cos 2\text{theta} = 1 - 2\sin^2\theta$. **(3)**

(b) Hence show that $\sin^2\theta = (1 - \cos 2\text{theta})/2$, and use this result to find the exact value of $\sin^2(15 \text{ degrees})$, giving your answer in the form $(a - \sqrt{b})/c$ where a, b, c are integers. **(3)**

(Total for Question 6 is 6 marks)

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7 Show that, for $\cos\theta$ not equal to 0, $\sec^2\theta - \tan^2\theta = 1$.

(Total for Question 7 is 2 marks)

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8 Write $\sec(\theta)$ in terms of $\cos(\theta)$.

(Total for Question 8 is 1 mark)

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9 Simplify $\sin(\theta) / \cos(\theta)$.

(Total for Question 9 is 1 mark)

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10 Given that $\cos A = 3/5$, where A is acute, and $\sin B = 5/13$, where B is obtuse.

(a) Find the exact value of $\sin(A + B)$. **(3)**

(b) Find the exact value of $\cos(A + B)$, and hence show that $180 \text{ degrees} < A + B < 270 \text{ degrees}$. **(4)**

(Total for Question 10 is 7 marks)

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11 For $\cos\theta$ not equal to 0 and $\sin\theta$ not equal to -1:

(a) Show that $(1 + \sin\theta)/\cos\theta + \cos\theta/(1 + \sin\theta) = 2\sec\theta$. **(4)**

(b) Hence, or otherwise, solve for $0 \text{ degrees} \leq \theta < 360 \text{ degrees}$, the equation **(4)**

$$(1 + \sin\theta)/\cos\theta + \cos\theta/(1 + \sin\theta) = 4$$

(Total for Question 11 is 8 marks)

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Mark scheme · 1.13 Pure: Trigonometry Depth: 1-hour Pack

Question 1

(a) uses $\sin^2 + \cos^2 = 1$ for each angle (or a right-angled triangle) **M1**

(a) cos A = 7/25, tan A = 24/7 **A1**

(a) sin B = 8/17 **A1**

(a) Answer: cos A = 7/25, sin B = 8/17, tan A = 24/7

(b) uses cos(A + B) = cos A cos B - sin A sin B **M1**

(b) substitutes the exact values **M1**

(b) -87/425 **A1**

(b) Answer: -87/425

(c) uses tan(A - B) = (tan A - tan B)/(1 + tan A tan B) **M1**

(c) substitutes the exact values **M1**

(c) 304/297 **A1**

(c) Answer: 304/297

(d) uses sin 2A = 2 sin A cos A and cos 2A = 1 - 2 $\sin^2$ A **M1**

(d) sin 2A = 336/625 **A1**

(d) cos 2A = -527/625 **A1**

(d) Answer: sin 2A = 336/625, cos 2A = -527/625

Question 2

uses $\cos^2 x = 1 - \sin^2 x$ to form an equation in sin x only **M1**

rearranges to the 3-term quadratic $2\sin^2 x + 3\sin x + 1 = 0$, oe **M1**

correct factorisation (2sin x + 1)(sin x + 1) = 0, or correct use of the quadratic formula **M1**

sin x = -1/2 leading to x = 210 and x = 330 (both), oe **A1**

sin x = -1 leading to x = 270, with no extra solutions given **A1**

Answer: x = 210, 270, 330 degrees

Question 3

1/2 cao **B1**

Answer: 1/2

Question 4

uses sin2theta = 2 sin θ cos θ to rewrite the equation **M1**

factorises correctly as cos θ (2 sin θ - 1) = 0 (does not divide through by cos θ) **M1**

cos θ = 0 giving θ = 90 and θ = 270 (both) **A1**

sin θ = 1/2 giving θ = 30 and θ = 150 (both) **A1**

all four solutions given, with none lost or repeated, cao **A1**

Answer: θ = 30, 90, 150, 270 degrees

Question 5

expands both sides using the compound angle formulae, sin(x+45)=sinx cos45+cosx sin45 and sin(x-45)=sinx cos45-cosx sin45 **M1**

uses cos45 = sin45 to cancel the common factor from every term **M1**

rearranges the resulting equation sinx+cosx = 2sinx-2cosx to 3cosx = sinx, i.e. tanx = 3 **M1**

x = awrt 71.6 **A1**

x = awrt 251.6 **A1**

Answer: x = 71.6, 251.6 degrees (awrt)

Question 6

(a) uses the compound angle formula with A = B = θ to write cos2theta = $\cos^2 \theta - \sin^2 \theta$ **M1**

(a) uses $\cos^2 \theta = 1 - \sin^2 \theta$ **M1**

(a) cso: cos2theta = 1 - 2$\sin^2 \theta$ **A1**

(a) Answer: Identity proved: cos2theta = 1 - 2$\sin^2 \theta$

(b) rearranges the result from (a) to give $\sin^2 \theta$ = (1-cos2theta)/2 **M1**

(b) substitutes θ = 15 and uses cos30 = $\sqrt{3}/2$ **M1**

(b) cso: $\sin^2(15) = (2 - \sqrt{3})/4$ **A1**

(b) Answer: $\sin^2$(15 degrees) = $(2 - \sqrt{3})/4$

Question 7

writes $\sec^2 \theta - \tan^2 \theta = (1 - \sin^2 \theta)/\cos^2 \theta = \cos^2 \theta / \cos^2 \theta$ **M1**

cso: = 1 **A1**

Answer: Proof: $\sec^2 \theta - \tan^2 \theta = 1$

Question 8

1/cos(θ) cao **B1**

Answer: 1/cos(θ)

Question 9

tan(θ) cao **B1**

Answer: tan(θ)

Question 10

(a) finds cos B = -12/13, using the 5-12-13 triangle with the correct negative sign for the obtuse angle B **M1**

(a) uses sin(A+B) = sinA cosB + cosA sinB with correct substitution **M1**

(a) sin(A+B) = -33/65, oe **A1**

(a) Answer: sin(A+B) = -33/65

(b) uses cos(A+B) = cosA cosB - sinA sinB **M1**

(b) correct substitution of values **M1**

(b) cos(A+B) = -56/65, oe **A1**

(b) cso: since sin(A+B) and cos(A+B) are both negative (from part (a) and above), A+B lies in the third quadrant, so 180 degrees < A+B < 270 degrees **A1**

(b) Answer: cos(A+B) = -56/65; A+B lies between 180 and 270 degrees

Question 11

(a) combines the two fractions over the common denominator cos θ (1 + sin θ) **M1**

(a) correct expanded numerator $(1+\sin \theta)^2 + \cos^2 \theta$ **A1**

(a) uses $\sin^2 \theta + \cos^2 \theta = 1$ to simplify the numerator to 2 + 2sin θ, or 2(1+sin θ) **M1**

(a) cso: cancels (1+sin θ) to obtain 2/cos θ = 2 sec θ **A1**

(a) Answer: Identity proved: (1+sin θ)/cos θ + cos θ/(1+sin θ) = 2 sec θ

(b) uses part (a) to write 2 sec θ = 4 (ft) **M1**

(b) sec θ = 2, so cos θ = 1/2 **M1**

(b) θ = 60 **A1**

(b) θ = 300 **A1**

(b) Answer: θ = 60, 300 degrees

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