1.13 Pure: Trigonometry Depth: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Show that, for sin x not equal to 0,

sin x / (1 + cos x) + (1 + cos x) / sin x = 2 / sin x

(Total for Question 1 is 4 marks)

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2 Given that θ is small and measured in radians, use small angle approximations to answer the following.

(a) Show that (1 - cos 6theta)/(θ sin 6theta) is approximately 3. **(3)**

(b) Evaluate (1 - cos 6theta)/(θ sin 6theta) at θ = 0.1 radians to 4 significant figures, and find the percentage error in the approximation 3, to 2 significant figures. **(4)**

(Total for Question 2 is 7 marks)

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3 f(x) = 24 sin x + 7 cos x, where x is in degrees.

(a) Express f(x) in the form R sin(x + α), where R > 0 and 0 < α < 90 degrees. Give R as an exact value and α to 1 decimal place. **(4)**

(b) Write down the maximum value of f(x), and the smallest positive value of x at which it occurs, to 1 decimal place. **(2)**

(c) Solve f(x) = 4 for 0 ≤ x < 360 degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 3 is 10 marks)

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4 The acute angles A and B satisfy sin A = 4/5 and cos B = 8/17.

(a) Find the exact values of cos A, sin B and tan A. **(3)**

(b) Find the exact value of cos(A - B). **(3)**

(c) Find the exact value of sin(A - B). **(3)**

(d) Find the exact value of sin 2A and of cos 2A. **(3)**

(Total for Question 4 is 12 marks)

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5 State the exact value of tan(45 degrees).

(Total for Question 5 is 1 mark)

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6 Solve, for 0 degrees ≤ x < 90 degrees, the equation cos x = 0.5, giving your answer in degrees.

(Total for Question 6 is 1 mark)

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7 Simplify $\sin^2(\theta) + \cos^2(\theta)$.

(Total for Question 7 is 1 mark)

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8 Solve, for 0 degrees ≤ x < 360 degrees, the equation 3 sin x = 2 cos x, giving your answers to 1 decimal place.

(Total for Question 8 is 3 marks)

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9 Starting from tan(A - B) = sin(A - B)/cos(A - B):

(a) Prove that tan(A - B) = (tanA - tanB)/(1 + tanA tanB), for cosA cosB not equal to 0 and 1 + tanA tanB not equal to 0. **(4)**

(b) Hence, using exact values for tan60 degrees and tan45 degrees, find the exact value of tan15 degrees, giving your answer in the form $a + b*\sqrt{c}$, where a, b and c are integers. **(4)**

(Total for Question 9 is 8 marks)

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Mark scheme · 1.13 Pure: Trigonometry Depth: Test Pack

Question 1

combines the two fractions over a common denominator sin x (1 + cos x) **M1**

correct expanded numerator $\sin^2 x + (1 + \cos x)^2$, oe **A1**

uses $\sin^2 x + \cos^2 x = 1$ to simplify the numerator to 2 + 2cos x, or 2(1 + cos x) **M1**

cso: cancels (1 + cos x) to obtain 2 / sin x **A1**

Answer: Identity shown (proof)

Question 2

(a) uses cos 6theta is approximately 1 - $(6\text{theta})^2/2$ **M1**

(a) numerator is approximately $(18)\theta^2$ and sin 6theta is approximately 6theta **M1**

(a) denominator is approximately 6theta^2, so the ratio is (18)/6 = 3 (cso) **A1**

(a) Answer: approximately 3

(b) calculates the value with θ = 0.1: 3.09336 **M1**

(b) 3.093 **A1**

(b) percentage error = |3.0000 - 3.0934|/3.0934 x 100 **M1**

(b) 3.0% **A1**

(b) Answer: 3.093, error 3.0%

Question 3

(a) R cos α = 24 and R sin α = 7 **M1**

(a) R = 25 **A1**

(a) tan α = 7/24 **M1**

(a) α = 16.3 degrees **A1**

(a) Answer: f(x) = 25 sin(x + 16.3)

(b) maximum 25 **B1**

(b) x + 16.3 = 90, so x = 73.7 **B1**

(b) Answer: maximum 25 at x = 73.7

(c) sin(x + 16.3) = 4/25 **M1**

(c) x + 16.3 = 9.2 or 170.8 (and adds 360 if needed) **M1**

(c) x = 154.5 **A1**

(c) x = 352.9 **A1**

(c) Answer: x = 154.5, 352.9

Question 4

(a) uses $\sin^2 + \cos^2 = 1$ for each angle (or a right-angled triangle) **M1**

(a) cos A = 3/5, tan A = 4/3 **A1**

(a) sin B = 15/17 **A1**

(a) Answer: cos A = 3/5, sin B = 15/17, tan A = 4/3

(b) uses cos(A - B) = cos A cos B + sin A sin B **M1**

(b) substitutes the exact values **M1**

(b) 84/85 **A1**

(b) Answer: 84/85

(c) uses sin(A - B) = sin A cos B - cos A sin B **M1**

(c) substitutes the exact values **M1**

(c) -13/85 **A1**

(c) Answer: -13/85

(d) uses sin 2A = 2 sin A cos A and cos 2A = 1 - 2 $\sin^2$ A **M1**

(d) sin 2A = 24/25 **A1**

(d) cos 2A = -7/25 **A1**

(d) Answer: sin 2A = 24/25, cos 2A = -7/25

Question 5

1 cao **B1**

Answer: 1

Question 6

x = 60 cao **B1**

Answer: x = 60

Question 7

1 cao **B1**

Answer: 1

Question 8

rearranges to tan x = 2/3 **M1**

reference angle awrt 33.7 **A1**

x = 33.7 or x = 213.7 (both values, since tan is positive in the 1st and 3rd quadrants) **A1**

Answer: x = 33.7 or x = 213.7

Question 9

(a) states tan(A-B) = sin(A-B)/cos(A-B) **B1**

(a) expands the numerator and denominator using the compound angle formulae, sinA cosB - cosA sinB and cosA cosB + sinA sinB **M1**

(a) divides both the numerator and the denominator by cosA cosB **M1**

(a) cso: tan(A-B) = (tanA - tanB)/(1 + tanA tanB) **A1**

(a) Answer: Identity proved: tan(A-B) = (tanA-tanB)/(1+tanA tanB)

(b) uses tan(A-B) with A=60, B=45, substituting $\tan 60 = \sqrt{3}$ and tan45=1 **M1**

(b) correct unsimplified expression $(\sqrt{3}-1)/(1+\sqrt{3})$ **M1**

(b) rationalises the denominator by multiplying the numerator and denominator by $(\sqrt{3}-1)$ **M1**

(b) cao: tan15 degrees = $2 - \sqrt{3}$ **A1**

(b) Answer: tan15 degrees = $2 - \sqrt{3}$

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