1.13 Pure: Trigonometry Depth

EDEXCEL 9MA0 · Calculator allowed · about 230 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question is about trigonometric identities and equations.

(a) Show that tan x + cot x = 2/sin 2x. **(3)**

(b) Hence solve tan x + cot x = 8 for 0 < x < 360 degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 1 is 7 marks)

2 In this question x is measured in degrees and $-180 < x \leq 180$.

(a) Solve the quadratic equation $y^2 - 1 = 0$ and hence state the possible values of cos x. **(2)**

(b) Hence solve $\cos^2 x - 1 = 0$ for $-180 < x \leq 180$, in degrees. **(4)**

(Total for Question 2 is 6 marks)

3 The acute angles A and B satisfy $\sin A = 24/25$ and $\cos B = 15/17$.

(a) Find the exact values of cos A, sin B and tan A. **(3)**

(b) Find the exact value of cos(A + B). **(3)**

(c) Find the exact value of tan(A - B). **(3)**

(d) Find the exact value of sin 2A and of cos 2A. **(3)**

(Total for Question 3 is 12 marks)

4 This question is about trigonometric identities and equations.

(a) Show that $(\sin x + \cos x)^2 = 1 + \sin 2x$. **(2)**

(b) Hence solve $(\sin x + \cos x)^2 = 1/2$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 4 is 6 marks)

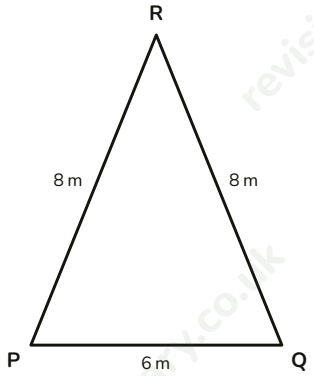
5 Show that, for $\sin \theta \cos \theta$ not equal to 0,

$\sec^2 \theta + \operatorname{cosec}^2 \theta = \sec^2 \theta \operatorname{cosec}^2 \theta$

(Total for Question 5 is 4 marks)

6 In triangle PQR, PQ = 6 m, QR = 8 m and PR = 8 m.

Diagram NOT accurately drawn



(a) Find the size of the largest angle of the triangle, giving your answer to 1 decimal place. **(3)**

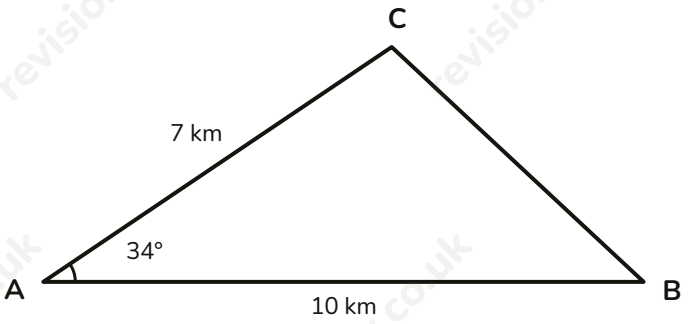
(b) Find the size of angle PRQ to 1 decimal place. **(2)**

(c) Find the area of triangle PQR to 3 significant figures. **(2)**

(Total for Question 6 is 7 marks)

7 In triangle ABC, AB = 10 km, AC = 7 km and angle BAC = 34 degrees.

Diagram NOT accurately drawn



(a) Calculate the length of BC, giving your answer to 3 significant figures. **(3)**

(b) Calculate the size of angle ABC, giving your answer to 1 decimal place. **(2)**

(c) Calculate the area of triangle ABC, giving your answer to 3 significant figures. **(2)**

(Total for Question 7 is 7 marks)

8 In this question x is measured in degrees and $-180 < x \leq 180$.

(a) Solve the quadratic equation $2y^2 - y - 1 = 0$ and hence state the possible values of $\cos x$. **(2)**

(b) Hence solve $2\cos^2 x - \cos x - 1 = 0$ for $-180 < x \leq 180$, in degrees. **(4)**

(Total for Question 8 is 6 marks)

9 $f(x) = 7 \sin x + 3 \cos x$, where x is in degrees.

(a) Express $f(x)$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < 90$ degrees. Give R as an exact value and α to 1 decimal place. **(4)**

(b) Write down the maximum value of $f(x)$, and the smallest positive value of x at which it occurs, to 1 decimal place. **(2)**

(c) Solve $f(x) = 1$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 9 is 10 marks)

10 Solve, for 0 degrees ≤ x < 360 degrees, the equation

$2\cos^2 x - 3 \sin x = 3$

giving all solutions as exact values.

(Total for Question 10 is 5 marks)

11 State the exact value of sin(30 degrees).

(Total for Question 11 is 1 mark)

12 State the exact value of cos(60 degrees).

(Total for Question 12 is 1 mark)

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13 Solve, for 0 degrees $\leq \theta <$ 360 degrees, the equation

sin 2theta = cos θ

giving all four solutions as exact values. Take care not to lose any solutions.

(Total for Question 13 is 5 marks)

14 Solve, for 0 degrees ≤ x < 360 degrees, the equation

$\sin(x + 45 \text{ degrees}) = 2 \sin(x - 45 \text{ degrees})$

giving your answers to 1 decimal place.

(Total for Question 14 is 5 marks)

15 Starting from the compound angle formula $\cos(\theta + \theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$:

(a) Prove that $\cos 2\text{theta} = 1 - 2\sin^2\theta$. **(3)**

(b) Hence show that $\sin^2\theta = (1 - \cos 2\text{theta})/2$, and use this result to find the exact value of $\sin^2(15 \text{ degrees})$, giving your answer in the form $(a - \sqrt{b})/c$ where a, b, c are integers. **(3)**

(Total for Question 15 is 6 marks)

16 Show that, for $\cos\theta$ not equal to 0, $\sec^2\theta - \tan^2\theta = 1$.

(Total for Question 16 is 2 marks)

17 Solve, for $0 \text{ degrees} \leq \theta < 360 \text{ degrees}$, the equation $\sin\theta = -0.5$, giving all solutions in degrees.

(Total for Question 17 is 2 marks)

18 Given that sin A = 3/5, where A is acute, find the exact value of cos A.

(Total for Question 18 is 2 marks)

19 Write sec(θ) in terms of cos(θ).

(Total for Question 19 is 1 mark)

20 Write cosec(θ) in terms of sin(θ).

(Total for Question 20 is 1 mark)

21 State the identity for cos(2theta) in terms of cos(θ) only.

(Total for Question 21 is 1 mark)

22 State the identity for sin(2theta) in terms of sin(θ) and cos(θ).

(Total for Question 22 is 1 mark)

23 Simplify $\sin(\theta) / \cos(\theta)$.

(Total for Question 23 is 1 mark)

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24 Solve, for 0 degrees $\leq \theta <$ 360 degrees, the equation

$4\cos 2\text{theta} + 8\cos\theta + 1 = 0$

giving your answers to 1 decimal place.

(Total for Question 24 is 6 marks)

25 Given that cos A = 3/5, where A is acute, and sin B = 5/13, where B is obtuse.

(a) Find the exact value of sin(A + B). **(3)**

(b) Find the exact value of cos(A + B), and hence show that 180 degrees < A + B < 270 degrees. **(4)**

(Total for Question 25 is 7 marks)

26 For $\cos\theta$ not equal to 0 and $\sin\theta$ not equal to -1:

(a) Show that $(1 + \sin\theta)/\cos\theta + \cos\theta/(1 + \sin\theta) = 2\sec\theta$. **(4)**

(b) Hence, or otherwise, solve for 0 degrees $\leq \theta <$ 360 degrees, the equation **(4)**

$(1 + \sin\theta)/\cos\theta + \cos\theta/(1 + \sin\theta) = 4$

(Total for Question 26 is 8 marks)

27 Solve, for 0 degrees $\leq x <$ 360 degrees, the equation $\cos(2x) = 0.5$, giving all solutions.

(Total for Question 27 is 2 marks)

28 Show that, for $\cos\theta$ not equal to 0 and $\sin\theta$ not equal to 1, $(1 - \sin\theta)/\cos\theta + \cos\theta/(1 - \sin\theta) = 2\sec\theta$.

(Total for Question 28 is 4 marks)

29 Solve, for 0 degrees $\leq \theta <$ 360 degrees, the equation $2\cos^2\theta + \sin\theta = 2$, giving your answers in degrees.

(Total for Question 29 is 4 marks)

30 $f(\theta) = 5 \sin \theta + 12 \cos \theta$, for $0 \text{ degrees} \leq \theta < 360 \text{ degrees}$.

(a) Express $f(\theta)$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90$, giving the value of α to 1 decimal place. **(2)**

(b) Hence solve, for $0 \text{ degrees} \leq \theta < 360 \text{ degrees}$, the equation $5 \sin \theta + 12 \cos \theta = 6.5$, giving your answers to 1 decimal place. **(2)**

(Total for Question 30 is 4 marks)

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31 The height, H metres, of a point on a large observation wheel above the ground, t minutes after the ride begins, is modelled by $H(t) = 65 + 60 \cos(12t)$, where the angle $12t$ is measured in degrees.

(a) State the maximum possible height given by the model. **(1)**

(b) Find the smallest positive value of t, in minutes, for which the height is at its minimum possible value. **(2)**

(Total for Question 31 is 3 marks)

32 For $\tan \theta$ not equal to plus or minus 1:

(a) Prove that $\tan 2\text{theta} = 2 \tan \theta / (1 - \tan^2 \theta)$. **(3)**

(b) Hence solve, for 0 degrees $\leq \theta \leq$ 180 degrees, the equation $\tan 2\text{theta} = 3\tan \theta$, giving non-exact answers to 1 decimal place. You should find four solutions, including $\theta = 0$ and $\theta = 180$. **(5)**

(Total for Question 32 is 8 marks)

33 $f(x) = 7 \sin x - 24 \cos x$, for $0 \text{ degrees} \leq x < 360 \text{ degrees}$.

(a) Express $f(x)$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < 90$ degrees, giving the value of α to 1 decimal place. **(4)**

(b) Hence solve, for $0 \text{ degrees} \leq x < 360 \text{ degrees}$, the equation $7 \sin x - 24 \cos x = 10$, giving your answers to 1 decimal place. **(5)**

(Total for Question 33 is 9 marks)

34 Prove that $\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$, using the addition formula for $\cos(A+B)$ together with the double angle identities for $\cos(2\theta)$ and $\sin(2\theta)$.

(Total for Question 34 is 4 marks)

35 $g(\theta) = 4\cos\theta + 3\sin\theta$, for 0 degrees $\leq \theta < 360$ degrees.

(a) Express $g(\theta)$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90$ degrees, giving α to 2 decimal places. **(4)**

(b) Using the form found in part (a), find the maximum value of $[g(\theta)]^2 - 2g(\theta)$, and state a value of θ, 0 degrees $\leq \theta < 360$ degrees, at which this maximum occurs. **(5)**

(Total for Question 35 is 9 marks)

36 The number of hours of daylight, H, in a UK town on a day t months after 1 January is modelled by

$H(t) = 12 + 2.5 \sin(30t) - 1.5 \cos(30t)$, for $0 \leq t < 12$

where the angle is measured in degrees.

(a) Express $2.5 \sin(30t) - 1.5 \cos(30t)$ in the form $R \sin(30t - \alpha)$, where $R > 0$ and $0 < \alpha < 90$ degrees, giving R to 3 significant figures and α to 1 decimal place. **(4)**

(b) Hence write down the maximum number of daylight hours given by the model, to 3 significant figures, and find the smallest positive value of t (to 2 decimal places) at which this maximum occurs. **(3)**

(c) Find the total length of time during the year ($0 \leq t < 12$) for which there are at least 13.5 hours of daylight, giving your answer in months to 2 decimal places. **(6)**

(Total for Question 36 is 13 marks)

Mark scheme · 1.13 Pure: Trigonometry Depth

Question 1

(a) writes tan x + cot x = sin x/cos x + cos x/sin x **M1**

(a) combines to $(\sin^2 x + \cos^2 x)/(\sin x \cos x)$ **M1**

(a) uses $\sin^2 x + \cos^2 x = 1$ and sin x cos x = (1/2) sin 2x to give 2/sin 2x (cso) **A1**

(a) Answer: tan x + cot x = 2/sin 2x

(b) forms sin 2x = 2/8 = 1/4 **M1**

(b) 2x = 14.5 or 180 - 14.5, then adds 360 **M1**

(b) two correct values **A1**

(b) all four values: 7.2, 82.8, 187.2, 262.8 **A1**

(b) Answer: x = 7.2, 82.8, 187.2, 262.8

Question 2

(a) factorises or uses the quadratic formula **M1**

(a) cos x = 1 or cos x = -1 **A1**

(a) Answer: cos x = 1 or -1

(b) finds a solution of cos x = 1 using inverse cos **M1**

(b) solutions from cos x = 1: 0 **A1**

(b) repeats for cos x = -1 **M1**

(b) all solutions: 0, 180 degrees **A1**

(b) Answer: 0, 180 degrees

Question 3

(a) uses $\sin^2 + \cos^2 = 1$ for each angle (or a right-angled triangle) **M1**

(a) cos A = 7/25, tan A = 24/7 **A1**

(a) sin B = 8/17 **A1**

(a) Answer: cos A = 7/25, sin B = 8/17, tan A = 24/7

(b) uses cos(A + B) = cos A cos B - sin A sin B **M1**

(b) substitutes the exact values **M1**

(b) -87/425 **A1**

(b) Answer: -87/425

(c) uses tan(A - B) = (tan A - tan B)/(1 + tan A tan B) **M1**

(c) substitutes the exact values **M1**

(c) 304/297 **A1**

(c) Answer: 304/297

(d) uses sin 2A = 2 sin A cos A and $\cos 2A = 1 - 2\sin^2 A$ **M1**

(d) sin 2A = 336/625 **A1**

(d) cos 2A = -527/625 **A1**

(d) Answer: sin 2A = 336/625, cos 2A = -527/625

Question 4

(a) expands to $\sin^2 x + 2 \sin x \cos x + \cos^2 x$ **M1**

(a) uses $\sin^2 x + \cos^2 x = 1$ and 2 sin x cos x = sin 2x (cso) **A1**

(a) Answer: $(\sin x + \cos x)^2 = 1 + \sin 2x$

(b) forms sin 2x = -1/2 **M1**

(b) finds 2x = -30.0 and the symmetric value 180 + 30.0, extended by 360 **M1**

(b) any two correct values of x **A1**

(b) all four values: 105.0, 165.0, 285.0, 345.0 **A1**

(b) Answer: x = 105.0, 165.0, 285.0, 345.0

Question 5

writes $\sec^2 \theta$ and $\text{cosec}^2 \theta$ in terms of sin and cos, and combines over the common denominator $\sin^2 \theta \cos^2 \theta$ **M1**

correct combined numerator $\sin^2 \theta + \cos^2 \theta$ **A1**

uses $\sin^2 \theta + \cos^2 \theta = 1$ to simplify to $1 / (\sin^2 \theta \cos^2 \theta)$ **M1**

cso: rewrites $1/(\sin^2 \theta \cos^2 \theta)$ as $\sec^2 \theta \text{cosec}^2 \theta$ **A1**

Answer: Identity shown (proof)

Question 6

(a) the largest angle is opposite the longest side PR, so uses $\cos Q = (PQ^2 + QR^2 - PR^2)/(2 PQ \times QR)$ **M1**

(a) cos Q = (36 + 64 - 64)/(2 x 6 x 8) = 0.3750 **A1**

(a) Q = 68.0 degrees **A1**

(a) Answer: 68.0 degrees

(b) cosine rule for the angle at R **M1**

(b) 44.0 degrees **A1**

(b) Answer: 44.0 degrees

(c) (1/2) PQ x QR x sin Q **M1**

(c) 22.2 m^2 **A1**

(c) Answer: 22.2 m^2

Question 7

(a) uses the cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ **M1**

(a) $BC^2 = 49 + 100 - 2(7)(10) \cos 34 = 32.935$ **A1**

(a) BC = 5.74 km **A1**

(a) Answer: BC = 5.74 km

(b) uses the sine rule or the cosine rule with the calculated side **M1**

(b) angle ABC = 43.0 degrees **A1**

(b) Answer: 43.0 degrees

(c) uses area = (1/2) bc sin A **M1**

(c) 19.6 km^2 **A1**

(c) Answer: 19.6 km^2

Question 8

(a) factorises or uses the quadratic formula **M1**

(a) cos x = -1/2 or cos x = 1 **A1**

(a) Answer: cos x = -1/2 or 1

(b) finds a solution of cos x = -1/2 using inverse cos **M1**

(b) solutions from cos x = -1/2: -120, 120 **A1**

(b) repeats for cos x = 1 **M1**

(b) all solutions: -120, 0, 120 degrees **A1**

(b) Answer: -120, 0, 120 degrees

Question 9

(a) R cos α = 7 and R sin α = 3 **M1**

(a) R = $\sqrt{58}$ **A1**

(a) tan α = 3/7 **M1**

(a) α = 23.2 degrees **A1**

(a) Answer: f(x) = $\sqrt{58}$ sin(x + 23.2)

(b) maximum $\sqrt{58}$ **B1**

(b) x + 23.2 = 90, so x = 66.8 **B1**

(b) Answer: maximum $\sqrt{58}$ at x = 66.8

(c) sin(x + 23.2) = 1/$\sqrt{58}$ **M1**

(c) x + 23.2 = 7.5 or 172.5 (and adds 360 if needed) **M1**

(c) x = 149.3 **A1**

(c) x = 344.3 **A1**

(c) Answer: x = 149.3, 344.3

Question 10

uses $\cos^2 x = 1 - \sin^2 x$ to form an equation in sin x only **M1**

rearranges to the 3-term quadratic $2\sin^2 x + 3\sin x + 1 = 0$, oe **M1**

correct factorisation (2sin x + 1)(sin x + 1) = 0, or correct use of the quadratic formula **M1**

sin x = -1/2 leading to x = 210 and x = 330 (both), oe **A1**

sin x = -1 leading to x = 270, with no extra solutions given **A1**

Answer: x = 210, 270, 330 degrees

Question 11

1/2 cao **B1**

Answer: 1/2

Question 12

1/2 cao **B1**

Answer: 1/2

Question 13

uses sin2theta = 2 sin θ cos θ to rewrite the equation **M1**

factorises correctly as cos θ (2 sin θ - 1) = 0 (does not divide through by cos θ) **M1**

cos θ = 0 giving θ = 90 and θ = 270 (both) **A1**

sin θ = 1/2 giving θ = 30 and θ = 150 (both) **A1**

all four solutions given, with none lost or repeated, cao **A1**

Answer: θ = 30, 90, 150, 270 degrees

Question 14

expands both sides using the compound angle formulae, sin(x+45)=sinx cos45+cosx sin45 and sin(x-45)=sinx cos45-cosx sin45 **M1**

uses cos45 = sin45 to cancel the common factor from every term **M1**

rearranges the resulting equation sinx+cosx = 2sinx-2cosx to 3cosx = sinx, i.e. tanx = 3 **M1**

x = awrt 71.6 **A1**

x = awrt 251.6 **A1**

Answer: x = 71.6, 251.6 degrees (awrt)

Question 15

(a) uses the compound angle formula with A = B = θ to write cos2theta = $\cos^2 \theta - \sin^2 \theta$ **M1**

(a) uses $\cos^2 \theta = 1 - \sin^2 \theta$ **M1**

(a) cso: $\cos2\text{theta} = 1 - 2\sin^2 \theta$ **A1**

(a) Answer: Identity proved: $\cos2\text{theta} = 1 - 2\sin^2 \theta$

(b) rearranges the result from (a) to give $\sin^2 \theta = (1-\cos2\text{theta})/2$ **M1**

(b) substitutes θ = 15 and uses cos30 = $\sqrt{3}/2$ **M1**

(b) cso: $\sin^2(15) = (2 - \sqrt{3})/4$ **A1**

(b) Answer: $\sin^2(15 \text{ degrees}) = (2 - \sqrt{3})/4$

Question 16

writes $\sec^2 \theta - \tan^2 \theta = (1 - \sin^2 \theta)/\cos^2 \theta = \cos^2 \theta / \cos^2 \theta$ **M1**

cso: = 1 **A1**

Answer: Proof: $\sec^2 \theta - \tan^2 \theta = 1$

Question 17

finds the reference angle arcsin(0.5) = 30 and identifies the 3rd and 4th quadrants **M1**

θ = 210 or θ = 330, both values **A1**

Answer: θ = 210 or θ = 330

Question 18

uses $\sin^2 A + \cos^2 A = 1$ to find $\cos^2 A = 1 - 9/25 = 16/25$ **M1**

cos A = 4/5 (positive, since A is acute) **A1**

Answer: cos A = 4/5

Question 19

1/cos(θ) cao **B1**

Answer: 1/cos(θ)

Question 20

1/sin(θ) cao **B1**

Answer: 1/sin(θ)

Question 21

$2\cos^2(\theta) - 1$, oe **B1**

Answer: $2\cos^2(\theta) - 1$

Question 22

2sin(θ)cos(θ) cao **B1**

Answer: 2sin(θ)cos(θ)

Question 23

tan(θ) cao **B1**

Answer: tan(θ)

Question 24

uses cos2theta = $2\cos^2\theta - 1$ to form an equation in cos θ only **M1**

rearranges to the 3-term quadratic $8\cos^2\theta + 8\cos\theta - 3 = 0$, oe **M1**

solves the quadratic (formula or completing the square), dependent on the previous method mark **dM1**

rejects cos θ = awrt -1.29 as impossible (with a reason given), retaining cos θ = awrt 0.291 **A1**

θ = awrt 73.1 **A1**

θ = awrt 286.9 **A1**

Answer: θ = 73.1, 286.9 degrees (awrt)

Question 25

(a) finds cos B = -12/13, using the 5-12-13 triangle with the correct negative sign for the obtuse angle B **M1**

(a) uses sin(A+B) = sinA cosB + cosA sinB with correct substitution **M1**

(a) sin(A+B) = -33/65, oe **A1**

(a) Answer: sin(A+B) = -33/65

(b) uses cos(A+B) = cosA cosB - sinA sinB **M1**

(b) correct substitution of values **M1**

(b) cos(A+B) = -56/65, oe **A1**

(b) cso: since sin(A+B) and cos(A+B) are both negative (from part (a) and above), A+B lies in the third quadrant, so 180 degrees < A+B < 270 degrees **A1**

(b) Answer: cos(A+B) = -56/65; A+B lies between 180 and 270 degrees

Question 26

(a) combines the two fractions over the common denominator cos θ (1 + sin θ) **M1**

(a) correct expanded numerator $(1+\sin\theta)^2 + \cos^2\theta$ **A1**

(a) uses $\sin^2\theta + \cos^2\theta = 1$ to simplify the numerator to 2 + 2sin θ, or 2(1+sin θ) **M1**

(a) cso: cancels (1+sin θ) to obtain 2/cos θ = 2 sec θ **A1**

(a) Answer: Identity proved: (1+sin θ)/cos θ + cos θ/(1+sin θ) = 2 sec θ

(b) uses part (a) to write 2 sec θ = 4 (ft) **M1**

(b) sec θ = 2, so cos θ = 1/2 **M1**

(b) θ = 60 **A1**

(b) θ = 300 **A1**

(b) Answer: θ = 60, 300 degrees

Question 27

solves for 2x over the extended range $0 \le 2x < 720$, giving 2x = 60, 300, 420, 660 **M1**

x = 30, 150, 210, 330, all four values **A1**

Answer: x = 30, 150, 210, 330

Question 28

combines the two fractions over the common denominator cos θ (1 - sin θ) **M1**

forms the numerator $(1-\sin\theta)^2 + \cos^2\theta = 2 - 2\sin\theta$, using $\sin^2+\cos^2=1$ **A1**

factorises the numerator as 2(1 - sin θ) **M1**

cso: cancels (1 - sin θ) to leave 2/cos θ = 2 sec θ **A1**

Answer: Proof: (1-sin θ)/cos θ + cos θ/(1-sin θ) = 2 sec θ

Question 29

uses $\cos^2\theta = 1 - \sin^2\theta$ to form the equation $-2\sin^2\theta + \sin\theta = 0$ **M1**

factorises to sin θ(1 - 2sin θ) = 0, giving sin θ = 0 or sin θ = 0.5 **A1**

θ = 0 or θ = 180 (from sin θ = 0) **A1**

θ = 30 or θ = 150 (from sin θ = 0.5), all four values found **A1**

Answer: θ = 0, 30, 150 or 180

Question 30

(a) $R = \sqrt{5^2 + 12^2} = 13$ **M1**

(a) α = awrt 67.4, from tan α = 12/5 **A1**

(a) Answer: f(θ) = 13 sin(θ + 67.4)

(b) uses the form from (a): 13 sin(θ + α) = 6.5, so sin(θ + α) = 0.5, giving θ + α = 150 or 390 **M1**

(b) θ = awrt 82.6 or θ = awrt 322.6 (ft from (a)) **A1**

(b) Answer: θ = 82.6 or θ = 322.6

Question 31

(a) 125 m **B1**

(a) Answer: 125 m

(b) recognises the minimum occurs when cos(12t) = -1, so 12t = 180 **M1**

(b) t = 15 minutes cao **A1**

(b) Answer: t = 15 minutes

Question 32

(a) writes tan2theta = sin2theta/cos2theta and uses sin2theta=2sintheta costheta, $\cos2\theta=\cos^2\theta-\sin^2\theta$ **M1**

(a) divides the numerator and denominator by $\cos^2\theta$ **M1**

(a) cso: $\tan2\theta = 2\tan\theta/(1-\tan^2\theta)$ **A1**

(a) Answer: Identity proved: $\tan2\theta = 2\tan\theta/(1-\tan^2\theta)$

(b) uses the result from (a) to form $2\tan\theta/(1-\tan^2\theta) = 3\tan\theta$ **M1**

(b) rearranges (letting t = tan θ) to $t(3t^2 - 1) = 0$, or equivalent, and factorises **M1**

(b) tan θ = 0 giving θ = 0 and θ = 180 **A1**

(b) tan θ = plus or minus $1/\sqrt{3}$ giving θ = 30 and θ = 150 **A1**

(b) all four solutions stated, with no extra or repeated values, cao **A1**

(b) Answer: θ = 0, 30, 150, 180 degrees

Question 33

(a) $R = \sqrt{7^2 + 24^2}$ **M1**

(a) R = 25 **A1**

(a) tan α = 24/7 (from R cos α = 7, R sin α = 24), or equivalent **M1**

(a) α = 73.7 (awrt) **A1**

(a) Answer: f(x) = 25 sin(x - 73.7 degrees) (awrt)

(b) uses the result from (a) to write 25 sin(x - 73.7...) = 10 (ft on their R, α) **M1**

(b) sin(x - α) = 0.4, leading to a reference angle of awrt 23.6 degrees **M1**

(b) correct method for the second solution: (180 - their reference angle) + their α **M1**

(b) x = awrt 97.3 **A1**

(b) x = awrt 230.2 **A1**

(b) Answer: x = 97.3, 230.2 degrees (awrt)

Question 34

writes cos(3theta) = cos(2theta + θ) and expands using the addition formula to cos2theta costheta - sin2theta sintheta **M1**

substitutes cos2theta = $2\cos^2\theta - 1$ and sin2theta = 2sinthetacostheta **M1**

simplifies to $2\cos^3\theta$ - costheta - $2\sin^2\theta$ costheta **A1**

cso: uses $\sin^2\theta = 1 - \cos^2\theta$ to obtain $4\cos^3\theta$ - 3costheta **A1**

Answer: Proof: cos(3theta) = $4\cos^3(\theta)$ - 3cos(θ)

Question 35

(a) $R = \sqrt{4^2 + 3^2}$ **M1**

(a) R = 5 **A1**

(a) tan α = 3/4 **M1**

(a) α = 36.87 (awrt) **A1**

(a) Answer: g(θ) = 5cos(θ - 36.87 degrees) (awrt)

(b) lets u = g(θ) = R cos(θ - α) (ft) and states the range $-5 \leq u \leq 5$ **M1**

(b) writes the expression as $u^2 - 2u = (u-1)^2 - 1$, or evaluates the expression at both endpoints u=5 and u=-5 **M1**

(b) correct values at both endpoints: value 15 (at u=5) and value 35 (at u=-5) **A1**

(b) maximum value = 35, cao (identifying u=-5, the endpoint further from the vertex u=1, as the maximum) **A1**

(b) θ = awrt 216.87 (ft), from cos(θ - α) = -1 **A1**

(b) Answer: Maximum value = 35, occurring at θ = 216.87 degrees (awrt)

Question 36

(a) $R = \sqrt{2.5^2 + 1.5^2}$ **M1**

(a) R = awrt 2.92 (exact value $\sqrt{34}/2$) **A1**

(a) tan α = 1.5/2.5 (= 0.6) **M1**

(a) α = awrt 31.0 **A1**

(a) Answer: H(t) = 12 + 2.92 sin(30t - 31.0) (awrt), R = $\sqrt{34}/2$, α = awrt 31.0 degrees

(b) maximum = 12 + R = awrt 14.9 hours **B1**

(b) sets 30t - α = 90 (using their α, ideally unrounded for accuracy) **M1**

(b) t = awrt 4.03 (ft) **A1**

(b) Answer: Maximum = 14.9 hours (3sf), occurring at t = 4.03 months after 1 January (awrt)

(c) forms the inequality 12 + R sin(30t - α) ≥ 13.5, i.e. sin(30t - α) ≥ 1.5/R (ft their R, α) **M1**

(c) recognises that the boundary sin(30t - α) = 1.5/R = sin(α) is satisfied when 30t - α = α or 30t - α = 180 - α **M1**

(c) t = 6 (exact), from the second boundary equation **A1**

(c) t = 2 x α/30, using their unrounded α, dependent on the previous M1 **M1**

(c) t = awrt 2.06 (ft), giving the interval [2.06, 6] during which the daylight is at least 13.5 hours **A1**

(c) duration = 6 - 2.06 = awrt 3.94 months, cao **A1**

(c) Answer: There are at least 13.5 hours of daylight for 3.94 months (between t = 2.06 and t = 6)