1.14 Pure: Differentiation Depth: 1-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

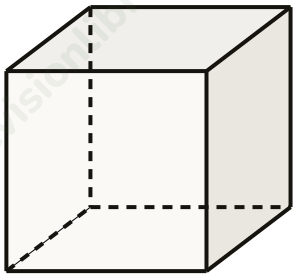
Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The side length x cm of a cube increases at a constant rate of 0.3 cm/s.

Diagram NOT accurately drawn



x cm

(a) Find the rate at which the volume $V = x^3$ is increasing when $x = 6$. **(3)**

(b) Find the rate at which the total surface area $S = 6x^2$ is increasing at that instant. **(2)**

(Total for Question 1 is 5 marks)

---

2 A curve C has parametric equations $x = t^2 - 1$, $y = t^3 - 3t$, t in R.

(a) Find dy/dx in terms of t. **(3)**

(b) Find an equation of the tangent to C at the point where t = -2, in the form ax + by + c = 0 with integer coefficients. **(3)**

(c) The tangent in part (b) meets the x-axis at the point Q. Find the x coordinate of Q. **(2)**

(Total for Question 2 is 8 marks)

---

3 A curve C has parametric equations $x = 3t^2 - 3$, $y = t^3 - 5t$, t in R.

(a) Find dy/dx in terms of t. **(3)**

(b) Find an equation of the tangent to C at the point where t = 2, in the form ax + by + c = 0 with integer coefficients. **(3)**

(c) The tangent in part (b) meets the x-axis at the point Q. Find the x coordinate of Q. **(2)**

(Total for Question 3 is 8 marks)

© Revision Library | revisionlibrary.co.uk

4 A farmer has 156 m of fencing to enclose a rectangular field. One side of the field is a straight wall and needs no fencing. The sides perpendicular to the wall have length x m.

(a) Show that the area of the field is $A = x(156 - 2x)\ m^2$. **(2)**

(b) Use calculus to find the maximum area, justifying that it is a maximum. **(4)**

(Total for Question 4 is 6 marks)

---

5 A curve C has equation $x^2 + y^2 - 4x + 6y = 12$.

(a) Show that $dy/dx = (2 - x)/(y + 3)$. **(3)**

(b) Verify that the point (2, 2) lies on C, and find the gradient of C at this point. **(2)**

(Total for Question 5 is 5 marks)

---

6 A curve has equation $y = x^2 * e^{-x}$.

(a) Find dy/dx, giving your answer fully factorised. **(3)**

(b) Show that the curve has stationary points where x = 0 and x = 2, and find the corresponding y-coordinates. **(3)**

(c) Find d^2y/dx^2 and hence determine the nature of each stationary point. **(2)**

(Total for Question 6 is 8 marks)

---

7 Water is poured into an inverted right circular conical tank at a constant rate of 30 cm^3 per second. The tank has height 40 cm and the radius of its circular top is 20 cm, so the radius r cm and depth h cm of the water are always in the same ratio as the tank's dimensions. Let V cm^3 be the volume of water in the tank when the depth is h cm.

(a) Show that $V = (\pi/12)*h^3$. **(3)**

(b) Find dh/dt in terms of h. **(3)**

(c) Find the rate at which the depth is increasing at the instant when h = 10 cm, giving your answer to 3 significant figures. **(3)**

(d) Explain, with justification, whether the rate at which the depth is increasing gets faster or slower as h increases. **(2)**

(Total for Question 7 is 11 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 1.14 Pure: Differentiation Depth: 1-hour Pack

Question 1

(a) $dV/dt = 3x^2\ dx/dt$	**M1**
(a) $3(6)^2(0.3)$	**A1**
(a) $32.4\ cm^3/s$	**A1**
(a) Answer: $32.4\ cm^3/s$	
(b) $dS/dt = 12x\ dx/dt$	**M1**
(b) $21.6\ cm^2/s$	**A1**
(b) Answer: $21.6\ cm^2/s$	

Question 2

(a) $dx/dt = 2t$ and $dy/dt = 3t^2 - 3$	**M1**
(a) $dy/dx = (dy/dt)/(dx/dt)$	**M1**
(a) $dy/dx = (3t^2 - 3)/(2t)$	**A1**
(a) Answer: $dy/dx = (3t^2 - 3)/(2t)$	
(b) the point is (3, -2)	**B1**
(b) gradient = -9/4	**M1**
(b) $9x + 4y - 19 = 0$	**A1**
(b) Answer: $9x + 4y - 19 = 0$	
(c) sets y = 0 in the tangent	**M1**
(c) x = 19/9	**A1**
(c) Answer: x = 19/9	

Question 3

(a) $dx/dt = 6t$ and $dy/dt = 3t^2 - 5$	**M1**
(a) $dy/dx = (dy/dt)/(dx/dt)$	**M1**
(a) $dy/dx = (3t^2 - 5)/(6t)$	**A1**
(a) Answer: $dy/dx = (3t^2 - 5)/(6t)$	
(b) the point is (9, -2)	**B1**
(b) gradient = 7/12	**M1**
(b) $7x - 12y - 87 = 0$	**A1**
(b) Answer: $7x - 12y - 87 = 0$	
(c) sets y = 0 in the tangent	**M1**
(c) x = 87/7	**A1**
(c) Answer: x = 87/7	

Question 4

(a) length parallel to the wall = 156 - 2x	**M1**
(a) A = x(156 - 2x) (cso)	**A1**
(a) Answer: A = x(156 - 2x)	
(b) $A = 156x - 2x^2$, $dA/dx = 156 - 4x$	**M1**
(b) dA/dx = 0 gives x = 39	**A1**
(b) $d^2A/dx^2 = -4 < 0$ so a maximum	**B1**
(b) maximum area = 39 x 78 = 3042 m^2	**A1**
(b) Answer: 3042 m^2	

Question 5

(a) differentiate implicitly term by term, with at least 3 of the 4 terms correct (2x, 2y*dy/dx, -4, 6*dy/dx)	**M1**
(a) fully correct implicit equation $2x + 2y*(dy/dx) - 4 + 6*(dy/dx) = 0$	**A1**
(a) rearrange to $dy/dx = (2-x)/(y+3)$ cso	**A1**
(a) Answer: $dy/dx = (2 - x)/(y + 3)$	
(b) verify (2,2) satisfies the equation (4+4-8+12=12) and substitute x=2, y=2 into the implicit derivative	**M1**
(b) dy/dx = 0 cao	**A1**
(b) Answer: Gradient at (2, 2) is 0 (a horizontal tangent)	

Question 6

(a) attempt at the product rule with $u=x^2$, $v=e^{-x}$	**M1**
(a) correct unsimplified derivative $2x*e^{-x} - x^2*e^{-x}$	**A1**
(a) $dy/dx = x*e^{-x}*(2-x)$ cao	**A1**
(a) Answer: $dy/dx = x * e^{-x} * (2 - x)$	
(b) set $dy/dx=0$ and use that e^{-x} is never zero to obtain $x(2-x)=0$	**M1**
(b) x=0 and x=2 both stated cso	**A1**
(b) y-coordinates y=0 (at x=0) and $y=4/e^2$, awrt 0.541 (at x=2), both cao	**A1**
(b) Answer: Stationary points at (0, 0) and $(2, 4/e^2)$, where $4/e^2$ is awrt 0.541	
(c) correct $d^2y/dx^2 = e^{-x}*(x^2-4x+2)$, found by differentiating dy/dx again using the product rule	**M1**
(c) correct classification with reasoning: (0,0) is a minimum since $d^2y/dx^2=2>0$ at x=0; $(2, 4/e^2)$ is a maximum since $d^2y/dx^2=-2/e^2<0$ at x=2, both cao	**A1**
(c) Answer: (0, 0) is a minimum point; $(2, 4/e^2)$ is a maximum point	

Question 7

(a) use similar triangles to write r in terms of h, r = h/2	**M1**
(a) substitute into $V=(1/3)*\pi*r^2*h$	**M1**
(a) $V = (\pi/12)*h^3$ cso, with full simplification shown	**A1**
(a) Answer: $V = (\pi/12)*h^3$	
(b) differentiate $V=(\pi/12)*h^3$ to obtain $dV/dh = (\pi/4)*h^2$	**M1**
(b) use the chain rule $dh/dt = (dV/dt)/(dV/dh)$ with dV/dt=30	**M1**
(b) $dh/dt = 120/(\pi*h^2)$ oe cao	**A1**
(b) Answer: $dh/dt = 120/(\pi*h^2)$	
(c) substitute h=10 into their dh/dt expression (ft)	**M1**
(c) exact value $6/(5*\pi)$ oe, e.g. $1.2/\pi$	**A1**
(c) awrt 0.382 cm/s cao	**A1**
(c) Answer: $dh/dt = 6/(5*\pi)$ cm/s, awrt 0.382 cm/s	
(d) correct statement that dh/dt decreases as h increases	**B1**
(d) valid justification, e.g. dh/dt is inversely proportional to h^2 (from $dh/dt=120/(\pi*h^2)$), so as h grows dh/dt must fall, consistent with the tank widening so the same inflow rate produces a slower rise in depth	**B1**
(d) Answer: dh/dt decreases as h increases	

© Revision Library | revisionlibrary.co.uk