1.14 Pure: Differentiation Depth: 2-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The curve C has equation $y = 2x^3 - 15x^2 + 24x$.

(a) Find dy/dx and d^2y/dx^2. **(3)**

(b) Find the coordinates of the stationary points of C. **(4)**

(c) Determine the nature of each stationary point. **(2)**

(d) Find the range of values of k for which the equation $2x^3 - 15x^2 + 24x = k$ has three distinct real roots. **(2)**

(Total for Question 1 is 11 marks)

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2 $y = x^4 \ln(5x)$, $x > 0$.

(a) Find dy/dx. **(3)**

(b) Show that the curve has a single stationary point and find its exact coordinates. **(4)**

(Total for Question 2 is 7 marks)

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3 Water runs into a vertical conical vessel, vertex downwards, at 40π cm^3/s. The semi-vertical angle is 45 degrees, so the volume of water at depth h cm is $V = (1/3)\pi h^3$.

(a) Find an expression for dV/dh. **(2)**

(b) Find the rate at which the depth is increasing when $h = 8$. **(3)**

(Total for Question 3 is 5 marks)

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4 $f(x) = 2x^3 - 30x^2 - 4x + 2$.

(a) Find $f'(x)$ and $f''(x)$. **(3)**

(b) Find the coordinates of the point of inflection of the curve $y = f(x)$, showing that the second derivative changes sign. **(4)**

(c) Find the range of values of x for which f is concave. **(2)**

(Total for Question 4 is 9 marks)

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5 $y = e^{-2x} \sin(2x)$.

(a) Find dy/dx. **(3)**

(b) Find the exact gradient of the curve at the point where $x = \pi/4$. **(2)**

(Total for Question 5 is 5 marks)

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6 The curve C has equation $y = 2x^3 - 9x^2$.

(a) Find dy/dx and d^2y/dx^2. **(3)**

(b) Find the coordinates of the stationary points of C. **(4)**

(c) Determine the nature of each stationary point. **(2)**

(d) Find the range of values of k for which the equation $2x^3 - 9x^2 = k$ has three distinct real roots. **(2)**

(Total for Question 6 is 11 marks)

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7 $y = (3x - 2)^5$. Find dy/dx.

(Total for Question 7 is 1 mark)

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8 $y = (5x + 2)^4$. Find dy/dx, and hence find the exact value of dy/dx when $x = 0$.

(Total for Question 8 is 2 marks)

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9 $y = (3x - 1)/(x + 2)$. Find dy/dx, giving your answer as a single simplified fraction.

(Total for Question 9 is 2 marks)

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10 A curve has parametric equations $x = t^3$, $y = 2t^2$, for t in R. Find dy/dx in terms of t.

(Total for Question 10 is 2 marks)

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11 A curve has equation $y = (4x^2 - 3)^3$.

(a) Find dy/dx, giving your answer in fully factorised form. **(2)**

(b) Hence find the exact value of dy/dx when $x = 1$. **(2)**

(Total for Question 11 is 4 marks)

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12 A curve has parametric equations $x = 2t - 1$, $y = t^2 + 3t$, for t in R. Find dy/dx in terms of t, and hence find the value of dy/dx when $t = 2$.

(Total for Question 12 is 3 marks)

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13 $y = e^{6x}$. Find dy/dx.

(Total for Question 13 is 1 mark)

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14 $y = (x^2 + 1)^3$. Find dy/dx.

(Total for Question 14 is 1 mark)

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15 $y = e^{x^2}$. Find dy/dx.

(Total for Question 15 is 1 mark)

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16 $y = \ln(4x)$, for $x > 0$. Find dy/dx.

(Total for Question 16 is 1 mark)

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17 $y = 1/x^2$, for x not equal to 0. Find dy/dx.

(Total for Question 17 is 1 mark)

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18 $y = \sqrt{x}$, for $x > 0$. Find dy/dx.

(Total for Question 18 is 1 mark)

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19 $y = x^3 - 4x$. Find the value of dy/dx when $x = 2$.

(Total for Question 19 is 1 mark)

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20 A curve C has parametric equations $x = 2t + 1$, $y = t^2 - 4t$, for t in R.

(a) Find dy/dx in terms of t. **(3)**

(b) Find the coordinates of the point on C where the tangent is parallel to the line $y = 2x + 5$. **(4)**

(c) Show that the cartesian equation of C can be written in the form $y = ax^2 + bx + c$, stating the values of a, b and c. **(3)**

(Total for Question 20 is 10 marks)

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21 A curve has equation $y = x^3 e^{-2x}$.

(a) Show that dy/dx can be written as $x^2 e^{-2x}(3 - 2x)$. **(2)**

(b) Hence find the exact value(s) of x for which the tangent to the curve is horizontal. **(2)**

(Total for Question 21 is 4 marks)

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22 A curve C is defined by the equation $x^2*y - y^2 = 14$. The point P(3, 2) lies on C.

(a) Find dy/dx in terms of x and y. **(4)**

(b) Show that the gradient of C at P is -12/5. **(2)**

(c) Find an equation of the tangent to C at P, giving your answer in the form ax + by + c = 0 with integer coefficients. **(3)**

(d) Find an equation of the normal to C at P, giving your answer in the form ax + by + c = 0 with integer coefficients. **(3)**

(Total for Question 22 is 12 marks)

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Mark scheme · 1.14 Pure: Differentiation Depth: 2-hour Pack

Question 1

(a) differentiates, reducing the power of each term by 1 **M1**

(a) $dy/dx = 6x^2 - 30x + 24$ **A1**

(a) $d^2y/dx^2 = 12x - 30$ **A1**

(a) Answer: $dy/dx = 6x^2 - 30x + 24$, $d^2y/dx^2 = 12x - 30$

(b) sets $dy/dx = 0$ **M1**

(b) $x = 1$ and $x = 4$ **A1**

(b) substitutes into the equation of the curve **M1**

(b) (1, 11) and (4, -16) **A1**

(b) Answer: (1, 11) and (4, -16)

(c) evaluates d^2y/dx^2 at each stationary point **M1**

(c) (1, 11) is a maximum ($d^2y/dx^2 = -18 < 0$) and (4, -16) is a minimum ($d^2y/dx^2 = 18 > 0$) **A1**

(c) Answer: maximum at x = 1, minimum at x = 4

(d) the horizontal line $y = k$ must cut the curve three times, so lies between the minimum and maximum values **M1**

(d) $-16 < k < 11$ **A1**

(d) Answer: $-16 < k < 11$

Question 2

(a) uses the product rule with $u = x^4$ and $v = \ln(5x)$ **M1**

(a) $du/dx = 4x^3$ and $dv/dx = 1/x$ **A1**

(a) $dy/dx = x^3(4\ln(5x) + 1)$ **A1**

(a) Answer: $dy/dx = x^3(4\ln(5x) + 1)$

(b) sets $dy/dx = 0$, with $x^3 > 0$ so $4\ln(5x) + 1 = 0$ **M1**

(b) $\ln(5x) = -1/4$ **A1**

(b) $x = e^{-1/4}/5$ **M1**

(b) $y = x^4 (-1/4) = -1/(2500e)$ **A1**

(b) Answer: $x = e^{-1/4}/5$, $y = -1/(2500e)$

Question 3

(a) differentiates $V = (1/3)\pi h^3$ **M1**

(a) $dV/dh = \pi h^2$ **A1**

(a) Answer: $dV/dh = \pi h^2$

(b) $dh/dt = (dV/dt)/(dV/dh)$ **M1**

(b) $40pi/(64pi)$ **A1**

(b) 5/8 cm/s **A1**

(b) Answer: 5/8 cm/s

Question 4

(a) differentiates **M1**

(a) $f'(x) = 6x^2 - 60x - 4$ **A1**

(a) $f''(x) = 12x - 60$ **A1**

(a) Answer: $f'(x) = 6x^2 - 60x - 4$, $f''(x) = 12x - 60$

(b) solves $f''(x) = 0$ **M1**

(b) $x = 5$ **A1**

(b) $f''(4) = -12 < 0$ and $f''(6) = 12 > 0$, a change of sign **B1**

(b) (5, -518) **A1**

(b) Answer: (5, -518)

(c) $f''(x) < 0$ **M1**

(c) $x < 5$ **A1**

(c) Answer: $x < 5$

Question 5

(a) uses the product rule **M1**

(a) $-2e^{-2x}\sin(2x)$ **A1**

(a) $+ 2e^{-2x}\cos(2x)$ **A1**

(a) Answer: $dy/dx = e^{-2x}(-2\sin(2x) + 2\cos(2x))$

(b) substitutes $x = \pi/4$, where $\sin(2x) = 1$ and $\cos(2x) = 0$ **M1**

(b) $-2e^{-1/2pi}$ **A1**

(b) Answer: $-2e^{-1/2pi}$

Question 6

(a) differentiates, reducing the power of each term by 1 **M1**

(a) $dy/dx = 6x^2 - 18x$ **A1**

(a) $d^2y/dx^2 = 12x - 18$ **A1**

(a) Answer: $dy/dx = 6x^2 - 18x$, $d^2y/dx^2 = 12x - 18$

(b) sets $dy/dx = 0$ **M1**

(b) $x = 0$ and $x = 3$ **A1**

(b) substitutes into the equation of the curve **M1**

(b) (0, 0) and (3, -27) **A1**

(b) Answer: (0, 0) and (3, -27)

(c) evaluates d^2y/dx^2 at each stationary point **M1**

(c) (0, 0) is a maximum ($d^2y/dx^2 = -18 < 0$) and (3, -27) is a minimum ($d^2y/dx^2 = 18 > 0$) **A1**

(c) Answer: maximum at x = 0, minimum at x = 3

(d) the horizontal line $y = k$ must cut the curve three times, so lies between the minimum and maximum values **M1**

(d) $-27 < k < 0$ **A1**

(d) Answer: $-27 < k < 0$

Question 7

$15(3x-2)^4$ cao **B1**

Answer: $15(3x-2)^4$

Question 8

$dy/dx = 20(5x+2)^3$ **M1**

at x=0: 160 cao **A1**

Answer: $dy/dx = 20(5x+2)^3$; at x=0, $dy/dx = 160$

Question 9

uses the quotient rule **M1**

$dy/dx = 7/(x+2)^2$ cao **A1**

Answer: $dy/dx = 7/(x+2)^2$

Question 10

finds $dy/dt = 4t$ and $dx/dt = 3t^2$ **M1**

$dy/dx = 4/(3t)$ cao **A1**

Answer: $dy/dx = 4/(3t)$

Question 11

(a) applies the chain rule: $dy/dx = 3(4x^2-3)^2 \times 8x$ **M1**

(a) $24x(4x^2-3)^2$ cao **A1**

(a) Answer: $dy/dx = 24x(4x^2-3)^2$

(b) substitutes x=1 into the expression from (a) **M1**

(b) 24 cao **A1**

(b) Answer: 24

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Question 12

finds dy/dt = 2t + 3 and dx/dt = 2 **M1**

dy/dx = (2t+3)/2 **A1**

at t=2: dy/dx = 3.5 cao **A1**

Answer: dy/dx = (2t+3)/2; at t=2, dy/dx = 3.5

Question 13

$6e^{6x}$ cao **B1**

Answer: $6e^{6x}$

Question 14

$6x(x^2+1)^2$ cao **B1**

Answer: $6x(x^2+1)^2$

Question 15

$2x\ e^{x^2}$ cao **B1**

Answer: $2x\ e^{x^2}$

Question 16

1/x cao **B1**

Answer: 1/x

Question 17

$-2/x^3$ cao **B1**

Answer: $-2/x^3$

Question 18

$1/(2\sqrt{x})$ cao **B1**

Answer: $1/(2\sqrt{x})$

Question 19

8 cao **B1**

Answer: 8

Question 20

(a) dx/dt = 2 **M1**

(a) dy/dt = 2t-4 **M1**

(a) dy/dx = t-2 oe cao **A1**

(a) Answer: dy/dx = t - 2

(b) set dy/dx equal to the gradient of the given line (2) **M1**

(b) t=4 **A1**

(b) substitute t=4 into both parametric equations **M1**

(b) point (9, 0) cao **A1**

(b) Answer: (9, 0)

(c) make t the subject of the x equation, t=(x-1)/2 **M1**

(c) substitute into $y=t^2-4t$ and expand **M1**

(c) $y = (1/4)x^2 - (5/2)x + 9/4$ cso (equivalently $4y=x^2-10x+9$), with a=1/4, b=-5/2, c=9/4 stated **A1**

(c) Answer: $y = (1/4)x^2 - (5/2)x + 9/4$, so a = 1/4, b = -5/2, c = 9/4

Question 21

(a) uses the product rule: $dy/dx = 3x^2\ e^{-2x} - 2x^3\ e^{-2x}$ **M1**

(a) cso: factorises to $x^2\ e^{-2x}(3-2x)$ **A1**

(a) Answer: $dy/dx = x^2\ e^{-2x}(3-2x)$

(b) sets $x^2\ e^{-2x}(3-2x) = 0$ and uses that e^{-2x} is never zero **M1**

(b) x = 0 or x = 3/2, both values **A1**

(b) Answer: x = 0 or x = 3/2

Question 22

(a) differentiate x^2*y implicitly using the product rule to obtain $2xy + x^2*(dy/dx)$ **M1**

(a) differentiate y^2 implicitly to obtain $2y*(dy/dx)$ **M1**

(a) correct full implicit equation $2xy + x^2*(dy/dx) - 2y*(dy/dx) = 0$ **A1**

(a) $dy/dx = 2xy/(2y-x^2)$ oe cao, e.g. $-2xy/(x^2-2y)$ **A1**

(a) Answer: $dy/dx = 2xy/(2y - x^2)$

(b) substitute x=3, y=2 into their dy/dx expression (ft) **M1**

(b) gradient = -12/5 cso **A1**

(b) Answer: Gradient at P = -12/5

(c) use y-2 = m(x-3) with m=-12/5 (ft their gradient from part (b)) **M1**

(c) multiply through by 5 and rearrange to clear fractions **M1**

(c) 12x + 5y - 46 = 0 cao (oe, any correct integer multiple) **A1**

(c) Answer: 12x + 5y - 46 = 0

(d) normal gradient = 5/12 (negative reciprocal of their tangent gradient, ft) **M1**

(d) use y-2 = (5/12)(x-3) and clear fractions **M1**

(d) 5x - 12y + 9 = 0 cao (oe) **A1**

(d) Answer: 5x - 12y + 9 = 0

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