1.14 Pure: Differentiation Depth: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A curve has equation $y = (3x^2 - 5)^4$.

(a) Find dy/dx, giving your answer in fully simplified factorised form. **(3)**

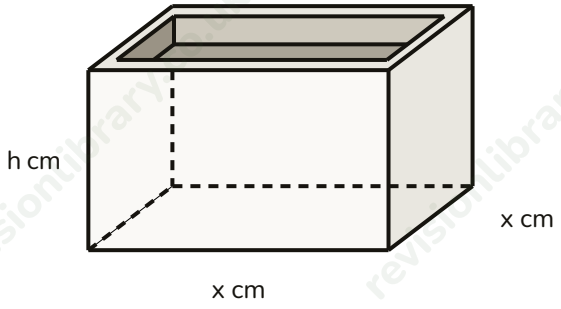
(b) Hence find the exact value of dy/dx when $x = 2$. **(2)**

(Total for Question 1 is 5 marks)

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2 An open-topped box is a cuboid with a square base of side x cm and height h cm. The volume of the box is 2916 cm^3.

Diagram NOT accurately drawn



(a) Show that the total external surface area, A cm^2, is $A = x^2 + 11664/x$. **(3)**

(b) Find the value of x for which A is a minimum. **(4)**

(c) Prove that this value of x gives a minimum and find the minimum surface area. **(3)**

(Total for Question 2 is 10 marks)

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3 A curve has equation $y = (2x - 1)/(x^2 + 4)$.

(a) Find dy/dx, giving your answer as a single simplified fraction. **(3)**

(b) Hence find the exact x-coordinates of the points where $dy/dx = 0$, giving your answers in the form $(a \pm \sqrt{b})/c$. **(2)**

(Total for Question 3 is 5 marks)

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4 A spherical balloon is inflated so that its volume V cm^3 increases at a constant rate of 200 cm^3 per second. At time t seconds the radius of the balloon is r cm.

(a) Show that $dr/dt = 50/(\pi * r^2)$. **(3)**

(b) Find the rate of increase of the radius at the instant when r = 10 cm, giving your answer to 3 significant figures. **(2)**

(c) Find the rate of increase of the surface area of the balloon at this instant. **(3)**

(Total for Question 4 is 8 marks)

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5 A curve satisfies $x^2 + y^2 = 25$. Differentiate implicitly with respect to x, giving your answer in the form ... = 0.

(Total for Question 5 is 1 mark)

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6 $y = x e^{4x}$. Show that dy/dx can be written as $e^{4x}(1 + 4x)$.

(Total for Question 6 is 2 marks)

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7 A cylindrical water tank has a fixed radius of 4 m. Water is pumped into the tank at a constant rate of 6 cubic metres per minute. At time t minutes, the depth of water in the tank is h metres.

(a) Show that $dh/dt = 3/(8\pi)$. **(2)**

(b) Find the time taken, in minutes to 3 significant figures, for the depth to increase from 0 m to 2.5 m. **(2)**

(Total for Question 7 is 4 marks)

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8 A curve has parametric equations $x = t^2$, $y = 3t$. State dx/dt and dy/dt.

(Total for Question 8 is 1 mark)

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9 A curve has equation $y = (5x + 1)/(x^2 + 3)$.

(a) Find dy/dx, giving your answer as a single simplified fraction. **(2)**

(b) Hence find the exact x-coordinates of the points where $dy/dx = 0$, giving your answers in the form $(a \pm \sqrt{b})/c$. **(2)**

(Total for Question 9 is 4 marks)

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10 A uniform ladder of length 5 m rests with its foot on horizontal ground and its top against a vertical wall. The foot of the ladder is pulled directly away from the wall at a constant rate of 0.4 m per second. At time t seconds, let x m be the distance of the foot from the wall and let θ radians be the angle between the ladder and the ground, so that $x = 5*\cos(\theta)$.

(a) Show that $dx/dt = -5*\sin(\theta)*(d\theta/dt)$. **(2)**

(b) Hence find the exact rate of change of θ at the instant when $\theta = \pi/3$, and interpret the sign of your answer. **(5)**

(Total for Question 10 is 7 marks)

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Mark scheme · 1.14 Pure: Differentiation Depth: Test Pack

Question 1

(a) attempt at the chain rule, differentiating to the form $k(3x^2-5)^3$ **M1**

(a) correct unsimplified derivative $4(3x^2-5)^3 * 6x$ identified **A1**

(a) $dy/dx = 24x(3x^2-5)^3$ cao **A1**

(a) Answer: $dy/dx = 24x(3x^2 - 5)^3$

(b) substitute x=2 into their dy/dx expression (ft) **M1**

(b) 16464 cao **A1**

(b) Answer: 16464

Question 2

(a) $h = 2916/x^2$ **M1**

(a) $A = x^2 + 4xh$ **M1**

(a) $A = x^2 + 11664/x$ (cso) **A1**

(a) Answer: $A = x^2 + 11664/x$

(b) $dA/dx = 2x - 11664/x^2$ **M1**

(b) sets $dA/dx = 0$ **M1**

(b) $x^3 = 5832$ **A1**

(b) x = 18 **A1**

(b) Answer: x = 18

(c) $d^2A/dx^2 = 2 + 23328/x^3$ **M1**

(c) = 6 > 0 at x = 18, so a minimum **A1**

(c) $A = 324 + 11664/18 = 972$ cm^2 **A1**

(c) Answer: 972 cm^2

Question 3

(a) attempt at the quotient rule in the form $(v*u' - u*v')/v^2$ or equivalent **M1**

(a) correct unsimplified numerator $2(x^2+4) - (2x-1)(2x)$ **A1**

(a) $dy/dx = (-2x^2+2x+8)/(x^2+4)^2$ oe cao, e.g. $-2(x^2-x-4)/(x^2+4)^2$ **A1**

(a) Answer: $dy/dx = (-2x^2 + 2x + 8)/(x^2+4)^2$, equivalently $-2(x^2 - x - 4)/(x^2+4)^2$

(b) set the numerator $-2x^2+2x+8=0$ (i.e. $x^2-x-4=0$) and apply the quadratic formula **M1**

(b) $x = (1+\sqrt{17})/2$ or $x = (1-\sqrt{17})/2$, both stated oe **A1**

(b) Answer: $x = (1 + \sqrt{17})/2$ or $x = (1 - \sqrt{17})/2$ (awrt 2.56 or -1.56)

Question 4

(a) differentiate $V=(4/3)*\pi*r^3$ to obtain $dV/dr = 4*\pi*r^2$ **M1**

(a) use the chain rule $dr/dt = (dV/dt)/(dV/dr)$ with dV/dt=200 **M1**

(a) $dr/dt = 200/(4*\pi*r^2) = 50/(\pi*r^2)$ cso **A1**

(a) Answer: $dr/dt = 50/(\pi*r^2)$

(b) substitute r=10 into the formula from part (a) **M1**

(b) dr/dt = awrt 0.159 cm/s cao **A1**

(b) Answer: dr/dt = 1/(2*π) cm/s, awrt 0.159 cm/s

(c) differentiate $S=4*\pi*r^2$ to obtain $dS/dr = 8*\pi*r$ **M1**

(c) use the chain rule $dS/dt = (dS/dr)*(dr/dt)$ with their dr/dt (ft) **M1**

(c) dS/dt = 40 cm^2 per second cao **A1**

(c) Answer: dS/dt = 40 cm^2 per second

Question 5

2x + 2y(dy/dx) = 0 oe **B1**

Answer: 2x + 2y(dy/dx) = 0

Question 6

uses the product rule: $dy/dx = e^{4x} + 4x\,e^{4x}$ **M1**

cso: factorises to $e^{4x}(1+4x)$ **A1**

Answer: $dy/dx = e^{4x}(1+4x)$

Question 7

(a) writes $V = \pi r^2 h = 16\pi h$ and differentiates to get $dV/dt = 16\pi (dh/dt)$ **M1**

(a) cso: $dh/dt = 6/(16\pi) = 3/(8\pi)$ **A1**

(a) Answer: dh/dt = 3/(8pi)

(b) recognises dh/dt is constant, so time = depth / (dh/dt) **M1**

(b) awrt 20.9 minutes **A1**

(b) Answer: awrt 20.9 minutes

Question 8

dx/dt = 2t, dy/dt = 3, both required **B1**

Answer: dx/dt = 2t, dy/dt = 3

Question 9

(a) uses the quotient rule **M1**

(a) $dy/dx = (-5x^2 - 2x + 15)/(x^2+3)^2$ cao **A1**

(a) Answer: $dy/dx = (-5x^2-2x+15)/(x^2+3)^2$

(b) sets the numerator $5x^2+2x-15 = 0$ and applies the quadratic formula **M1**

(b) $x = (-1 \pm 2\sqrt{19})/5$ cao **A1**

(b) Answer: $x = (-1 \pm 2\sqrt{19})/5$

Question 10

(a) differentiate $x=5*\cos(\theta)$ implicitly with respect to t using the chain rule **M1**

(a) $dx/dt = -5*\sin(\theta)*(d\theta/dt)$ cso **A1**

(a) Answer: dx/dt = -5*sin(θ)*(dtheta/dt)

(b) substitute dx/dt=0.4 (the rate the foot moves away from the wall) and rearrange for dtheta/dt **M1**

(b) $d\theta/dt = -0.4/(5*\sin(\theta)) = -2/(25*\sin(\theta))$ oe **A1**

(b) substitute $\theta=\pi/3$ using the exact value $\sin(\pi/3)=\sqrt{3}/2$ **M1**

(b) $d\theta/dt = -4*\sqrt{3}/75$ oe (awrt -0.0924 rad/s) **A1**

(b) correct interpretation: the negative sign shows θ is decreasing, i.e. the ladder's angle with the ground is reducing as the foot slides away from the wall **B1**

(b) Answer: dtheta/dt = -4*√3/75 rad/s (awrt -0.0924 rad/s); θ is decreasing

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