1.14 Pure: Differentiation Depth

EDEXCEL 9MA0 · Calculator allowed · about 225 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A curve has parametric equations $x = t^2 - 1$, $y = 3t - t^3$, for t in R.

(a) Find dy/dx in terms of t. **(3)**

(b) Find the value of dy/dx when $t = 2$. **(2)**

(Total for Question 1 is 5 marks)

2 A curve C has parametric equations $x = 2t^2 - 3$, $y = 3t^3 - t$, t in R.

(a) Find dy/dx in terms of t. **(3)**

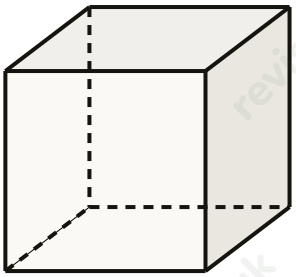
(b) Find an equation of the tangent to C at the point where t = 1, in the form ax + by + c = 0 with integer coefficients. **(3)**

(c) The tangent in part (b) meets the x-axis at the point Q. Find the x coordinate of Q. **(2)**

(Total for Question 2 is 8 marks)

3 The side length x cm of a cube increases at a constant rate of 0.3 cm/s.

Diagram NOT accurately drawn



(a) Find the rate at which the volume $V = x^3$ is increasing when $x = 6$. **(3)**

(b) Find the rate at which the total surface area $S = 6x^2$ is increasing at that instant. **(2)**

(Total for Question 3 is 5 marks)

4 A curve C has parametric equations $x = t^2 - 1$, $y = t^3 - 3t$, t in R.

(a) Find dy/dx in terms of t. **(3)**

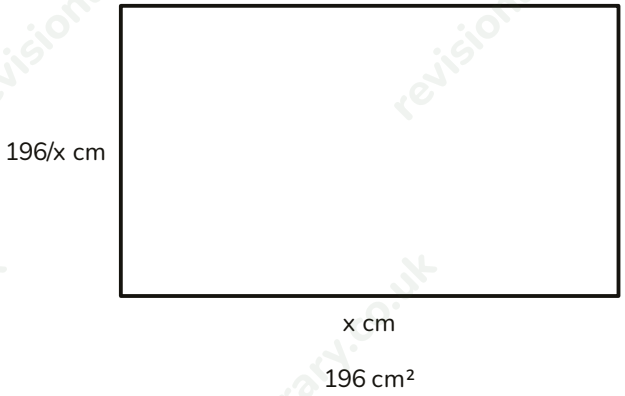
(b) Find an equation of the tangent to C at the point where $t = -2$, in the form $ax + by + c = 0$ with integer coefficients. **(3)**

(c) The tangent in part (b) meets the x-axis at the point Q. Find the x coordinate of Q. **(2)**

(Total for Question 4 is 8 marks)

5 A rectangle has area 196 cm^2 and width x cm.

Diagram NOT accurately drawn



(a) Show that the perimeter of the rectangle is $p = 2x + 392/x$. **(2)**

(b) Find the minimum value of the perimeter, justifying your answer. **(4)**

(Total for Question 5 is 6 marks)

6 A curve C has parametric equations $x = 3t^2 - 3$, $y = t^3 - 5t$, t in R.

(a) Find dy/dx in terms of t. **(3)**

(b) Find an equation of the tangent to C at the point where $t = 2$, in the form $ax + by + c = 0$ with integer coefficients. **(3)**

(c) The tangent in part (b) meets the x-axis at the point Q. Find the x coordinate of Q. **(2)**

(Total for Question 6 is 8 marks)

7 The curve C has equation $y = 2x^3 - 15x^2 + 24x$.

(a) Find dy/dx and d^2y/dx^2. **(3)**

(b) Find the coordinates of the stationary points of C. **(4)**

(c) Determine the nature of each stationary point. **(2)**

(d) Find the range of values of k for which the equation $2x^3 - 15x^2 + 24x = k$ has three distinct real roots. **(2)**

(Total for Question 7 is 11 marks)

8 A curve has equation $y = x^2 * e^{3x}$.

(a) Show that dy/dx can be written as $x * e^{3x} * (3x + 2)$. **(4)**

(b) Hence find the exact value(s) of x for which the tangent to the curve is horizontal. **(2)**

(Total for Question 8 is 6 marks)

9 $y = x^4 \ln(5x)$, $x > 0$.

(a) Find dy/dx. **(3)**

(b) Show that the curve has a single stationary point and find its exact coordinates. **(4)**

(Total for Question 9 is 7 marks)

10 Water runs into a vertical conical vessel, vertex downwards, at 40pi cm^3/s. The semi-vertical angle is 45 degrees, so the volume of water at depth h cm is $V = (1/3)\pi h^3$.

(a) Find an expression for dV/dh. **(2)**

(b) Find the rate at which the depth is increasing when h = 8. **(3)**

(Total for Question 10 is 5 marks)

11 $f(x) = 2x^3 - 30x^2 - 4x + 2$.

(a) Find $f'(x)$ and $f''(x)$. **(3)**

(b) Find the coordinates of the point of inflection of the curve $y = f(x)$, showing that the second derivative changes sign. **(4)**

(c) Find the range of values of x for which f is concave. **(2)**

(Total for Question 11 is 9 marks)

12 $y = e^{-2x} \sin(2x)$.

(a) Find dy/dx. **(3)**

(b) Find the exact gradient of the curve at the point where $x = \pi/4$. **(2)**

(Total for Question 12 is 5 marks)

13 The curve C has equation $y = 2x^3 - 9x^2$.

(a) Find dy/dx and d^2y/dx^2. **(3)**

(b) Find the coordinates of the stationary points of C. **(4)**

(c) Determine the nature of each stationary point. **(2)**

(d) Find the range of values of k for which the equation $2x^3 - 9x^2 = k$ has three distinct real roots. **(2)**

(Total for Question 13 is 11 marks)

14 A farmer has 156 m of fencing to enclose a rectangular field. One side of the field is a straight wall and needs no fencing. The sides perpendicular to the wall have length x m.

(a) Show that the area of the field is $A = x(156 - 2x)$ m^2. **(2)**

(b) Use calculus to find the maximum area, justifying that it is a maximum. **(4)**

(Total for Question 14 is 6 marks)

15 A curve C has equation $x^2 + y^2 - 4x + 6y = 12$.

(a) Show that $dy/dx = (2 - x)/(y + 3)$. **(3)**

(b) Verify that the point (2, 2) lies on C, and find the gradient of C at this point. **(2)**

(Total for Question 15 is 5 marks)

16 A curve has equation $y = e^{\cos(2x)}$.

(a) Find dy/dx. **(2)**

(b) Find the exact value of dy/dx when $x = \pi/6$, giving your answer in the form $-\sqrt{ke}$ for an integer k. **(3)**

(Total for Question 16 is 5 marks)

17 A curve has equation $y = x^2 * e^{-x}$.

(a) Find dy/dx, giving your answer fully factorised. **(3)**

(b) Show that the curve has stationary points where $x = 0$ and $x = 2$, and find the corresponding y-coordinates. **(3)**

(c) Find d^2y/dx^2 and hence determine the nature of each stationary point. **(2)**

(Total for Question 17 is 8 marks)

18 $y = (3x - 2)^5$. Find dy/dx.

(Total for Question 18 is 1 mark)

19 $y = (5x + 2)^4$. Find dy/dx, and hence find the exact value of dy/dx when x = 0.

(Total for Question 19 is 2 marks)

20 $y = (3x - 1)/(x + 2)$. Find dy/dx, giving your answer as a single simplified fraction.

(Total for Question 20 is 2 marks)

21 A curve has parametric equations $x = t^3$, $y = 2t^2$, for t in R. Find dy/dx in terms of t.

(Total for Question 21 is 2 marks)

22 A curve has equation $y = (4x^2 - 3)^3$.

(a) Find dy/dx, giving your answer in fully factorised form. **(2)**

(b) Hence find the exact value of dy/dx when $x = 1$. **(2)**

(Total for Question 22 is 4 marks)

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23 A curve C has equation $x^2 + y^2 + 6x - 4y = 12$. Show that $dy/dx = -(x+3)/(y-2)$.

(Total for Question 23 is 3 marks)

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24 A curve has parametric equations $x = 2t - 1$, $y = t^2 + 3t$, for t in R. Find dy/dx in terms of t, and hence find the value of dy/dx when $t = 2$.

(Total for Question 24 is 3 marks)

25 $y = e^{6x}$. Find dy/dx.

(Total for Question 25 is 1 mark)

26 $y = (x^2 + 1)^3$. Find dy/dx.

(Total for Question 26 is 1 mark)

27 $y = e^{x^2}$. Find dy/dx.

(Total for Question 27 is 1 mark)

28 $y = \ln(4x)$, for $x > 0$. Find dy/dx.

(Total for Question 28 is 1 mark)

29 $y = 1/x^2$, for x not equal to 0. Find dy/dx.

(Total for Question 29 is 1 mark)

30 $y = \sqrt{x}$, for $x > 0$. Find dy/dx.

(Total for Question 30 is 1 mark)

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31 $y = x^3 - 4x$. Find the value of dy/dx when $x = 2$.

(Total for Question 31 is 1 mark)

32 Water is poured into an inverted right circular conical tank at a constant rate of 30 cm^3 per second. The tank has height 40 cm and the radius of its circular top is 20 cm, so the radius r cm and depth h cm of the water are always in the same ratio as the tank's dimensions. Let V cm^3 be the volume of water in the tank when the depth is h cm.

(a) Show that $V = (\pi/12)*h^3$. **(3)**

(b) Find dh/dt in terms of h. **(3)**

(c) Find the rate at which the depth is increasing at the instant when h = 10 cm, giving your answer to 3 significant figures. **(3)**

(d) Explain, with justification, whether the rate at which the depth is increasing gets faster or slower as h increases. **(2)**

(Total for Question 32 is 11 marks)

33 A curve C has parametric equations $x = 2t + 1$, $y = t^2 - 4t$, for t in R.

(a) Find dy/dx in terms of t. **(3)**

(b) Find the coordinates of the point on C where the tangent is parallel to the line $y = 2x + 5$. **(4)**

(c) Show that the cartesian equation of C can be written in the form $y = ax^2 + bx + c$, stating the values of a, b and c. **(3)**

(Total for Question 33 is 10 marks)

34 A curve has equation $y = x^3 e^{-2x}$.

(a) Show that dy/dx can be written as $x^2 e^{-2x}(3 - 2x)$. **(2)**

(b) Hence find the exact value(s) of x for which the tangent to the curve is horizontal. **(2)**

(Total for Question 34 is 4 marks)

35 A curve C is defined by the equation $x^2*y - y^2 = 14$. The point P(3, 2) lies on C.

(a) Find dy/dx in terms of x and y. **(4)**

(b) Show that the gradient of C at P is -12/5. **(2)**

(c) Find an equation of the tangent to C at P, giving your answer in the form ax + by + c = 0 with integer coefficients. **(3)**

(d) Find an equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$ with integer coefficients. **(3)**

(Total for Question 35 is 12 marks)

Mark scheme · 1.14 Pure: Differentiation Depth

Question 1

(a) $dx/dt = 2t$	**M1**
(a) $dy/dt = 3 - 3t^2$	**M1**
(a) $dy/dx = (3-3t^2)/(2t)$ oe cao, e.g. $3(1-t^2)/(2t)$	**A1**
(a) Answer: $dy/dx = (3 - 3t^2)/(2t)$	
(b) substitute t=2 into their dy/dx expression (ft)	**M1**
(b) $dy/dx = -9/4$ oe cao (accept -2.25)	**A1**
(b) Answer: $dy/dx = -9/4$	

Question 2

(a) $dx/dt = 4t$ and $dy/dt = 9t^2 - 1$	**M1**
(a) $dy/dx = (dy/dt)/(dx/dt)$	**M1**
(a) $dy/dx = (9t^2 - 1)/(4t)$	**A1**
(a) Answer: $dy/dx = (9t^2 - 1)/(4t)$	
(b) the point is (-1, 2)	**B1**
(b) gradient = 2	**M1**
(b) $2x - y + 4 = 0$	**A1**
(b) Answer: $2x - y + 4 = 0$	
(c) sets y = 0 in the tangent	**M1**
(c) x = -2	**A1**
(c) Answer: x = -2	

Question 3

(a) $dV/dt = 3x^2\, dx/dt$	**M1**
(a) $3(6)^2(0.3)$	**A1**
(a) 32.4 cm^3/s	**A1**
(a) Answer: 32.4 cm^3/s	
(b) $dS/dt = 12x\, dx/dt$	**M1**
(b) 21.6 cm^2/s	**A1**
(b) Answer: 21.6 cm^2/s	

Question 4

(a) $dx/dt = 2t$ and $dy/dt = 3t^2 - 3$	**M1**
(a) $dy/dx = (dy/dt)/(dx/dt)$	**M1**
(a) $dy/dx = (3t^2 - 3)/(2t)$	**A1**
(a) Answer: $dy/dx = (3t^2 - 3)/(2t)$	
(b) the point is (3, -2)	**B1**
(b) gradient = -9/4	**M1**
(b) $9x + 4y - 19 = 0$	**A1**
(b) Answer: $9x + 4y - 19 = 0$	
(c) sets y = 0 in the tangent	**M1**
(c) x = 19/9	**A1**
(c) Answer: x = 19/9	

Question 5

(a) length = 196/x	**M1**
(a) $p = 2x + 2(196/x) = 2x + 392/x$ (cso)	**A1**
(a) Answer: $p = 2x + 392/x$	
(b) $dp/dx = 2 - 392/x^2$	**M1**
(b) = 0 gives $x^2 = 196$, x = 14	**A1**
(b) $d^2p/dx^2 = 784/x^3 > 0$, so a minimum	**B1**
(b) minimum perimeter 56 cm (the rectangle is a square)	**A1**
(b) Answer: 56 cm	

Question 6

(a) $dx/dt = 6t$ and $dy/dt = 3t^2 - 5$	**M1**
(a) $dy/dx = (dy/dt)/(dx/dt)$	**M1**
(a) $dy/dx = (3t^2 - 5)/(6t)$	**A1**
(a) Answer: $dy/dx = (3t^2 - 5)/(6t)$	
(b) the point is (9, -2)	**B1**
(b) gradient = 7/12	**M1**
(b) $7x - 12y - 87 = 0$	**A1**
(b) Answer: $7x - 12y - 87 = 0$	
(c) sets y = 0 in the tangent	**M1**
(c) x = 87/7	**A1**
(c) Answer: x = 87/7	

Question 7

(a) differentiates, reducing the power of each term by 1	**M1**
(a) $dy/dx = 6x^2 - 30x + 24$	**A1**
(a) $d^2y/dx^2 = 12x - 30$	**A1**
(a) Answer: $dy/dx = 6x^2 - 30x + 24$, $d^2y/dx^2 = 12x - 30$	
(b) sets dy/dx = 0	**M1**
(b) x = 1 and x = 4	**A1**
(b) substitutes into the equation of the curve	**M1**
(b) (1, 11) and (4, -16)	**A1**
(b) Answer: (1, 11) and (4, -16)	
(c) evaluates d^2y/dx^2 at each stationary point	**M1**
(c) (1, 11) is a maximum ($d^2y/dx^2 = -18 < 0$) and (4, -16) is a minimum ($d^2y/dx^2 = 18 > 0$)	**A1**
(c) Answer: maximum at x = 1, minimum at x = 4	
(d) the horizontal line y = k must cut the curve three times, so lies between the minimum and maximum values	**M1**
(d) $-16 < k < 11$	**A1**
(d) Answer: $-16 < k < 11$	

Question 8

(a) attempt at the product rule with $u=x^2$, $v=e^{3x}$	**M1**
(a) correct unsimplified derivative $2x*e^{3x} + 3x^2*e^{3x}$	**A1**
(a) factorise out the common factor $x*e^{3x}$, dependent on the previous M1	**dM1**
(a) $dy/dx = x*e^{3x}*(3x+2)$ cso	**A1**
(a) Answer: $dy/dx = x * e^{3x} * (3x + 2)$	
(b) set dy/dx=0 and use that e^{3x} is never zero to obtain $x(3x+2)=0$	**M1**
(b) x = 0 or x = -2/3 both stated oe	**A1**
(b) Answer: x = 0 or x = -2/3	

Question 9

(a) uses the product rule with $u = x^4$ and $v = \ln(5x)$	**M1**
(a) $du/dx = 4x^3$ and $dv/dx = 1/x$	**A1**
(a) $dy/dx = x^3(4 \ln(5x) + 1)$	**A1**
(a) Answer: $dy/dx = x^3(4 \ln(5x) + 1)$	
(b) sets dy/dx = 0, with $x^3 > 0$ so $4 \ln(5x) + 1 = 0$	**M1**
(b) $\ln(5x) = -1/4$	**A1**
(b) $x = e^{-1/4}/5$	**M1**
(b) $y = x^4 (-1/4) = -1/(2500e)$	**A1**
(b) Answer: $x = e^{-1/4}/5$, $y = -1/(2500e)$	

Question 10

(a) differentiates $V = (1/3)\pi h^3$ **M1**

(a) $dV/dh = \pi h^2$ **A1**

(a) Answer: $dV/dh = \pi h^2$

(b) $dh/dt = (dV/dt)/(dV/dh)$ **M1**

(b) $40pi/(64pi)$ **A1**

(b) 5/8 cm/s **A1**

(b) Answer: 5/8 cm/s

Question 11

(a) differentiates **M1**

(a) $f'(x) = 6x^2 - 60x - 4$ **A1**

(a) $f''(x) = 12x - 60$ **A1**

(a) Answer: $f'(x) = 6x^2 - 60x - 4$, $f''(x) = 12x - 60$

(b) solves $f''(x) = 0$ **M1**

(b) $x = 5$ **A1**

(b) $f''(4) = -12 < 0$ and $f''(6) = 12 > 0$, a change of sign **B1**

(b) (5, -518) **A1**

(b) Answer: (5, -518)

(c) $f''(x) < 0$ **M1**

(c) $x < 5$ **A1**

(c) Answer: $x < 5$

Question 12

(a) uses the product rule **M1**

(a) $-2e^{-2x} \sin(2x)$ **A1**

(a) $+ 2e^{-2x} \cos(2x)$ **A1**

(a) Answer: $dy/dx = e^{-2x}(-2\sin(2x) + 2\cos(2x))$

(b) substitutes $x = \pi/4$, where $\sin(2x) = 1$ and $\cos(2x) = 0$ **M1**

(b) $-2e^{-1/2pi}$ **A1**

(b) Answer: $-2e^{-1/2pi}$

Question 13

(a) differentiates, reducing the power of each term by 1 **M1**

(a) $dy/dx = 6x^2 - 18x$ **A1**

(a) $d^2y/dx^2 = 12x - 18$ **A1**

(a) Answer: $dy/dx = 6x^2 - 18x$, $d^2y/dx^2 = 12x - 18$

(b) sets $dy/dx = 0$ **M1**

(b) $x = 0$ and $x = 3$ **A1**

(b) substitutes into the equation of the curve **M1**

(b) (0, 0) and (3, -27) **A1**

(b) Answer: (0, 0) and (3, -27)

(c) evaluates d^2y/dx^2 at each stationary point **M1**

(c) (0, 0) is a maximum ($d^2y/dx^2 = -18 < 0$) and (3, -27) is a minimum ($d^2y/dx^2 = 18 > 0$) **A1**

(c) Answer: maximum at x = 0, minimum at x = 3

(d) the horizontal line $y = k$ must cut the curve three times, so lies between the minimum and maximum values **M1**

(d) $-27 < k < 0$ **A1**

(d) Answer: $-27 < k < 0$

Question 14

(a) length parallel to the wall = 156 - 2x **M1**

(a) $A = x(156 - 2x)$ (cso) **A1**

(a) Answer: $A = x(156 - 2x)$

(b) $A = 156x - 2x^2$, $dA/dx = 156 - 4x$ **M1**

(b) $dA/dx = 0$ gives $x = 39$ **A1**

(b) $d^2A/dx^2 = -4 < 0$ so a maximum **B1**

(b) maximum area = $39 \times 78 = 3042$ m^2 **A1**

(b) Answer: 3042 m^2

Question 15

(a) differentiate implicitly term by term, with at least 3 of the 4 terms correct (2x, 2y*dy/dx, -4, 6*dy/dx) **M1**

(a) fully correct implicit equation $2x + 2y*(dy/dx) - 4 + 6*(dy/dx) = 0$ **A1**

(a) rearrange to $dy/dx = (2-x)/(y+3)$ cso **A1**

(a) Answer: $dy/dx = (2 - x)/(y + 3)$

(b) verify (2,2) satisfies the equation (4+4-8+12=12) and substitute x=2, y=2 into the implicit derivative **M1**

(b) $dy/dx = 0$ cao **A1**

(b) Answer: Gradient at (2, 2) is 0 (a horizontal tangent)

Question 16

(a) attempt at the chain rule to the form $k*\sin(2x)*e^{\cos(2x)}$ **M1**

(a) $dy/dx = -2*\sin(2x)*e^{\cos(2x)}$ cao **A1**

(a) Answer: $dy/dx = -2 * \sin(2x) * e^{\cos(2x)}$

(b) substitute $x=\pi/6$ using the exact values $\sin(\pi/3)=\sqrt{3}/2$ and $\cos(\pi/3)=1/2$ **M1**

(b) correct unsimplified exact value $-\sqrt{3} * e^{1/2}$ **A1**

(b) $dy/dx = -\sqrt{3e}$ cao **A1**

(b) Answer: $dy/dx = -\sqrt{3e}$, equivalently $-\sqrt{3} * e^{1/2}$ (awrt -2.856)

Question 17

(a) attempt at the product rule with $u=x^2$, $v=e^{-x}$ **M1**

(a) correct unsimplified derivative $2x*e^{-x} - x^2*e^{-x}$ **A1**

(a) $dy/dx = x*e^{-x}*(2-x)$ cao **A1**

(a) Answer: $dy/dx = x * e^{-x} * (2 - x)$

(b) set $dy/dx=0$ and use that e^{-x} is never zero to obtain $x(2-x)=0$ **M1**

(b) x=0 and x=2 both stated cso **A1**

(b) y-coordinates y=0 (at x=0) and $y=4/e^2$, awrt 0.541 (at x=2), both cao **A1**

(b) Answer: Stationary points at (0, 0) and (2, $4/e^2$), where $4/e^2$ is awrt 0.541

(c) correct $d^2y/dx^2 = e^{-x}*(x^2-4x+2)$, found by differentiating dy/dx again using the product rule **M1**

(c) correct classification with reasoning: (0,0) is a minimum since $d^2y/dx^2=2>0$ at x=0; (2, $4/e^2$) is a maximum since $d^2y/dx^2=-2/e^2<0$ at x=2, both cao **A1**

(c) Answer: (0, 0) is a minimum point; (2, $4/e^2$) is a maximum point

Question 18

$15(3x-2)^4$ cao **B1**

Answer: $15(3x-2)^4$

Question 19

$dy/dx = 20(5x+2)^3$ **M1**

at x=0: 160 cao **A1**

Answer: $dy/dx = 20(5x+2)^3$; at x=0, $dy/dx = 160$

Question 20

uses the quotient rule **M1**

$dy/dx = 7/(x+2)^2$ cao **A1**

Answer: $dy/dx = 7/(x+2)^2$

Question 21

finds $dy/dt = 4t$ and $dx/dt = 3t^2$ **M1**

$dy/dx = 4/(3t)$ cao **A1**

Answer: $dy/dx = 4/(3t)$

Question 22

(a) applies the chain rule: $dy/dx = 3(4x^2-3)^2 \times 8x$ **M1**

(a) $24x(4x^2-3)^2$ cao **A1**

(a) Answer: $dy/dx = 24x(4x^2-3)^2$

(b) substitutes x=1 into the expression from (a) **M1**

(b) 24 cao **A1**

(b) Answer: 24

Question 23

differentiates implicitly: $2x + 2y(dy/dx) + 6 - 4(dy/dx) = 0$ **M1**

rearranges to $dy/dx(2y - 4) = -(2x + 6)$ **A1**

cso: $dy/dx = -(x+3)/(y-2)$ **A1**

Answer: $dy/dx = -(x+3)/(y-2)$

Question 24

finds $dy/dt = 2t + 3$ and $dx/dt = 2$ **M1**

$dy/dx = (2t+3)/2$ **A1**

at t=2: $dy/dx = 3.5$ cao **A1**

Answer: $dy/dx = (2t+3)/2$; at t=2, $dy/dx = 3.5$

Question 25

$6e^{6x}$ cao **B1**

Answer: $6e^{6x}$

Question 26

$6x(x^2+1)^2$ cao **B1**

Answer: $6x(x^2+1)^2$

Question 27

$2x\ e^{x^2}$ cao **B1**

Answer: $2x\ e^{x^2}$

Question 28

$1/x$ cao **B1**

Answer: $1/x$

Question 29

$-2/x^3$ cao **B1**

Answer: $-2/x^3$

Question 30

$1/(2\sqrt{x})$ cao **B1**

Answer: $1/(2\sqrt{x})$

Question 31

8 cao **B1**

Answer: 8

Question 32

(a) use similar triangles to write r in terms of h, r = h/2 **M1**

(a) substitute into $V=(1/3)*\pi*r^2*h$ **M1**

(a) $V = (\pi/12)*h^3$ cso, with full simplification shown **A1**

(a) Answer: $V = (\pi/12)*h^3$

(b) differentiate $V=(\pi/12)*h^3$ to obtain $dV/dh = (\pi/4)*h^2$ **M1**

(b) use the chain rule $dh/dt = (dV/dt)/(dV/dh)$ with $dV/dt=30$ **M1**

(b) $dh/dt = 120/(\pi*h^2)$ oe cao **A1**

(b) Answer: $dh/dt = 120/(\pi*h^2)$

(c) substitute h=10 into their dh/dt expression (ft) **M1**

(c) exact value $6/(5*\pi)$ oe, e.g. $1.2/\pi$ **A1**

(c) awrt 0.382 cm/s cao **A1**

(c) Answer: $dh/dt = 6/(5*\pi)$ cm/s, awrt 0.382 cm/s

(d) correct statement that dh/dt decreases as h increases **B1**

(d) valid justification, e.g. dh/dt is inversely proportional to h^2 (from $dh/dt=120/(\pi*h^2)$), so as h grows dh/dt must fall, consistent with the tank widening so the same inflow rate produces a slower rise in depth **B1**

(d) Answer: dh/dt decreases as h increases

Question 33

(a) $dx/dt = 2$ **M1**

(a) $dy/dt = 2t-4$ **M1**

(a) $dy/dx = t-2$ oe cao **A1**

(a) Answer: $dy/dx = t - 2$

(b) set dy/dx equal to the gradient of the given line (2) **M1**

(b) t=4 **A1**

(b) substitute t=4 into both parametric equations **M1**

(b) point (9, 0) cao **A1**

(b) Answer: (9, 0)

(c) make t the subject of the x equation, $t=(x-1)/2$ **M1**

(c) substitute into $y=t^2-4t$ and expand **M1**

(c) $y = (1/4)x^2 - (5/2)x + 9/4$ cso (equivalently $4y=x^2-10x+9$), with a=1/4, b=-5/2, c=9/4 stated **A1**

(c) Answer: $y = (1/4)x^2 - (5/2)x + 9/4$, so a = 1/4, b = -5/2, c = 9/4

Question 34

(a) uses the product rule: $dy/dx = 3x^2\ e^{-2x} - 2x^3\ e^{-2x}$ **M1**

(a) cso: factorises to $x^2\ e^{-2x}(3-2x)$ **A1**

(a) Answer: $dy/dx = x^2\ e^{-2x}(3-2x)$

(b) sets $x^2\ e^{-2x}(3-2x) = 0$ and uses that e^{-2x} is never zero **M1**

(b) x = 0 or x = 3/2, both values **A1**

(b) Answer: x = 0 or x = 3/2

Question 35

(a) differentiate x^2*y implicitly using the product rule to obtain $2xy + x^2*(dy/dx)$ **M1**

(a) differentiate y^2 implicitly to obtain $2y*(dy/dx)$ **M1**

(a) correct full implicit equation $2xy + x^2*(dy/dx) - 2y*(dy/dx) = 0$ **A1**

(a) $dy/dx = 2xy/(2y-x^2)$ oe cao, e.g. $-2xy/(x^2-2y)$ **A1**

(a) Answer: $dy/dx = 2xy/(2y - x^2)$

(b) substitute x=3, y=2 into their dy/dx expression (ft) **M1**

(b) gradient = -12/5 cso **A1**

(b) Answer: Gradient at P = -12/5

(c) use y-2 = m(x-3) with m=-12/5 (ft their gradient from part (b)) **M1**

(c) multiply through by 5 and rearrange to clear fractions **M1**

(c) 12x + 5y - 46 = 0 cao (oe, any correct integer multiple) **A1**

(c) Answer: 12x + 5y - 46 = 0

(d) normal gradient = 5/12 (negative reciprocal of their tangent gradient, ft) **M1**

(d) use y-2 = (5/12)(x-3) and clear fractions **M1**

(d) 5x - 12y + 9 = 0 cao (oe) **A1**

(d) Answer: 5x - 12y + 9 = 0