1.15 Pure: Integration Depth: 1-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $f(x) = (5x + 13)/((x + 2)(x + 3))$, $x > -2$.

(a) Express $f(x)$ in partial fractions. **(3)**

(b) Hence find the exact value of the integral from $x = 1$ to $x = 5$ of $f(x)\,dx$. **(4)**

(Total for Question 1 is 7 marks)

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2 This question is about integrating $\sin^2(3x)$ with respect to x.

(a) Use a double angle formula to show that $\sin^2(3x) = (1 - \cos(6x))/2$. **(2)**

(b) Hence find the exact value of the integral from $x = 0$ to $x = \pi/12$ of $\sin^2(3x)\,dx$. **(3)**

(Total for Question 2 is 5 marks)

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3 A curve satisfies $x\,dy/dx = y$ for $x > 0$, $y > 0$, and passes through $(1, 2)$.

(a) Find y in terms of x. **(4)**

(b) Find the value of y when $x = 4$. **(2)**

(Total for Question 3 is 6 marks)

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4 The curve C has equation $y = 1/\sqrt{9 - x^2}$, for $-3 < x < 3$.

(a) Find integral of $1/\sqrt{9 - x^2}\,dx$. **(2)**

(b) Hence show that the definite integral, from $x = 0$ to $x = 3/2$, of $1/\sqrt{9 - x^2}\,dx$, equals $\pi/6$. **(3)**

(Total for Question 4 is 5 marks)

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5 The curve C has equation $y = x/(1 + x^2)$, for $x \geq 0$.

(a) Show that integral of $x/(1 + x^2)\ dx = (1/2)*\ln(1 + x^2) + c$. **(2)**

(b) Hence find the exact area of the region bounded by C, the x-axis and the lines $x = 0$ and $x = \sqrt{3}$, giving your answer in the form $k*\ln(2)$, where k is a constant to be found. **(3)**

(c) Find the exact area of the region bounded by C, the x-axis and the lines $x = \sqrt{3}$ and $x = 3$, giving your answer in the form $(1/2)*\ln(p/q)$, where p and q are integers in lowest terms. **(3)**

(Total for Question 5 is 8 marks)

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6 $f(x) = (8x + 7) / ((x - 1)(x + 4))$.

(a) Express f(x) in the form $A/(x - 1) + B/(x + 4)$, where A and B are constants to be found. **(3)**

(b) Hence find integral of f(x) dx. **(3)**

(c) Find the exact value of the definite integral, from $x = 2$ to $x = 6$, of f(x) dx, giving your answer in the form $p*\ln(5) + q*\ln(3)$, where p and q are integers to be found. **(4)**

(Total for Question 6 is 10 marks)

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7 The curve C has equation $y = 1/\sqrt{16-x^2}$, for $-4 < x < 4$.

(a) Find the integral of $1/\sqrt{16-x^2}\ dx$. **(2)**

(b) Hence show that the definite integral, from $x = 0$ to $x = 2$, of $1/\sqrt{16-x^2}\ dx$, equals $\pi/6$. **(2)**

(Total for Question 7 is 4 marks)

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8 The number of bacteria, N, in a culture is modelled by the differential equation $dN/dt = N/50$, where t is measured in hours. Initially there are 200 bacteria.

(a) By separating the variables, show that $N = 200\ e^{t/50}$. **(2)**

(b) Find the time taken, in hours to 3 significant figures, for the population to reach 500. **(2)**

(Total for Question 8 is 4 marks)

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Mark scheme · 1.15 Pure: Integration Depth: 1-hour Pack

Question 1

(a) A/(x - p) + B/(x - q) and equating numerators **M1**

(a) A = 3 **A1**

(a) B = 2 **A1**

(a) Answer: 3/(x + 2) + 2/(x + 3)

(b) integrates each term to a logarithm: A ln|x - p| + B ln|x - q| **M1**

(b) 3 ln|x + 2| + 2 ln|x + 3| (+ c) **A1**

(b) substitutes the limits and combines logs **M1**

(b) 3 ln(7/3) + 2 ln(2) **A1**

(b) Answer: 3 ln(7/3) + 2 ln(2)

Question 2

(a) $\cos(6x) = 1 - 2\sin^2(3x)$ **M1**

(a) rearranges to the printed identity (cso) **A1**

(a) Answer: $\sin^2(3x) = (1 - \cos(6x))/2$

(b) uses the identity and integrates: x/2 - sin(6x)/12 **M1**

(b) substitutes the limits **M1**

(b) π/24 - 1/12 **A1**

(b) Answer: π/24 - 1/12

Question 3

(a) separates variables: integral of 1/y dy = integral of 1/x dx **M1**

(a) ln y = 1 ln x + c **A1**

(a) uses (1, 2): c = ln 2 **M1**

(a) y = 2x **A1**

(a) Answer: y = 2x

(b) substitutes x = 4 **M1**

(b) 8 **A1**

(b) Answer: 8

Question 4

(a) recognise the standard result leading to an arcsin(x/a) form, with a=3 identified **B1**

(a) arcsin(x/3) + c cao **A1**

(a) Answer: arcsin(x/3) + c

(b) substitute x=3/2 and x=0 into their answer from part (a) (ft) **M1**

(b) arcsin(1/2) - arcsin(0) obtained **A1**

(b) = π/6 cso **A1**

(b) Answer: π/6

Question 5

(a) recognise the integrand as (1/2) times the derivative of $\ln(1+x^2)$ (or use the substitution $u=1+x^2$, $du=2x\,dx$) **M1**

(a) $(1/2)*\ln(1+x^2)$ + c cso **A1**

(a) Answer: $(1/2)*\ln(1 + x^2)$ + c

(b) substitute limits $x=\sqrt{3}$ and x=0 into the result from part (a) (ft) **M1**

(b) (1/2)*ln(4) - (1/2)*ln(1) obtained **A1**

(b) k = 1, i.e. ln(2), cao **A1**

(b) Answer: ln(2), i.e. k = 1

(c) substitute limits x=3 and $x=\sqrt{3}$ into the result from part (a) (ft) **M1**

(c) (1/2)*ln(10) - (1/2)*ln(4) obtained **A1**

(c) (1/2)*ln(5/2), i.e. p=5, q=2, cao **A1**

(c) Answer: (1/2)*ln(5/2), i.e. p = 5, q = 2

Question 6

(a) form the identity 8x+7 = A(x+4) + B(x-1) (or use the cover-up method directly) **M1**

(a) A = 3 **A1**

(a) B = 5 **A1**

(a) Answer: f(x) = 3/(x - 1) + 5/(x + 4)

(b) integrate each partial fraction term from (a) to a ln form (ft their A, B) **M1**

(b) 3*ln|x-1| (ft) **A1**

(b) 3*ln|x-1| + 5*ln|x+4| (+c) cao **A1**

(b) Answer: 3*ln|x - 1| + 5*ln|x + 4| + c

(c) substitute limits x=6 and x=2 into their answer from (b) (ft) **M1**

(c) use the laws of logarithms to combine terms, e.g. writing ln(10) and ln(6) in terms of ln(2), ln(3) and ln(5) **M1**

(c) correct simplification to 8*ln(5) - 5*ln(3) (unsimplified intermediate working may be seen) **A1**

(c) p = 8, q = -5 cao **A1**

(c) Answer: 8*ln(5) - 5*ln(3), i.e. p = 8, q = -5

Question 7

(a) uses the standard result for integral of $1/\sqrt{a^2-x^2}$ dx with a=4 **M1**

(a) arcsin(x/4) + c cao **A1**

(a) Answer: arcsin(x/4) + c

(b) substitutes the limits into the result of part (a): arcsin(0.5) - arcsin(0) **M1**

(b) cso: = π/6, as required **A1**

(b) Answer: π/6

Question 8

(a) separates variables and integrates: ln N = t/50 + C **M1**

(a) cso: uses N=200 at t=0 to obtain $N = 200e^{t/50}$ **A1**

(a) Answer: $N = 200e^{t/50}$

(b) sets $200e^{t/50} = 500$ and takes logarithms of both sides **M1**

(b) awrt 45.8 hours **A1**

(b) Answer: awrt 45.8 hours

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