# 1.15 Pure: Integration Depth: 2-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

**Answer ALL questions. Show all your working.**

**1** The curve C has equation $y = -2x^2 - x - 4$ and the line l has equation $y = -15x + 16$. The curve and the line meet at the points A and B.

**(a)** Find the coordinates of A and B. **(3)**

**(b)** Find the area of the finite region bounded by the curve and the line. **(4)**

**(Total for Question 1 is 7 marks)**

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**2** $f(x) = (-6x - 3)/((x - 1)(x + 2))$, $x > 1$.

**(a)** Express f(x) in partial fractions. **(3)**

**(b)** Hence find the exact value of the integral from $x = 2$ to $x = 4$ of $f(x)\,dx$. **(4)**

**(Total for Question 2 is 7 marks)**

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**3** Use integration by parts to answer this question.

**(a)** Find integral of $x\cos(x)\,dx$. **(3)**

**(b)** Hence find the exact value of the integral from $x = 0$ to $x = \pi/2$ of $x\cos(x)\,dx$. **(2)**

**(Total for Question 3 is 5 marks)**

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**4** Use the substitution $u = 4x^2 + 6$ to answer this question.

**(a)** Show that integral of $x(4x^2 + 6)^3\,dx = (4x^2 + 6)^4/32 + c$. **(3)**

**(b)** Hence find the exact value of the integral from $x = 0$ to $x = 2$ of $x(4x^2 + 6)^3\,dx$. **(3)**

**(Total for Question 4 is 6 marks)**

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**5** This question is about integrating $\sin^2(2x)$ with respect to x.

**(a)** Use a double angle formula to show that $\sin^2(2x) = (1 - \cos(4x))/2$. **(2)**

**(b)** Hence find the exact value of the integral from $x = 0$ to $x = \pi/2$ of $\sin^2(2x)\,dx$. **(3)**

**(Total for Question 5 is 5 marks)**


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**6** Use the substitution $u = x^2 + 4$ to answer this question.

**(a)** Show that integral of $x(x^2 + 4)^2\ dx = (x^2 + 4)^3/6 + c$. **(3)**

**(b)** Hence find the exact value of the integral from $x = 0$ to $x = 1$ of $x(x^2 + 4)^2\ dx$. **(3)**

**(Total for Question 6 is 6 marks)**

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**7** This question is about integrating $\cos^2 x$ with respect to x.

**(a)** Use a double angle formula to show that $\cos^2(x) = (1 + \cos(2x))/2$. **(2)**

**(b)** Hence find the exact value of the integral from $x = 0$ to $x = \pi/4$ of $\cos^2(x)\ dx$. **(3)**

**(Total for Question 7 is 5 marks)**

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**8** Find integral of $(\sin(x))^3 * (\cos(x))^2\ dx$.

**(Total for Question 8 is 5 marks)**

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**9** Find the exact value of the definite integral, from $x = 1$ to $x = e$, of $x^2 * \ln(x)\ dx$, giving your answer in the form $(1/9)*(a*e^3 + b)$, where a and b are integers to be found.

**(Total for Question 9 is 6 marks)**

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**10** Find the integral of $4/(x+2)\ dx$, for $x > -2$.

**(Total for Question 10 is 1 mark)**

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**11** Find the exact value of the definite integral, from $x = 1$ to $x = e$, of $1/x\ dx$.

**(Total for Question 11 is 2 marks)**

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**12** Find the integral of $e^{3x}\ dx$.

**(Total for Question 12 is 1 mark)**

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**13** Find the integral of $\cos(4x)\ dx$.

**(Total for Question 13 is 1 mark)**

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**14** Find the integral of $1/x\ dx$, for $x > 0$.

**(Total for Question 14 is 1 mark)**

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**15** Find the integral of $\sec^2(x)\ dx$.

**(Total for Question 15 is 1 mark)**


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**16** Find the exact value of the definite integral, from $x = 0$ to $x = 2$, of $3x^2$ dx.

**(Total for Question 16 is 1 mark)**

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**17** Find the integral of $5e^{2x}$ dx.

**(Total for Question 17 is 1 mark)**

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**18** Find the integral of $8x(x^2+3)^3$ dx.

**(Total for Question 18 is 1 mark)**

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**19** A curve satisfies $dy/dx = 6x^2/y$, where $y > 0$, and passes through the point (0, 3). By separating the variables, find y in terms of x.

**(Total for Question 19 is 2 marks)**

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**20** Using the substitution $u = \sin(x)$, find the integral of $\sin^2(x)\cos(x)$ dx.

**(Total for Question 20 is 2 marks)**

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**21** $f(x) = x\, e^{3x}$.

**(a)** Find the integral of f(x) dx, using integration by parts. **(2)**

**(b)** Hence find the exact value of the definite integral, from $x = 0$ to $x = 1$, of f(x) dx. **(2)**

**(Total for Question 21 is 4 marks)**

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**22** Find the integral of $\sin^2(x)\cos^3(x)$ dx.

**(Total for Question 22 is 3 marks)**

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**23** The curve C has equation $y = 6x - x^2$. The line l has equation $y = 2x$. Find the area of the finite region bounded by C and l.

**(Total for Question 23 is 3 marks)**


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**24** Find integral of $e^{2x}*\sin(x)$ dx. You may use integration by parts twice.

**(Total for Question 24 is 6 marks)**

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**25** The number of members, P, of a newly-launched online chess club is modelled by the differential equation $dP/dt = P(80 - P)/200$, where t is the time in weeks after the club's launch and $0 < P < 80$. When $t = 0$, $P = 10$.

**(a)** Show that $1/(P(80 - P)) = (1/80)*(1/P + 1/(80 - P))$. **(2)**

**(b)** By separating the variables and using the result from part (a), show that the general solution of the differential equation can be written as $\ln(P/(80 - P)) = 0.4*t + c$, where c is an arbitrary constant. **(4)**

**(c)** Given that $P = 10$ when $t = 0$, find the value of c, and hence show that $P = 80/(1 + 7*e^{-0.4*t})$. **(4)**

**(d)** Find the time, in weeks, to 3 significant figures, for the club's membership to reach 40. **(3)**

**(e)** State one limitation of this model for predicting the club's membership over a long period of time. **(1)**

**(Total for Question 25 is 14 marks)**


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# Mark scheme · 1.15 Pure: Integration Depth: 2-hour Pack

## Question 1

(a) equates the expressions for y and rearranges to a quadratic **M1**

(a) x = 2 and x = 5 **A1**

(a) (2, -14) and (5, -59) **A1**

**(a) Answer: A(2, -14), B(5, -59)**

(b) area = integral of (line - curve) or of (curve - line) between the x values **M1**

(b) integrand $2x^2 - 14x + 20$ **A1**

(b) integrates and substitutes the limits **M1**

(b) 9 **A1**

**(b) Answer: 9**

## Question 2

(a) A/(x - p) + B/(x - q) and equating numerators **M1**

(a) A = -3 **A1**

(a) B = -3 **A1**

**(a) Answer: -3/(x - 1) - 3/(x + 2)**

(b) integrates each term to a logarithm: A ln|x - p| + B ln|x - q| **M1**

(b) -3 ln|x - 1| - 3 ln|x + 2| (+ c) **A1**

(b) substitutes the limits and combines logs **M1**

(b) -3 ln(3) - 3 ln(3/2) **A1**

**(b) Answer: -3 ln(3) - 3 ln(3/2)**

## Question 3

(a) u = x, dv/dx = cos(x) **M1**

(a) x sin(x) - integral of sin(x) dx **A1**

(a) x sin(x) + cos(x) + c **A1**

**(a) Answer: x sin(x) + cos(x) + c**

(b) substitutes the limits: sin(π/2) = 1 and cos(π/2) = 0, and cos 0 = 1 **M1**

(b) π/2 - 1 **A1**

**(b) Answer: π/2 - 1**

## Question 4

(a) du/dx = 8x, so x dx = du/8 **M1**

(a) the integral becomes (1/8) integral of $u^3$ du **M1**

(a) = $u^4/32$ + c and substitutes back (cso) **A1**

**(a) Answer: $(4x^2 + 6)^4/32$ + c**

(b) substitutes the limits into the result of part (a) **M1**

(b) $[(22)^4 - 6^4]/32$ **A1**

(b) 7280 **A1**

**(b) Answer: 7280**

## Question 5

(a) $\cos(4x) = 1 - 2\sin^2(2x)$ **M1**

(a) rearranges to the printed identity (cso) **A1**

**(a) Answer: $\sin^2(2x) = (1 - \cos(4x))/2$**

(b) uses the identity and integrates: x/2 - sin(4x)/8 **M1**

(b) substitutes the limits **M1**

(b) π/4 **A1**

**(b) Answer: π/4**

## Question 6

(a) du/dx = 2x, so x dx = du/2 **M1**

(a) the integral becomes (1/2) integral of $u^2$ du **M1**

(a) = $u^3/6$ + c and substitutes back (cso) **A1**

**(a) Answer: $(x^2 + 4)^3/6$ + c**

(b) substitutes the limits into the result of part (a) **M1**

(b) $[(5)^3 - 4^3]/6$ **A1**

(b) 61/6 **A1**

**(b) Answer: 61/6**

## Question 7

(a) $\cos(2x) = 2\cos^2 - 1(x)$ **M1**

(a) rearranges to the printed identity (cso) **A1**

**(a) Answer: $\cos^2(x) = (1 + \cos(2x))/2$**

(b) uses the identity and integrates: x/2 + sin(2x)/4 **M1**

(b) substitutes the limits **M1**

(b) π/8 + 1/4 **A1**

**(b) Answer: π/8 + 1/4**

## Question 8

use $(\sin(x))^2 = 1 - (\cos(x))^2$ to rewrite the integrand as $\sin(x)*((\cos(x))^2 - (\cos(x))^4)$ oe **M1**

substitute u = cos(x), du = -sin(x) dx (oe) **M1**

integrate to the form $u^5/5 - u^3/3$ (or equivalent sign-consistent form) **M1**

correct unsimplified answer in terms of u or x **A1**

$(\cos(x))^5/5 - (\cos(x))^3/3$ + c cao **A1**

**Answer: $(\cos(x))^5/5 - (\cos(x))^3/3$ + c**

## Question 9

attempt integration by parts with u=ln(x), $dv/dx=x^2$ **M1**

$v = x^3/3$ identified (du=(1/x)dx may be implied) **A1**

correct application: $(x^3/3)*\ln(x)$ - integral of $(x^3/3)*(1/x)$ dx **M1**

$(x^3/3)*\ln(x) - x^3/9$ (+c) oe **A1**

substitute limits x=e and x=1, using ln(e)=1 and ln(1)=0 **M1**

$(1/9)*(2*e^3 + 1)$, i.e. a=2 and b=1, cao **A1**

**Answer: $(2e^3 + 1)/9$, i.e. a = 2, b = 1**

## Question 10

4ln(x+2) + c cao **B1**

**Answer: 4ln(x+2) + c**

## Question 11

integrates to [ln x] between 1 and e **M1**

= ln(e) - ln(1) = 1 cao **A1**

**Answer: 1**

## Question 12

$(1/3)e^{3x}$ + c cao **B1**

**Answer: $(1/3)e^{3x}$ + c**

## Question 13

(1/4)sin(4x) + c cao **B1**

**Answer: (1/4)sin(4x) + c**


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## Question 14

ln(x) + c cao **B1**

**Answer: ln(x) + c**

## Question 15

tan(x) + c cao **B1**

**Answer: tan(x) + c**

## Question 16

8 cao **B1**

**Answer: 8**

## Question 17

$(5/2)e^{2x}$ + c cao **B1**

**Answer: $(5/2)e^{2x}$ + c**

## Question 18

$(x^2+3)^4$ + c cao **B1**

**Answer: $(x^2+3)^4$ + c**

## Question 19

separates variables and integrates: $y^2/2 = 2x^3 + C$ **M1**

uses (0,3) to find C = 4.5, giving $y = \sqrt{4x^3+9}$ **A1**

**Answer: $y = \sqrt{4x^3 + 9}$**

## Question 20

substitutes u = sin(x), du = cos(x) dx, to get integral of $u^2$ du **M1**

$= \sin^3(x)/3$ + c cao **A1**

**Answer: $\sin^3(x)/3$ + c**

## Question 21

(a) applies integration by parts with u=x, $dv=e^{3x}dx$ **M1**

(a) $(x/3)e^{3x} - (1/9)e^{3x}$ + c **A1**

**(a) Answer: $(x/3)e^{3x} - (1/9)e^{3x}$ + c**

(b) substitutes the limits x=0 and x=1 into the result of part (a) **M1**

(b) $(2e^3 + 1)/9$ cao (exact) **A1**

**(b) Answer: $(2e^3 + 1)/9$**

## Question 22

rewrites $\cos^3(x) = \cos(x)(1-\sin^2(x))$ and substitutes u = sin(x) **M1**

integrates to $u^3/3 - u^5/5$ **A1**

$= \sin^3(x)/3 - \sin^5(x)/5$ + c cao **A1**

**Answer: $\sin^3(x)/3 - \sin^5(x)/5$ + c**

## Question 23

finds the intersections x=0 and x=4 by solving $6x-x^2=2x$ **M1**

sets up the integral of $(4x-x^2)$ dx between 0 and 4 **M1**

area = 32/3 cao **A1**

**Answer: 32/3**

## Question 24

let I = integral of $e^{2x}*\sin(x)$ dx and apply integration by parts once, e.g. u=sin(x), $dv/dx=e^{2x}$, to reach $I=(1/2)e^{2x}\sin(x)$ - (1/2)*integral of $e^{2x}*\cos(x)$ dx **M1**

apply integration by parts a second time to the integral of $e^{2x}*\cos(x)$ dx, e.g. u=cos(x), $dv/dx=e^{2x}$, to reach $(1/2)e^{2x}\cos(x)$ + (1/2)*integral of $e^{2x}*\sin(x)$ dx **M1**

substitute the second result back to form an equation in I, e.g. $I = (1/2)e^{2x}\sin(x) - (1/4)e^{2x}\cos(x) - (1/4)I$, dependent on both previous method marks **dM1**

collect the I terms and solve for I, e.g. $(5/4)I = (1/2)e^{2x}\sin(x) - (1/4)e^{2x}\cos(x)$ **M1**

correct unsimplified form $I = (2/5)e^{2x}\sin(x) - (1/5)e^{2x}\cos(x)$ (+c) **A1**

$I = (1/5)*e^{2x}*(2*\sin(x) - \cos(x))$ + c, fully factorised, cao **A1**

**Answer: $(1/5)*e^{2x}*(2*\sin(x) - \cos(x))$ + c**

## Question 25

(a) combine the right-hand side over the common denominator 80*P*(80-P), reaching ((80-P)+P)/(80*P*(80-P)) **M1**

(a) simplify the numerator to 80, giving 1/(P(80-P)) cso **A1**

**(a) Answer: 1/(P(80 - P)) = (1/80)*(1/P + 1/(80 - P))**

(b) separate the variables: integral of 1/(P(80-P)) dP = integral of (1/200) dt (oe) **M1**

(b) use part (a) to integrate the left-hand side: (1/80)*(ln(P) - ln(80-P)) **M1**

(b) correct right-hand side integration, t/200 (+c) **A1**

(b) rearrange (multiply both sides by 80) to ln(P/(80-P)) = 0.4*t + c (oe), cso **A1**

**(b) Answer: ln(P/(80 - P)) = 0.4*t + c**

(c) substitute P=10, t=0 into the general solution to obtain c = ln(10/70) = ln(1/7), i.e. c = -ln(7) **M1**

(c) rearrange to the exponential form $P/(80-P) = e^{0.4t - \ln(7)} = (1/7)*e^{0.4t}$ (oe) **M1**

(c) rearrange algebraically to make P the subject, e.g. $7P = (80-P)*e^{0.4t}$, leading to $P*(e^{0.4t}+7) = 80*e^{0.4t}$ **M1**

(c) divide through by $e^{0.4t}$ to reach $P = 80/(1 + 7*e^{-0.4t})$ cso **A1**

**(c) Answer: c = -ln(7); $P = 80/(1 + 7*e^{-0.4*t})$**

(d) set P=40 in the formula from part (c) and rearrange to $7*e^{-0.4t} = 1$ (oe) **M1**

(d) take natural logs correctly to isolate t, e.g. t = ln(7)/0.4 **M1**

(d) awrt 4.86 (weeks) cao **A1**

**(d) Answer: t = ln(7)/0.4 = 4.86 weeks (3 s.f.)**

(e) any valid limitation with brief justification, e.g. the model assumes membership approaches a fixed maximum of 80 and never exceeds it, but real membership could plateau at a different level, fluctuate seasonally, or exceed 80 if demand grows (accept any sensible, justified critique of the constant-capacity assumption) **B1**

**(e) Answer: e.g. the model assumes a fixed maximum membership of 80 that can never be exceeded, which may not reflect real growth if the club expands its capacity or membership fluctuates due to factors the model does not capture**


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