1.15 Pure: Integration Depth: 30-minute Pack

EDEXCEL 9MA0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Use the substitution $u = 3x^2$ to answer this question.

(a) Find integral of $2x\ e^{3x^2}\ dx$. **(3)**

(b) Hence find the exact value of the integral from $x = 0$ to $x = 1$ of $2x\ e^{3x^2}\ dx$. **(2)**

(Total for Question 1 is 5 marks)

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2 The curve C has equation $y = x(x - 3)(x - 9)$. The curve crosses the x-axis at the origin and at the points A(3, 0) and B(9, 0). The finite region R_1 lies between the origin and A, and the finite region R_2 lies between A and B.

(a) Show that $y = x^3 - 12x^2 + 27x$. **(2)**

(b) Find integral of $y\ dx$. **(2)**

(c) Find the exact area of R_1 and the exact area of R_2. **(4)**

(d) Hence find the total area enclosed between the curve and the x-axis from $x = 0$ to $x = 9$. **(2)**

(Total for Question 2 is 10 marks)

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3 Using the substitution $u = 4 + x^2$, find the exact value of the definite integral, from $x = 0$ to $x = \sqrt{5}$, of $x/\sqrt{4 + x^2}\ dx$, showing each step of your working.

(Total for Question 3 is 5 marks)

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4 $f(x) = x * e^{2x}$.

(a) Find integral of $f(x)\ dx$. **(4)**

(b) Hence find the exact value of the definite integral, from $x = 0$ to $x = 1$, of $f(x)\ dx$. **(3)**

(c) Hence, or otherwise, write down the value of the definite integral, from $x = 0$ to $x = 1$, of $f(x)\ dx$, correct to 3 significant figures. **(1)**

(Total for Question 4 is 8 marks)

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Mark scheme · 1.15 Pure: Integration Depth: 30-minute Pack

Question 1

(a) du = 6x dx	**M1**
(a) integral of e^u du x 2/6	**M1**
(a) $1/3e^{3x^2}$ + c	**A1**
(a) Answer: $1/3e^{3x^2}$ + c	
(b) substitutes the limits	**M1**
(b) $1/3(e^3 - 1)$	**A1**
(b) Answer: $1/3(e^3 - 1)$	

Question 2

(a) multiplies out two brackets	**M1**
(a) $x^3 - 12x^2 + 27x$ (cso)	**A1**
(a) Answer: $y = x^3 - 12x^2 + 27x$	
(b) integrates term by term, raising each power by 1	**M1**
(b) $x^4/4 - 4x^3 + 27/2x^2$ (+ c)	**A1**
(b) Answer: $(1/4)x^4 - 4x^3 + (27/2)x^2 + c$	
(c) evaluates the integral between 0 and 3 for R_1	**M1**
(c) area of R_1 = 135/4 (the integral is 135/4)	**A1**
(c) evaluates the integral between 3 and 9 for R_2	**M1**
(c) area of R_2 = 216 (the integral is -216, negative as the curve is below the axis)	**A1**
(c) Answer: $R_1 = 135/4$, $R_2 = 216$	
(d) adds the magnitudes of the two areas	**M1**
(d) 999/4	**A1**
(d) Answer: 999/4	

Question 3

du/dx = 2x, so x dx = (1/2) du (oe)	**M1**
rewrite the integral in terms of u as (1/2) * integral of $u^{-1/2}$ du	**M1**
integrate to obtain $\sqrt{u}$, i.e. $\sqrt{4+x^2}$	**M1**
correctly change/back-substitute the limits: u=9 when x=$\sqrt{5}$, u=4 when x=0 (or equivalent evaluation in x)	**A1**
1 cso	**A1**
Answer: 1	

Question 4

(a) attempt integration by parts with u=x, dv/dx=e^{2x}	**M1**
(a) v = (1/2)e^{2x} identified (du=dx may be implied)	**A1**
(a) correct application: (x/2)*e^{2x} - integral of (1/2)*e^{2x} dx	**M1**
(a) (x/2)*e^{2x} - (1/4)*e^{2x} (+c) oe cao	**A1**
(a) Answer: (x/2)*e^{2x} - (1/4)*e^{2x} + c	
(b) substitute x=1 into their answer from part (a) (ft)	**M1**
(b) substitute x=0 into their answer from part (a) to obtain -1/4 (ft)	**M1**
(b) (e^2 + 1)/4 cao	**A1**
(b) Answer: (e^2 + 1)/4	
(c) awrt 2.10 (ft from part (b))	**B1**
(c) Answer: 2.10 (3 s.f.)	

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