1.15 Pure: Integration Depth: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Find integral of $6x(x^2 + 5)^2$ dx.

(Total for Question 1 is 3 marks)

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2 The curve C has equation $y = x(x - 5)(x - 6)$. The curve crosses the x-axis at the origin and at the points A(5, 0) and B(6, 0). The finite region R_1 lies between the origin and A, and the finite region R_2 lies between A and B.

(a) Show that $y = x^3 - 11x^2 + 30x$. **(2)**

(b) Find integral of y dx. **(2)**

(c) Find the exact area of R_1 and the exact area of R_2. **(4)**

(d) Hence find the total area enclosed between the curve and the x-axis from x = 0 to x = 6. **(2)**

(Total for Question 2 is 10 marks)

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3 A curve satisfies the differential equation $dy/dx = (2x + 1)/y^2$, where $y > 0$. The curve passes through the point (0, 2). Find y in terms of x, giving your answer in the form $y = (f(x))^{1/3}$.

(Total for Question 3 is 5 marks)

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4 The curve C has equation $y = 4x^2 - x^3$. The line l has equation $y = 3x$.

(a) Show that C and l intersect at x = 0, x = 1 and x = 3. **(3)**

(b) Find the area of the finite region bounded by C and l for $1 \leq x \leq 3$. **(6)**

(c) Find the area of the finite region bounded by C and l for $0 \leq x \leq 1$. **(4)**

(Total for Question 4 is 13 marks)

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5 $g(x) = (x^2 + 5x - 5) / ((x - 2)^2 * (x + 1))$.

(a) Express g(x) in the form $A/(x - 2) + B/(x - 2)^2 + C/(x + 1)$, where A, B and C are constants to be found. **(4)**

(b) Hence find integral of g(x) dx. **(4)**

(c) Find the exact value of the definite integral, from x = 3 to x = 4, of g(x) dx, giving your answer in the form a + ln(b), where a is a rational number and b is a fraction in lowest terms. **(3)**

(Total for Question 5 is 11 marks)

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6 $f(x) = (5x - 7) / ((x-3)(x+1))$.

(a) Express f(x) in the form A/(x-3) + B/(x+1), where A and B are constants to be found. **(2)**

(b) Hence find the integral of f(x) dx. **(2)**

(Total for Question 6 is 4 marks)

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Mark scheme · 1.15 Pure: Integration Depth: Test Pack

Question 1

recognise the integrand as a multiple of the derivative of (x^2+5), i.e. attempt the substitution $u=x^2+5$ or reverse chain rule — **M1**

integrate to the form $k(x^2+5)^3$ — **M1**

$(x^2+5)^3 + c$ cao — **A1**

Answer: $(x^2 + 5)^3 + c$

Question 2

(a) multiplies out two brackets — **M1**

(a) $x^3 - 11x^2 + 30x$ (cso) — **A1**

(a) Answer: $y = x^3 - 11x^2 + 30x$

(b) integrates term by term, raising each power by 1 — **M1**

(b) $x^4/4 - 11/3x^3 + 15x^2$ (+ c) — **A1**

(b) Answer: $(1/4)x^4 - (11/3)x^3 + 15x^2 + c$

(c) evaluates the integral between 0 and 5 for R_1 — **M1**

(c) area of R_1 = 875/12 (the integral is 875/12) — **A1**

(c) evaluates the integral between 5 and 6 for R_2 — **M1**

(c) area of R_2 = 11/12 (the integral is -11/12, negative as the curve is below the axis) — **A1**

(c) Answer: $R_1 = 875/12$, $R_2 = 11/12$

(d) adds the magnitudes of the two areas — **M1**

(d) 443/6 — **A1**

(d) Answer: 443/6

Question 3

separate the variables: $y^2\,dy = (2x+1)\,dx$ (oe, condone missing differentials) — **M1**

integrate correctly: $y^3/3 = x^2 + x$ (+c) — **A1**

substitute x=0, y=2 to find the constant, c = 8/3 — **M1**

$y^3 = 3x^2 + 3x + 8$ oe (e.g. multiplying through by 3) — **A1**

$y = (3x^2 + 3x + 8)^{1/3}$ cao — **A1**

Answer: $y = (3x^2 + 3x + 8)^{1/3}$

Question 4

(a) set $4x^2-x^3=3x$ and rearrange to $x^3-4x^2+3x=0$ (oe) — **M1**

(a) factorise to $x(x-1)(x-3)=0$ (or equivalent correct factorisation) — **M1**

(a) x = 0, x = 1 and x = 3 all stated cso — **A1**

(a) Answer: x = 0, x = 1, x = 3

(b) set up the definite integral, from x = 1 to x = 3, of $(4x^2-x^3-3x)\,dx$ — **M1**

(b) integrate to $(4/3)x^3 - (1/4)x^4 - (3/2)x^2$ — **M1**

(b) correct antiderivative, all three terms correct — **A1**

(b) substitute limits x=3 and x=1 into their antiderivative — **M1**

(b) correct evaluation: 9/4 at x=3 and -5/12 at x=1 (or equivalent decimals), seen — **A1**

(b) 8/3 cao — **A1**

(b) Answer: 8/3

(c) recognise that l is above C on this interval and set up the definite integral, from x = 0 to x = 1, of $(3x-4x^2+x^3)\,dx$ (oe, condone use of |curve-line| reasoning) — **M1**

(c) integrate to $(3/2)x^2 - (4/3)x^3 + (1/4)x^4$ — **M1**

(c) correct antiderivative, all three terms correct — **A1**

(c) 5/12 cao — **A1**

(c) Answer: 5/12

Question 5

(a) form the identity $x^2+5x-5 = A(x-2)(x+1) + B(x+1) + C(x-2)^2$ (or use the cover-up method for B and C) — **M1**

(a) B = 3 (from substituting x=2) — **A1**

(a) C = -1 (from substituting x=-1) — **A1**

(a) A = 2 (e.g. by comparing coefficients of x^2), all three values stated — **A1**

(a) Answer: $g(x) = 2/(x - 2) + 3/(x - 2)^2 - 1/(x + 1)$

(b) integrate the $A/(x-2)$ term to $A*\ln|x-2|$ (ft their A) — **M1**

(b) integrate the $B/(x-2)^2$ term to $-B/(x-2)$ (ft their B) — **M1**

(b) integrate the $C/(x+1)$ term to $C*\ln|x+1|$ (ft their C) — **A1**

(b) $2*\ln|x-2| - 3/(x-2) - \ln|x+1|$ (+c) cao — **A1**

(b) Answer: $2*\ln|x - 2| - 3/(x - 2) - \ln|x + 1| + c$

(c) substitute limits x=4 and x=3 into their answer from (b) (ft) — **M1**

(c) use the laws of logarithms to combine the ln terms into a single logarithm — **M1**

(c) 3/2 + ln(16/5) cao (equivalently 3/2 + 4ln(2) - ln(5)) — **A1**

(c) Answer: 3/2 + ln(16/5)

Question 6

(a) sets up $A(x+1) + B(x-3) = 5x-7$ and compares coefficients (or substitutes x=3, x=-1) — **M1**

(a) A = 2, B = 3 — **A1**

(a) Answer: A = 2, B = 3

(b) integrates each partial fraction term (ft from (a)) — **M1**

(b) $2\ln(x-3) + 3\ln(x+1) + c$ cao — **A1**

(b) Answer: $2\ln(x-3) + 3\ln(x+1) + c$

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