1.15 Pure: Integration Depth

EDEXCEL 9MA0 · Calculator allowed · about 235 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The curve C has equation $y = 2x^2 - x$ and the line l has equation $y = 5x$. The curve and the line meet at the points A and B.

(a) Find the coordinates of A and B. **(3)**

(b) Find the area of the finite region bounded by the curve and the line. **(4)**

(Total for Question 1 is 7 marks)

2 Use the substitution $u = 3x^2$ to answer this question.

(a) Find integral of $2x\, e^{3x^2}\, dx$. **(3)**

(b) Hence find the exact value of the integral from $x = 0$ to $x = 1$ of $2x\, e^{3x^2}\, dx$. **(2)**

(Total for Question 2 is 5 marks)

3 The curve C has equation $y = x(x - 3)(x - 9)$. The curve crosses the x-axis at the origin and at the points A(3, 0) and B(9, 0). The finite region R_1 lies between the origin and A, and the finite region R_2 lies between A and B.

(a) Show that $y = x^3 - 12x^2 + 27x$. **(2)**

(b) Find integral of y dx. **(2)**

(c) Find the exact area of R_1 and the exact area of R_2. **(4)**

(d) Hence find the total area enclosed between the curve and the x-axis from $x = 0$ to $x = 9$. **(2)**

(Total for Question 3 is 10 marks)

4 $f(x) = (5x + 13)/((x + 2)(x + 3))$, $x > -2$.

(a) Express $f(x)$ in partial fractions. **(3)**

(b) Hence find the exact value of the integral from $x = 1$ to $x = 5$ of $f(x)\,dx$. **(4)**

(Total for Question 4 is 7 marks)

5 This question is about integrating $\sin^2(3x)$ with respect to x.

(a) Use a double angle formula to show that $\sin^2(3x) = (1 - \cos(6x))/2$. **(2)**

(b) Hence find the exact value of the integral from $x = 0$ to $x = \pi/12$ of $\sin^2(3x)\,dx$. **(3)**

(Total for Question 5 is 5 marks)

6 The curve C has equation $y = -2x^2 - x - 4$ and the line l has equation $y = -15x + 16$. The curve and the line meet at the points A and B.

(a) Find the coordinates of A and B. **(3)**

(b) Find the area of the finite region bounded by the curve and the line. **(4)**

(Total for Question 6 is 7 marks)

7 Using the substitution $u = 4 + x^2$, find the exact value of the definite integral, from $x = 0$ to $x = \sqrt{5}$, of $x/\sqrt{4 + x^2}\ dx$, showing each step of your working.

(Total for Question 7 is 5 marks)

8 $f(x) = (-6x - 3)/((x - 1)(x + 2))$, $x > 1$.

(a) Express $f(x)$ in partial fractions. **(3)**

(b) Hence find the exact value of the integral from $x = 2$ to $x = 4$ of $f(x)\,dx$. **(4)**

(Total for Question 8 is 7 marks)

9 A curve satisfies $x \, dy/dx = y$ for $x > 0$, $y > 0$, and passes through $(1, 2)$.

(a) Find y in terms of x. **(4)**

(b) Find the value of y when $x = 4$. **(2)**

(Total for Question 9 is 6 marks)

10 Use integration by parts to answer this question.

(a) Find integral of $x \cos(x) \, dx$. **(3)**

(b) Hence find the exact value of the integral from $x = 0$ to $x = \pi/2$ of $x \cos(x) \, dx$. **(2)**

(Total for Question 10 is 5 marks)

11 Use the substitution $u = 4x^2 + 6$ to answer this question.

(a) Show that integral of $x(4x^2 + 6)^3\ dx = (4x^2 + 6)^4/32 + c$. **(3)**

(b) Hence find the exact value of the integral from $x = 0$ to $x = 2$ of $x(4x^2 + 6)^3\ dx$. **(3)**

(Total for Question 11 is 6 marks)

12 This question is about integrating $\sin^2(2x)$ with respect to x.

(a) Use a double angle formula to show that $\sin^2(2x) = (1 - \cos(4x))/2$. **(2)**

(b) Hence find the exact value of the integral from $x = 0$ to $x = \pi/2$ of $\sin^2(2x)\ dx$. **(3)**

(Total for Question 12 is 5 marks)

13 Use the substitution $u = x^2 + 4$ to answer this question.

(a) Show that integral of $x(x^2 + 4)^2\ dx = (x^2 + 4)^3/6 + c$. **(3)**

(b) Hence find the exact value of the integral from $x = 0$ to $x = 1$ of $x(x^2 + 4)^2\ dx$. **(3)**

(Total for Question 13 is 6 marks)

14 This question is about integrating $\cos^2 x$ with respect to x.

(a) Use a double angle formula to show that $\cos^2(x) = (1 + \cos(2x))/2$. **(2)**

(b) Hence find the exact value of the integral from $x = 0$ to $x = \pi/4$ of $\cos^2(x)\ dx$. **(3)**

(Total for Question 14 is 5 marks)

15 Find integral of $(\sin(x))^3 * (\cos(x))^2$ dx.

(Total for Question 15 is 5 marks)

16 $f(x) = x * e^{2x}$.

(a) Find integral of f(x) dx. **(4)**

(b) Hence find the exact value of the definite integral, from x = 0 to x = 1, of f(x) dx. **(3)**

(c) Hence, or otherwise, write down the value of the definite integral, from x = 0 to x = 1, of f(x) dx, correct to 3 significant figures. **(1)**

(Total for Question 16 is 8 marks)

17 Find the exact value of the definite integral, from x = 1 to x = e, of $x^2 * \ln(x)\, dx$, giving your answer in the form $(1/9)*(a*e^3 + b)$, where a and b are integers to be found.

(Total for Question 17 is 6 marks)

18 The curve C has equation $y = 1/\sqrt{9 - x^2}$, for $-3 < x < 3$.

(a) Find integral of $1/\sqrt{9 - x^2}$ dx. **(2)**

(b) Hence show that the definite integral, from $x = 0$ to $x = 3/2$, of $1/\sqrt{9 - x^2}$ dx, equals $\pi/6$. **(3)**

(Total for Question 18 is 5 marks)

19 Find the integral of $4/(x+2)$ dx, for $x > -2$.

(Total for Question 19 is 1 mark)

20 Find the exact value of the definite integral, from $x = 1$ to $x = e$, of $1/x$ dx.

(Total for Question 20 is 2 marks)

21 Find the integral of e^{3x} dx.

(Total for Question 21 is 1 mark)

22 Find the integral of cos(4x) dx.

(Total for Question 22 is 1 mark)

23 Find the integral of 1/x dx, for x > 0.

(Total for Question 23 is 1 mark)

24 Find the integral of $\sec^2(x)$ dx.

(Total for Question 24 is 1 mark)

25 Find the exact value of the definite integral, from x = 0 to x = 2, of $3x^2$ dx.

(Total for Question 25 is 1 mark)

26 Find the integral of $5e^{2x}$ dx.

(Total for Question 26 is 1 mark)

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27 The curve C has equation $y = x/(1 + x^2)$, for $x \geq 0$.

(a) Show that integral of $x/(1 + x^2)$ dx = (1/2)*ln$(1 + x^2)$ + c. **(2)**

(b) Hence find the exact area of the region bounded by C, the x-axis and the lines x = 0 and $x = \sqrt{3}$, giving your answer in the form k*ln(2), where k is a constant to be found. **(3)**

(c) Find the exact area of the region bounded by C, the x-axis and the lines $x = \sqrt{3}$ and x = 3, giving your answer in the form (1/2)*ln(p/q), where p and q are integers in lowest terms. **(3)**

(Total for Question 27 is 8 marks)

28 $f(x) = (8x + 7) / ((x - 1)(x + 4))$.

(a) Express $f(x)$ in the form $A/(x - 1) + B/(x + 4)$, where A and B are constants to be found. **(3)**

(b) Hence find integral of $f(x)\,dx$. **(3)**

(c) Find the exact value of the definite integral, from $x = 2$ to $x = 6$, of $f(x)\,dx$, giving your answer in the form $p*\ln(5) + q*\ln(3)$, where p and q are integers to be found. **(4)**

(Total for Question 28 is 10 marks)

29 Find the integral of $8x(x^2+3)^3$ dx.

(Total for Question 29 is 1 mark)

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30 Using the substitution $u = 3 + x^2$, find the integral of $x\sqrt{3+x^2}$ dx.

(Total for Question 30 is 2 marks)

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31 A curve satisfies $dy/dx = 6x^2/y$, where $y > 0$, and passes through the point (0, 3). By separating the variables, find y in terms of x.

(Total for Question 31 is 2 marks)

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32 Using the substitution $u = \sin(x)$, find the integral of $\sin^2(x)\cos(x)$ dx.

(Total for Question 32 is 2 marks)

33 $f(x) = x\,e^{3x}$.

(a) Find the integral of f(x) dx, using integration by parts. **(2)**

(b) Hence find the exact value of the definite integral, from x = 0 to x = 1, of f(x) dx. **(2)**

(Total for Question 33 is 4 marks)

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34 Find the integral of $\sin^2(x)\cos^3(x)$ dx.

(Total for Question 34 is 3 marks)

35 The curve C has equation $y = 1/\sqrt{16-x^2}$, for $-4 < x < 4$.

(a) Find the integral of $1/\sqrt{16-x^2}$ dx. **(2)**

(b) Hence show that the definite integral, from $x = 0$ to $x = 2$, of $1/\sqrt{16-x^2}$ dx, equals $\pi/6$. **(2)**

(Total for Question 35 is 4 marks)

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36 The curve C has equation $y = 6x - x^2$. The line l has equation $y = 2x$. Find the area of the finite region bounded by C and l.

(Total for Question 36 is 3 marks)

37 The number of bacteria, N, in a culture is modelled by the differential equation $dN/dt = N/50$, where t is measured in hours. Initially there are 200 bacteria.

(a) By separating the variables, show that $N = 200\,e^{t/50}$. **(2)**

(b) Find the time taken, in hours to 3 significant figures, for the population to reach 500. **(2)**

(Total for Question 37 is 4 marks)

38 Find the integral of $1/(2x+1)\,dx$, for $x > -1/2$.

(Total for Question 38 is 1 mark)

39 Find the integral of $6x^2(x^3-1)^2$ dx.

(Total for Question 39 is 1 mark)

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40 Find integral of $e^{2x}*\sin(x)$ dx. You may use integration by parts twice.

(Total for Question 40 is 6 marks)

41 The number of members, P, of a newly-launched online chess club is modelled by the differential equation $dP/dt = P(80 - P)/200$, where t is the time in weeks after the club's launch and $0 < P < 80$. When $t = 0$, $P = 10$.

(a) Show that $1/(P(80 - P)) = (1/80)*(1/P + 1/(80 - P))$. **(2)**

(b) By separating the variables and using the result from part (a), show that the general solution of the differential equation can be written as $\ln(P/(80 - P)) = 0.4*t + c$, where c is an arbitrary constant. **(4)**

(c) Given that $P = 10$ when $t = 0$, find the value of c, and hence show that $P = 80/(1 + 7*e^{-0.4*t})$. **(4)**

(d) Find the time, in weeks, to 3 significant figures, for the club's membership to reach 40. **(3)**

(e) State one limitation of this model for predicting the club's membership over a long period of time. **(1)**

(Total for Question 41 is 14 marks)

Mark scheme · 1.15 Pure: Integration Depth

Question 1

(a) equates the expressions for y and rearranges to a quadratic **M1**

(a) x = 0 and x = 3 **A1**

(a) (0, 0) and (3, 15) **A1**

(a) Answer: A(0, 0), B(3, 15)

(b) area = integral of (line - curve) or of (curve - line) between the x values **M1**

(b) integrand $2x^2 - 6x$ **A1**

(b) integrates and substitutes the limits **M1**

(b) 9 **A1**

(b) Answer: 9

Question 2

(a) du = 6x dx **M1**

(a) integral of e^u du x 2/6 **M1**

(a) $1/3e^{3x^2} + c$ **A1**

(a) Answer: $1/3e^{3x^2} + c$

(b) substitutes the limits **M1**

(b) $1/3(e^3 - 1)$ **A1**

(b) Answer: $1/3(e^3 - 1)$

Question 3

(a) multiplies out two brackets **M1**

(a) $x^3 - 12x^2 + 27x$ (cso) **A1**

(a) Answer: $y = x^3 - 12x^2 + 27x$

(b) integrates term by term, raising each power by 1 **M1**

(b) $x^4/4 - 4x^3 + 27/2x^2$ (+ c) **A1**

(b) Answer: $(1/4)x^4 - 4x^3 + (27/2)x^2 + c$

(c) evaluates the integral between 0 and 3 for R_1 **M1**

(c) area of R_1 = 135/4 (the integral is 135/4) **A1**

(c) evaluates the integral between 3 and 9 for R_2 **M1**

(c) area of R_2 = 216 (the integral is -216, negative as the curve is below the axis) **A1**

(c) Answer: $R_1 = 135/4$, $R_2 = 216$

(d) adds the magnitudes of the two areas **M1**

(d) 999/4 **A1**

(d) Answer: 999/4

Question 4

(a) A/(x - p) + B/(x - q) and equating numerators **M1**

(a) A = 3 **A1**

(a) B = 2 **A1**

(a) Answer: 3/(x + 2) + 2/(x + 3)

(b) integrates each term to a logarithm: A ln|x - p| + B ln|x - q| **M1**

(b) 3 ln|x + 2| + 2 ln|x + 3| (+ c) **A1**

(b) substitutes the limits and combines logs **M1**

(b) 3 ln(7/3) + 2 ln(2) **A1**

(b) Answer: 3 ln(7/3) + 2 ln(2)

Question 5

(a) $\cos(6x) = 1 - 2\sin^2(3x)$ **M1**

(a) rearranges to the printed identity (cso) **A1**

(a) Answer: $\sin^2(3x) = (1 - \cos(6x))/2$

(b) uses the identity and integrates: x/2 - sin(6x)/12 **M1**

(b) substitutes the limits **M1**

(b) π/24 - 1/12 **A1**

(b) Answer: π/24 - 1/12

Question 6

(a) equates the expressions for y and rearranges to a quadratic **M1**

(a) x = 2 and x = 5 **A1**

(a) (2, -14) and (5, -59) **A1**

(a) Answer: A(2, -14), B(5, -59)

(b) area = integral of (line - curve) or of (curve - line) between the x values **M1**

(b) integrand $2x^2 - 14x + 20$ **A1**

(b) integrates and substitutes the limits **M1**

(b) 9 **A1**

(b) Answer: 9

Question 7

du/dx = 2x, so x dx = (1/2) du (oe) **M1**

rewrite the integral in terms of u as (1/2) * integral of $u^{-1/2}$ du **M1**

integrate to obtain $\sqrt{u}$, i.e. $\sqrt{4+x^2}$ **M1**

correctly change/back-substitute the limits: u=9 when x=$\sqrt{5}$, u=4 when x=0 (or equivalent evaluation in x) **A1**

1 cso **A1**

Answer: 1

Question 8

(a) A/(x - p) + B/(x - q) and equating numerators **M1**

(a) A = -3 **A1**

(a) B = -3 **A1**

(a) Answer: -3/(x - 1) - 3/(x + 2)

(b) integrates each term to a logarithm: A ln|x - p| + B ln|x - q| **M1**

(b) -3 ln|x - 1| - 3 ln|x + 2| (+ c) **A1**

(b) substitutes the limits and combines logs **M1**

(b) -3 ln(3) - 3 ln(3/2) **A1**

(b) Answer: -3 ln(3) - 3 ln(3/2)

Question 9

(a) separates variables: integral of 1/y dy = integral of 1/x dx **M1**

(a) ln y = 1 ln x + c **A1**

(a) uses (1, 2): c = ln 2 **M1**

(a) y = 2x **A1**

(a) Answer: y = 2x

(b) substitutes x = 4 **M1**

(b) 8 **A1**

(b) Answer: 8

Question 10

(a) u = x, dv/dx = cos(x) **M1**

(a) x sin(x) - integral of sin(x) dx **A1**

(a) x sin(x) + cos(x) + c **A1**

(a) Answer: x sin(x) + cos(x) + c

(b) substitutes the limits: sin(π/2) = 1 and cos(π/2) = 0, and cos 0 = 1 **M1**

(b) π/2 - 1 **A1**

(b) Answer: π/2 - 1

Question 11

(a) du/dx = 8x, so x dx = du/8 **M1**

(a) the integral becomes (1/8) integral of u^3 du **M1**

(a) = $u^4/32$ + c and substitutes back (cso) **A1**

(a) Answer: $(4x^2 + 6)^4/32$ + c

(b) substitutes the limits into the result of part (a) **M1**

(b) $[(22)^4 - 6^4]/32$ **A1**

(b) 7280 **A1**

(b) Answer: 7280

Question 12

(a) $\cos(4x) = 1 - 2\sin^2(2x)$ **M1**

(a) rearranges to the printed identity (cso) **A1**

(a) Answer: $\sin^2(2x) = (1 - \cos(4x))/2$

(b) uses the identity and integrates: x/2 - sin(4x)/8 **M1**

(b) substitutes the limits **M1**

(b) π/4 **A1**

(b) Answer: π/4

Question 13

(a) du/dx = 2x, so x dx = du/2 **M1**

(a) the integral becomes (1/2) integral of u^2 du **M1**

(a) = $u^3/6$ + c and substitutes back (cso) **A1**

(a) Answer: $(x^2 + 4)^3/6$ + c

(b) substitutes the limits into the result of part (a) **M1**

(b) $[(5)^3 - 4^3]/6$ **A1**

(b) 61/6 **A1**

(b) Answer: 61/6

Question 14

(a) $\cos(2x) = 2\cos^2 - 1(x)$ **M1**

(a) rearranges to the printed identity (cso) **A1**

(a) Answer: $\cos^2(x) = (1 + \cos(2x))/2$

(b) uses the identity and integrates: x/2 + sin(2x)/4 **M1**

(b) substitutes the limits **M1**

(b) π/8 + 1/4 **A1**

(b) Answer: π/8 + 1/4

Question 15

use $(\sin(x))^2 = 1 - (\cos(x))^2$ to rewrite the integrand as $\sin(x)*((\cos(x))^2 - (\cos(x))^4)$ oe **M1**

substitute u = cos(x), du = -sin(x) dx (oe) **M1**

integrate to the form $u^5/5 - u^3/3$ (or equivalent sign-consistent form) **M1**

correct unsimplified answer in terms of u or x **A1**

$(\cos(x))^5/5 - (\cos(x))^3/3$ + c cao **A1**

Answer: $(\cos(x))^5/5 - (\cos(x))^3/3$ + c

Question 16

(a) attempt integration by parts with u=x, dv/dx=e^{2x} **M1**

(a) v = $(1/2)e^{2x}$ identified (du=dx may be implied) **A1**

(a) correct application: $(x/2)*e^{2x}$ - integral of $(1/2)*e^{2x}$ dx **M1**

(a) $(x/2)*e^{2x} - (1/4)*e^{2x}$ (+c) oe cao **A1**

(a) Answer: $(x/2)*e^{2x} - (1/4)*e^{2x}$ + c

(b) substitute x=1 into their answer from part (a) (ft) **M1**

(b) substitute x=0 into their answer from part (a) to obtain -1/4 (ft) **M1**

(b) $(e^2 + 1)/4$ cao **A1**

(b) Answer: $(e^2 + 1)/4$

(c) awrt 2.10 (ft from part (b)) **B1**

(c) Answer: 2.10 (3 s.f.)

Question 17

attempt integration by parts with u=ln(x), dv/dx=x^2 **M1**

v = $x^3/3$ identified (du=(1/x)dx may be implied) **A1**

correct application: $(x^3/3)*\ln(x)$ - integral of $(x^3/3)*(1/x)$ dx **M1**

$(x^3/3)*\ln(x) - x^3/9$ (+c) oe **A1**

substitute limits x=e and x=1, using ln(e)=1 and ln(1)=0 **M1**

(1/9)*(2*e^3 + 1), i.e. a=2 and b=1, cao **A1**

Answer: $(2e^3 + 1)/9$, i.e. a = 2, b = 1

Question 18

(a) recognise the standard result leading to an arcsin(x/a) form, with a=3 identified **B1**

(a) arcsin(x/3) + c cao **A1**

(a) Answer: arcsin(x/3) + c

(b) substitute x=3/2 and x=0 into their answer from part (a) (ft) **M1**

(b) arcsin(1/2) - arcsin(0) obtained **A1**

(b) = π/6 cso **A1**

(b) Answer: π/6

Question 19

4ln(x+2) + c cao **B1**

Answer: 4ln(x+2) + c

Question 20

integrates to [ln x] between 1 and e **M1**

= ln(e) - ln(1) = 1 cao **A1**

Answer: 1

Question 21

$(1/3)e^{3x}$ + c cao **B1**

Answer: $(1/3)e^{3x}$ + c

Question 22

(1/4)sin(4x) + c cao **B1**

Answer: (1/4)sin(4x) + c

Question 23

ln(x) + c cao **B1**

Answer: ln(x) + c

Question 24

tan(x) + c cao **B1**

Answer: tan(x) + c

Question 25

8 cao **B1**

Answer: 8

Question 26

$(5/2)e^{2x}$ + c cao **B1**

Answer: $(5/2)e^{2x}$ + c

Question 27

(a) recognise the integrand as (1/2) times the derivative of $\ln(1+x^2)$ (or use the substitution $u=1+x^2$, $du=2x\,dx$) **M1**

(a) $(1/2)*\ln(1+x^2)$ + c cso **A1**

(a) Answer: $(1/2)*\ln(1 + x^2)$ + c

(b) substitute limits $x=\sqrt{3}$ and x=0 into the result from part (a) (ft) **M1**

(b) $(1/2)*\ln(4) - (1/2)*\ln(1)$ obtained **A1**

(b) k = 1, i.e. ln(2), cao **A1**

(b) Answer: ln(2), i.e. k = 1

(c) substitute limits x=3 and $x=\sqrt{3}$ into the result from part (a) (ft) **M1**

(c) $(1/2)*\ln(10) - (1/2)*\ln(4)$ obtained **A1**

(c) $(1/2)*\ln(5/2)$, i.e. p=5, q=2, cao **A1**

(c) Answer: $(1/2)*\ln(5/2)$, i.e. p = 5, q = 2

Question 28

(a) form the identity 8x+7 = A(x+4) + B(x-1) (or use the cover-up method directly) **M1**

(a) A = 3 **A1**

(a) B = 5 **A1**

(a) Answer: f(x) = 3/(x - 1) + 5/(x + 4)

(b) integrate each partial fraction term from (a) to a ln form (ft their A, B) **M1**

(b) $3*\ln|x-1|$ (ft) **A1**

(b) $3*\ln|x-1| + 5*\ln|x+4|$ (+c) cao **A1**

(b) Answer: $3*\ln|x - 1| + 5*\ln|x + 4|$ + c

(c) substitute limits x=6 and x=2 into their answer from (b) (ft) **M1**

(c) use the laws of logarithms to combine terms, e.g. writing ln(10) and ln(6) in terms of ln(2), ln(3) and ln(5) **M1**

(c) correct simplification to $8*\ln(5) - 5*\ln(3)$ (unsimplified intermediate working may be seen) **A1**

(c) p = 8, q = -5 cao **A1**

(c) Answer: $8*\ln(5) - 5*\ln(3)$, i.e. p = 8, q = -5

Question 29

$(x^2+3)^4$ + c cao **B1**

Answer: $(x^2+3)^4$ + c

Question 30

substitutes $u = 3+x^2$, $du = 2x\,dx$, to get integral of $(1/2)\sqrt{u}\,du$ **M1**

$= (1/3)(3+x^2)^{3/2}$ + c cao **A1**

Answer: $(1/3)(3+x^2)^{3/2}$ + c

Question 31

separates variables and integrates: $y^2/2 = 2x^3 + C$ **M1**

uses (0,3) to find C = 4.5, giving $y = \sqrt{4x^3+9}$ **A1**

Answer: $y = \sqrt{4x^3 + 9}$

Question 32

substitutes u = sin(x), du = cos(x) dx, to get integral of u^2 du **M1**

$= \sin^3(x)/3$ + c cao **A1**

Answer: $\sin^3(x)/3$ + c

Question 33

(a) applies integration by parts with u=x, $dv=e^{3x}dx$ **M1**

(a) $(x/3)e^{3x} - (1/9)e^{3x}$ + c **A1**

(a) Answer: $(x/3)e^{3x} - (1/9)e^{3x}$ + c

(b) substitutes the limits x=0 and x=1 into the result of part (a) **M1**

(b) $(2e^3 + 1)/9$ cao (exact) **A1**

(b) Answer: $(2e^3 + 1)/9$

Question 34

rewrites $\cos^3(x) = \cos(x)(1-\sin^2(x))$ and substitutes u = sin(x) **M1**

integrates to $u^3/3 - u^5/5$ **A1**

$= \sin^3(x)/3 - \sin^5(x)/5$ + c cao **A1**

Answer: $\sin^3(x)/3 - \sin^5(x)/5$ + c

Question 35

(a) uses the standard result for integral of $1/\sqrt{a^2-x^2}$ dx with a=4 **M1**

(a) arcsin(x/4) + c cao **A1**

(a) Answer: arcsin(x/4) + c

(b) substitutes the limits into the result of part (a): arcsin(0.5) - arcsin(0) **M1**

(b) cso: = π/6, as required **A1**

(b) Answer: π/6

Question 36

finds the intersections x=0 and x=4 by solving $6x-x^2=2x$ **M1**

sets up the integral of $(4x-x^2)$ dx between 0 and 4 **M1**

area = 32/3 cao **A1**

Answer: 32/3

Question 37

(a) separates variables and integrates: ln N = t/50 + C **M1**

(a) cso: uses N=200 at t=0 to obtain $N = 200e^{t/50}$ **A1**

(a) Answer: $N = 200e^{t/50}$

(b) sets $200e^{t/50} = 500$ and takes logarithms of both sides **M1**

(b) awrt 45.8 hours **A1**

(b) Answer: awrt 45.8 hours

Question 38

(1/2)ln(2x+1) + c cao **B1**

Answer: (1/2)ln(2x+1) + c

Question 39

$(2/3)(x^3-1)^3$ + c cao **B1**

Answer: $(2/3)(x^3-1)^3$ + c

Question 40

let I = integral of $e^{2x}*\sin(x)$ dx and apply integration by parts once, e.g. u=sin(x), dv/dx=e^{2x}, to reach I=(1/2)e^{2x}sin(x) - (1/2)*integral of $e^{2x}*\cos(x)$ dx **M1**

apply integration by parts a second time to the integral of $e^{2x}*\cos(x)$ dx, e.g. u=cos(x), dv/dx=e^{2x}, to reach (1/2)e^{2x}cos(x) + (1/2)*integral of $e^{2x}*\sin(x)$ dx **M1**

substitute the second result back to form an equation in I, e.g. I = (1/2)e^{2x}sin(x) - (1/4)e^{2x}cos(x) - (1/4)I, dependent on both previous method marks **dM1**

collect the I terms and solve for I, e.g. (5/4)I = (1/2)e^{2x}sin(x) - (1/4)e^{2x}cos(x) **M1**

correct unsimplified form I = (2/5)e^{2x}sin(x) - (1/5)e^{2x}cos(x) (+c) **A1**

I = (1/5)*e^{2x}*(2*sin(x) - cos(x)) + c, fully factorised, cao **A1**

Answer: (1/5)*e^{2x}*(2*sin(x) - cos(x)) + c

Question 41

(a) combine the right-hand side over the common denominator 80*P*(80-P), reaching ((80-P)+P)/(80*P*(80-P)) **M1**

(a) simplify the numerator to 80, giving 1/(P(80-P)) cso **A1**

(a) Answer: 1/(P(80 - P)) = (1/80)*(1/P + 1/(80 - P))

(b) separate the variables: integral of 1/(P(80-P)) dP = integral of (1/200) dt (oe) **M1**

(b) use part (a) to integrate the left-hand side: (1/80)*(ln(P) - ln(80-P)) **M1**

(b) correct right-hand side integration, t/200 (+c) **A1**

(b) rearrange (multiply both sides by 80) to ln(P/(80-P)) = 0.4*t + c (oe), cso **A1**

(b) Answer: ln(P/(80 - P)) = 0.4*t + c

(c) substitute P=10, t=0 into the general solution to obtain c = ln(10/70) = ln(1/7), i.e. c = -ln(7) **M1**

(c) rearrange to the exponential form P/(80-P) = $e^{0.4t - \ln(7)}$ = (1/7)*$e^{0.4t}$ (oe) **M1**

(c) rearrange algebraically to make P the subject, e.g. 7P = (80-P)*$e^{0.4t}$, leading to P*($e^{0.4t}$+7) = 80*$e^{0.4t}$ **M1**

(c) divide through by $e^{0.4t}$ to reach P = 80/(1 + 7*$e^{-0.4t}$) cso **A1**

(c) Answer: c = -ln(7); P = 80/(1 + 7*$e^{-0.4*t}$)

(d) set P=40 in the formula from part (c) and rearrange to 7*$e^{-0.4t}$ = 1 (oe) **M1**

(d) take natural logs correctly to isolate t, e.g. t = ln(7)/0.4 **M1**

(d) awrt 4.86 (weeks) cao **A1**

(d) Answer: t = ln(7)/0.4 = 4.86 weeks (3 s.f.)

(e) any valid limitation with brief justification, e.g. the model assumes membership approaches a fixed maximum of 80 and never exceeds it, but real membership could plateau at a different level, fluctuate seasonally, or exceed 80 if demand grows (accept any sensible, justified critique of the constant-capacity assumption) **B1**

(e) Answer: e.g. the model assumes a fixed maximum membership of 80 that can never be exceeded, which may not reflect real growth if the club expands its capacity or membership fluctuates due to factors the model does not capture