1.16 Pure: Vectors and Numerical Methods Depth: 1-hour Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Relative to a fixed origin O, the points A, B and C have position vectors OA = -i - 3k, OB = 3i - 3j and OC = -2i + j + k.

(a) Find the exact lengths of the sides |AB|, |AC| and |BC| of triangle ABC. **(3)**

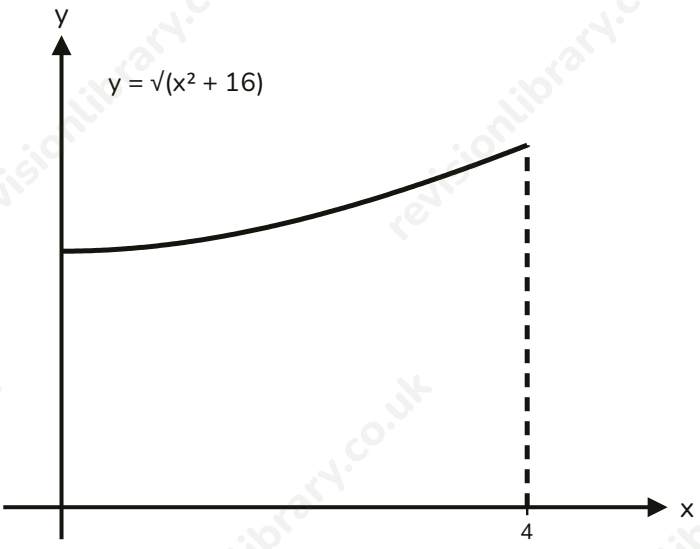
(b) Find the size of angle BAC to 1 decimal place. **(3)**

(c) Find the area of triangle ABC to 3 significant figures. **(2)**

(Total for Question 1 is 8 marks)

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2 The curve C has equation $y = \sqrt{x^2 + 16}$, for $0 \leq x \leq 4$.



(a) Using the trapezium rule with 4 strips of equal width, find an estimate for the integral from x=0 to x=4 of $\sqrt{x^2+16}$ dx, giving your answer to 3 decimal places. You should show the value of y at each of the 5 values of x used. **(5)**

(b) State, with a reason, whether your estimate in part (a) is an overestimate or an underestimate of the true value of the integral. **(2)**

(Total for Question 2 is 7 marks)

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3 $g(x) = e^x - 3x - 2$. The equation $g(x) = 0$ has a root α in the interval [2,3].

(a) Show that α lies between $x = 2.1$ and $x = 2.2$. **(2)**

(b) By further systematic decimal search (or otherwise), show that $\alpha = 2.13$, correct to 2 decimal places. **(3)**

(Total for Question 3 is 5 marks)

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4 State the formula for the scalar (dot) product of vectors $a = a1\ i + a2\ j + a3\ k$ and $b = b1\ i + b2\ j + b3\ k$.

(Total for Question 4 is 1 mark)

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5 $f(x) = x^3 - 2x - 5$. Given that $f(2) = -1$ and $f(3) = 16$, state whether $f(x) = 0$ has a root between $x = 2$ and $x = 3$, with a reason.

(Total for Question 5 is 1 mark)

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6 The line l has vector equation $r = t(i + 2j + 2k)$, where t is a scalar parameter. The point P has position vector $8i + j + 4k$.

(a) Find the position vector of N, the foot of the perpendicular from P to l. **(6)**

(b) Find the distance PN, giving your answer in the form $k*\sqrt{5}$, where k is an integer. **(3)**

(c) The point P' is the reflection of P in the line l. Find the position vector of P'. **(3)**

(Total for Question 6 is 12 marks)

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7 $g(x) = x^3 + 2x - 6$. The equation $g(x) = 0$ has a root α in the interval [1,2].

(a) Using $x0 = 1.5$, apply the Newton-Raphson method once to g(x) to find x1, giving your answer to 4 decimal places. **(2)**

(b) Apply the Newton-Raphson method a second time to find x2, giving your answer to 4 decimal places. **(2)**

(Total for Question 7 is 4 marks)

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8 J = the definite integral, from x = 0 to x = 1, of 1/(1+x) dx. The exact value of J is ln(2).

(a) Using the trapezium rule with 2 strips of equal width, find an estimate for J, giving your answer to 3 decimal places. **(2)**

(b) Given that the exact value of J is ln(2), find the percentage error in your estimate from part (a), giving your answer to 2 significant figures. **(1)**

(Total for Question 8 is 3 marks)

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9 A drone follows a path such that its position vector, in km, relative to a fixed point O, at time t hours ($t \geq 0$) is given by $r(t) = t\,i + t^2\,j + t\,k$. A ground station is located at the point P with position vector $4i + 2j + 5k$.

(a) Show that the square of the distance between the drone and the ground station at time t is given by $D(t) = t^4 - 2t^2 - 18t + 45$. **(4)**

(b) Show that the value of t at which the distance is a minimum satisfies $2t^3 - 2t - 9 = 0$. **(3)**

(c) Show that this equation has a root between t = 1 and t = 2. **(2)**

(d) Taking $t0 = 1.85$ as a first approximation, use the Newton-Raphson method once to find an improved estimate t1 for the root, giving your answer to 4 decimal places. **(4)**

(Total for Question 9 is 13 marks)

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Mark scheme · 1.16 Pure: Vectors and Numerical Methods Depth: 1-hour Pack

Question 1

(a) finds AB, AC and BC by subtracting position vectors **M1**

(a) $|AB| = \sqrt{34}$ **A1**

(a) $|AC| = 3\sqrt{2}$, $|BC| = \sqrt{42}$ **A1**

(a) Answer: $|AB| = \sqrt{34}$, $|AC| = 3\sqrt{2}$, $|BC| = \sqrt{42}$

(b) cosine rule: $\cos A = (AB^2 + AC^2 - BC^2)/(2\,|AB||AC|)$ **M1**

(b) $\cos A = (34 + 18 - 42)/(2 \times 5.831 \times 4.243) = 0.2021$ **A1**

(b) angle BAC = 78.3 degrees **A1**

(b) Answer: 78.3 degrees

(c) $(1/2)|AB||AC| \sin A$ **M1**

(c) 12.1 **A1**

(c) Answer: 12.1

Question 2

(a) correct strip width h = 1 **M1**

(a) attempts at least 3 correct y-values (y0=4, y1=4.1231, y2=4.4721, y3=5, y4=5.6569, or ft) **M1**

(a) all 5 y-values correct awrt 4dp **A1**

(a) correct trapezium structure $(h/2)[y0+y4+2(y1+y2+y3)]$ **M1**

(a) estimate = 18.424 awrt **A1**

(a) Answer: Integral (approx) = 18.424

(b) states overestimate **B1**

(b) valid reason referencing the shape of the curve, e.g. $y = \sqrt{x^2+16}$ is concave up (convex) on [0,4], so the chords used by the trapezium rule lie above the curve **B1**

(b) Answer: Overestimate, because the curve is concave up (convex) over [0,4], so the trapezium chords lie above the curve.

Question 3

(a) evaluates g(2.1) and g(2.2) **M1**

(a) g(2.1) = -0.1338 (<0), g(2.2) = 0.4250 (>0); sign change confirms root in (2.1,2.2) cso **A1**

(a) Answer: g(2.1) = -0.1338, g(2.2) = 0.4250; sign change confirms root in (2.1,2.2)

(b) evaluates g(2.125) and g(2.135), or an equivalent bounding interval for 2.13 **M1**

(b) g(2.125) = -0.0021 (awrt, negative) and g(2.135) = 0.0520 (awrt, positive), both correct **A1**

(b) sign change on (2.125,2.135), g continuous, hence α = 2.13 correct to 2dp cso **A1**

(b) Answer: α = 2.13 (2dp)

Question 4

a.b = a1 b1 + a2 b2 + a3 b3 **B1**

Answer: a.b = a1b1 + a2b2 + a3b3

Question 5

yes, since f changes sign (from negative to positive) and f is continuous **B1**

Answer: Yes, by the change of sign

Question 6

(a) writes a general point on l as (t, 2t, 2t) **M1**

(a) forms the vector from P to this general point (or vice versa) in terms of t **M1**

(a) sets the dot product of this vector with the direction vector (1,2,2) equal to zero **M1**

(a) correct equation 9t - 18 = 0 oe **A1**

(a) t = 2 **A1**

(a) N = 2i + 4j + 4k **A1**

(a) Answer: N = 2i + 4j + 4k

(b) forms vector PN = N - P **M1**

(b) attempts magnitude $\sqrt{(-6)^2+3^2+0^2}$ **M1**

(b) PN = $3\sqrt{5}$ cso, exact form required **A1**

(b) Answer: PN = $3\sqrt{5}$

(c) recognises that N is the midpoint of P and P' **M1**

(c) correct method P' = 2N - P, or equivalent **M1**

(c) P' = -4i + 7j + 4k **A1**

(c) Answer: P' = -4i + 7j + 4k

Question 7

(a) uses $g'(x)=3x^2+2$ and substitutes x0=1.5 into the Newton-Raphson formula **M1**

(a) x1 = 1.4571 cao **A1**

(a) Answer: x1 = 1.4571

(b) applies the formula again using x1 from (a) (ft) **M1**

(b) x2 = 1.4562 cao (ft) **A1**

(b) Answer: x2 = 1.4562

Question 8

(a) uses the three correct ordinates 1, 2/3, 0.5 with strip width 0.5 **M1**

(a) estimate = 0.708 (3dp) **A1**

(a) Answer: 0.708

(b) awrt 2.2% (ft from (a)) **B1**

(b) Answer: awrt 2.2%

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Question 9

(a) forms the vector from P to the drone (or vice versa): $(t-4, t^2-2, t-5)$ **M1**

(a) squares and sums the three components **M1**

(a) correct unsimplified expression $(t-4)^2+(t^2-2)^2+(t-5)^2$ **A1**

(a) simplifies to $D(t) = t^4-2t^2-18t+45$ cso **A1**

(a) Answer: $D(t) = t^4 - 2t^2 - 18t + 45$

(b) differentiates D(t): $D'(t) = 4t^3-4t-18$ **M1**

(b) sets $D'(t) = 0$ **M1**

(b) divides by 2 to obtain $2t^3-2t-9 = 0$ cso **A1**

(b) Answer: $2t^3 - 2t - 9 = 0$

(c) evaluates $g(1) = 2(1)-2(1)-9$ and $g(2) = 2(8)-2(2)-9$ **M1**

(c) sign change ($g(1) = -9 < 0$, $g(2) = 3 > 0$), g continuous, hence root in (1,2) cso **A1**

(c) Answer: g(1) = -9, g(2) = 3; sign change confirms a root between t=1 and t=2

(d) correct derivative $g'(t) = 6t^2-2$ **M1**

(d) correct Newton-Raphson substitution using $g(1.85) = -0.0368$ and $g'(1.85) = 18.535$ **M1**

(d) correct unrounded value t1 = 1.85198... **A1**

(d) t1 = 1.8520 awrt **A1**

(d) Answer: t1 = 1.8520

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