1.16 Pure: Vectors and Numerical Methods Depth: Test Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 65 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Relative to a fixed origin O, a = 2i - j + 2k and b = i + 2j - 2k.

(a) Find |a|. **(2)**

(b) Find the vector c = a + 2b, giving your answer in terms of i, j and k. **(2)**

(c) Find |c|, giving your answer to 2 decimal places. **(2)**

(d) Find the angle between a and b, giving your answer in degrees to 1 decimal place. **(3)**

(Total for Question 1 is 9 marks)

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2 $f(x) = x^2 - 7$. Find $f'(x)$.

(Total for Question 2 is 1 mark)

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3 I = integral from x=0 to x=1 of $1/(1+x^2)$ dx.

(a) Using the trapezium rule with 2 strips of equal width, find an estimate for I, giving your answer to 3 decimal places. **(4)**

(b) Using the trapezium rule with 4 strips of equal width, find a second estimate for I, giving your answer to 4 decimal places. **(4)**

(c) Given that the exact value of I is π/4, find the percentage error in each of your estimates from parts (a) and (b), giving your answers to 2 significant figures. **(3)**

(d) Comment on how the percentage error changes as the number of strips is doubled, with reference to your answers to part (c). **(1)**

(Total for Question 3 is 12 marks)

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4 $f(x) = 2x^3 - 9x^2 + 5$.

(a) Show that $x0 = 3$ is not a suitable first approximation for solving $f(x) = 0$ using the Newton-Raphson method, explaining what goes wrong. **(3)**

(b) Using $x0 = 4.4$ instead, apply the Newton-Raphson method once to find $x1$, giving your answer to 4 decimal places. **(3)**

(Total for Question 4 is 6 marks)

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5 Show that the equation $x^3 - 5x - 2 = 0$ has a root between $x = 2$ and $x = 3$.

(Total for Question 5 is 2 marks)

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6 $f(x) = x^3 - 4x - 1$. Evaluate $f(2)$ and $f(3)$.

(Total for Question 6 is 1 mark)

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7 State the Newton-Raphson iteration formula for finding an improved estimate x_{n+1} for a root of $f(x) = 0$.

(Total for Question 7 is 1 mark)

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8 Lines l1 and l2 have vector equations

l1: $\mathbf{r} = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + t(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

l2: $\mathbf{r} = (5\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + s(-2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$

where s and t are scalar parameters.

(a) Show that l1 and l2 intersect, and find the position vector of their point of intersection. **(6)**

(b) Find the angle between l1 and l2, giving your answer in degrees to 1 decimal place. **(4)**

(Total for Question 8 is 10 marks)

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9 $f(x) = x^3 - 5x - 2$. Using $x0 = 3$, apply the Newton-Raphson method once to $f(x)$ to find $x1$, giving your answer to 4 decimal places.

(Total for Question 9 is 2 marks)

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10 Using the iteration formula $x_{n+1} = 4 - 2/x_n^2$, with x0 = 3.7, find the values of x1, x2 and x3, each to 4 decimal places, and hence state an estimate for the root of $x^3 - 4x^2 + 2 = 0$, to 2 decimal places.

(Total for Question 10 is 4 marks)

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11 $h(x) = e^x - 4x$. Show that h(x) = 0 has a root between x = 0.3 and x = 0.4. By further decimal search, show that this root is x = 0.36, correct to 2 decimal places.

(Total for Question 11 is 3 marks)

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Mark scheme · 1.16 Pure: Vectors and Numerical Methods Depth: Test Pack

Question 1

(a) attempts $2^2 + (-1)^2 + 2^2$ **M1**

(a) $|a| = 3$ cao **A1**

(a) Answer: $|a| = 3$

(b) attempts 2b = 2i + 4j - 4k, or the full component sum a + 2b **M1**

(b) c = 4i + 3j - 2k oe **A1**

(b) Answer: c = 4i + 3j - 2k

(c) attempts $4^2 + 3^2 + (-2)^2$ **M1**

(c) $|c| = \sqrt{29} = 5.39$ awrt **A1**

(c) Answer: $|c| = \sqrt{29}$, awrt 5.39

(d) attempts a.b = (2)(1) + (-1)(2) + (2)(-2) **M1**

(d) attempts $\cos(\theta) = (a.b)/(|a||b|)$ with correct |a| and |b| **M1**

(d) θ = 116.4 degrees awrt **A1**

(d) Answer: θ = awrt 116.4 degrees

Question 2

2x cao **B1**

Answer: 2x

Question 3

(a) correct strip width h = 0.5 **M1**

(a) correct y-values y0=1, y1=0.8 (at x=0.5), y2=0.5 **M1**

(a) all y-values correct **A1**

(a) estimate = 0.775 using (h/2)[y0+y2+2y1] **A1**

(a) Answer: Estimate = 0.775

(b) correct strip width h = 0.25 **M1**

(b) at least 3 correct y-values (1, 0.9412, 0.8, 0.64, 0.5) **M1**

(b) all 5 y-values correct **A1**

(b) estimate = 0.7828 awrt **A1**

(b) Answer: Estimate = 0.7828

(c) correct method: percentage error = |estimate - π/4|/(π/4) x 100 **M1**

(c) 2-strip percentage error = 1.3% awrt **A1**

(c) 4-strip percentage error = 0.33% awrt **A1**

(c) Answer: 2-strip error = awrt 1.3%; 4-strip error = awrt 0.33%

(d) valid comment, e.g. doubling the number of strips (halving h) reduces the percentage error by a factor of about 4 (from 1.3% to 0.33%), consistent with the trapezium rule's error decreasing in proportion to h^2 **B1**

(d) Answer: The error falls by a factor of about 4 when the number of strips doubles (1.3% to 0.33%), consistent with trapezium-rule error being proportional to h^2.

Question 4

(a) finds $f'(x) = 6x^2 - 18x$ **M1**

(a) evaluates f'(3) = 54 - 54 = 0 **M1**

(a) valid explanation: since f'(3) = 0, the tangent to the curve at x0 = 3 is horizontal, so the Newton-Raphson formula requires dividing by f'(x0) = 0, which is undefined; x1 cannot be found (geometrically, the tangent never meets the x-axis) **A1**

(a) Answer: f'(3) = 0, so the tangent at x0=3 is horizontal and the Newton-Raphson formula fails (division by zero)

(b) correct Newton-Raphson substitution using f(4.4) = 1.128 and f'(4.4) = 36.96 **M1**

(b) correct unrounded working x1 = 4.4 - 1.128/36.96 **A1**

(b) x1 = 4.3695 awrt **A1**

(b) Answer: x1 = 4.3695

Question 5

evaluates f(2) = -4 and f(3) = 10 **M1**

cso: sign change (negative to positive) with f continuous confirms a root in (2,3) **A1**

Answer: f(2) = -4, f(3) = 10; sign change confirms a root

Question 6

f(2) = -1, f(3) = 14, both required **B1**

Answer: f(2) = -1, f(3) = 14

Question 7

$x_{n+1} = x_n - f(x_n)/f'(x_n)$ **B1**

Answer: $x_{n+1} = x_n - f(x_n)/f'(x_n)$

Question 8

(a) equates two components (e.g. i and k) to form a pair of simultaneous equations in s and t **M1**

(a) correct pair of equations, e.g. t + 2s = 4 and t - s = 1 oe **A1**

(a) solves the equations simultaneously **M1**

(a) t = 2, s = 1 oe **A1**

(a) verifies using the remaining (j) component: both sides equal 0, confirming consistency, so the lines intersect **B1**

(a) position vector of intersection = 3i + 7k oe cso **A1**

(a) Answer: Lines intersect at the point with position vector 3i + 7k

(b) attempts dot product of direction vectors (1)(-2) + (-1)(2) + (2)(2) **M1**

(b) d1.d2 = 0 **A1**

(b) recognises $\cos(\theta) = 0$ implies the lines are perpendicular **M1**

(b) θ = 90 degrees cao **A1**

(b) Answer: θ = 90 degrees (the lines are perpendicular)

Question 9

substitutes x0=3 into x1 = x0 - f(x0)/f'(x0), using $f'(x)=3x^2-5$ **M1**

x1 = 2.5455 cao **A1**

Answer: x1 = 2.5455

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Question 10

substitutes x0 = 3.7 into the iteration formula	**M1**
x1 = 3.8539	**A1**
x2 = 3.8653, x3 = 3.8661 (ft)	**A1**
root estimate = 3.87 (2dp, ft)	**A1**

Answer: x1 = 3.8539, x2 = 3.8653, x3 = 3.8661; root awrt 3.87

Question 11

h(0.3) = awrt 0.150 (positive) and h(0.4) = awrt -0.108 (negative); sign change confirms a root in (0.3, 0.4)	**B1**
evaluates h(0.355) = awrt 0.006 (positive) and h(0.365) = awrt -0.019 (negative)	**M1**
cso: since h(0.355) > 0 and h(0.365) < 0, the root lies between 0.355 and 0.365, so x = 0.36 correct to 2dp	**A1**

Answer: x = 0.36 (2dp)

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