1.16 Pure: Vectors and Numerical Methods Depth

EDEXCEL 9MA0 · Calculator allowed · about 225 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 In triangle OAB, OA = a and OB = b. The point P lies on AB such that AP : PB = 1 : 2. The point Q lies on OP such that OQ : QP = 4 : 2.

(a) Find AB in terms of a and b, and hence find OP in terms of a and b. **(3)**

(b) Find OQ in terms of a and b. **(2)**

(c) Find AQ in terms of a and b, giving your answer in its simplest form. **(2)**

(Total for Question 1 is 7 marks)

2 Relative to a fixed origin O, the points A and B have position vectors OA = -5i + 7j and OB = 2i - 3j + 8k.

(a) Find the vector AB and the exact length |AB|. **(4)**

(b) Find the position vector of the midpoint M of AB. **(2)**

(c) The point C lies on the line AB, with AC = 1/2 AB. Find the position vector of C. **(2)**

(d) The point D has position vector -12i + 17j - 8k. Show that D lies on the line AB and find the ratio AD : AB. **(3)**

(Total for Question 2 is 11 marks)

3 Relative to a fixed origin O, the points A, B and C have position vectors OA = -i - 3k, OB = 3i - 3j and OC = -2i + j + k.

(a) Find the exact lengths of the sides |AB|, |AC| and |BC| of triangle ABC. **(3)**

(b) Find the size of angle BAC to 1 decimal place. **(3)**

(c) Find the area of triangle ABC to 3 significant figures. **(2)**

(Total for Question 3 is 8 marks)

4 $f(x) = x^3 + 5x - 9$. The equation $f(x) = 0$ has a root α near $x = 1$.

(a) Find $f'(x)$. **(2)**

(b) Use the Newton-Raphson method with $x_0 = 1$ to find x_1, x_2 and x_3, giving each answer to 4 decimal places. **(4)**

(c) Hence state the value of α correct to 3 decimal places, and justify your answer by evaluating f at two suitable values. **(3)**

(Total for Question 4 is 9 marks)

5 The vectors a and b are given by a = 3i - 7j and b = -6i - 7j.

(a) Find a + 3b, and its magnitude to 3 significant figures. **(4)**

(b) Find the angle that a + 3b makes with the positive i direction, measured anticlockwise, to 1 decimal place. **(2)**

(c) Given that a + λ b is parallel to j, find the value of λ. **(2)**

(Total for Question 5 is 8 marks)

6 Relative to a fixed origin O, the points A, B and C have position vectors OA = -3i + 3j - 2k, OB = 3i + 5j - 8k and OC = 9i + 7j + pk, where p is a constant. A, B and C are collinear.

(a) Find the vectors AB and BC, giving BC in terms of p. **(3)**

(b) Use the fact that A, B and C are collinear to find the value of p. **(3)**

(c) Hence write down the ratio AB : BC. **(1)**

(Total for Question 6 is 7 marks)

7 Relative to a fixed origin O, the points A and B have position vectors OA = 4i - j + 2k and OB = -4i + 7j + 5k.

(a) Find the vector AB and the exact length |AB|. **(4)**

(b) Find the position vector of the midpoint M of AB. **(2)**

(c) The point C lies on the line AB, with AC = 3/2 AB. Find the position vector of C. **(2)**

(d) The point D has position vector -12i + 15j + 8k. Show that D lies on the line AB and find the ratio AD : AB. **(3)**

(Total for Question 7 is 11 marks)

8 In triangle OAB, OA = a and OB = b. The point P lies on AB such that AP : PB = 1 : 1. The point Q lies on OP such that OQ : QP = 2 : 2.

(a) Find AB in terms of a and b, and hence find OP in terms of a and b. **(3)**

(b) Find OQ in terms of a and b. **(2)**

(c) Find AQ in terms of a and b, giving your answer in its simplest form. **(2)**

(Total for Question 8 is 7 marks)

9 The table shows values of $y = \ln(x + 3)$, rounded to 4 decimal places, for equally spaced values of x.

x	0	1	2	3	4
y	1.0986	1.3863	1.6094	1.7918	1.9459

(a) Use the trapezium rule with 4 strips to estimate the integral from $x = 0$ to $x = 4$ of $\ln(x + 3)\,dx$, giving your answer to 4 decimal places. **(4)**

(b) State, with a reason, whether the trapezium rule gives an overestimate or an underestimate. **(2)**

(c) Find the percentage error in your estimate, given that the exact value is 6.3255, to 2 significant figures. **(2)**

(Total for Question 9 is 8 marks)

10 Relative to a fixed origin O, the points A, B and C have position vectors OA = -3i - 3j, OB = -j - 2k and OC = i + 3j - k.

(a) Find the exact lengths of the sides |AB|, |AC| and |BC| of triangle ABC. **(3)**

(b) Find the size of angle BAC to 1 decimal place. **(3)**

(c) Find the area of triangle ABC to 3 significant figures. **(2)**

(Total for Question 10 is 8 marks)

11 Relative to a fixed origin O, the points P and Q have position vectors $\overrightarrow{OP} = 3\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{OQ} = 11\mathbf{i} - 3\mathbf{j}$.

(a) Find the vector PQ. **(2)**

(b) The point R lies on PQ such that PR:RQ = 3:1. Find the position vector of R. **(3)**

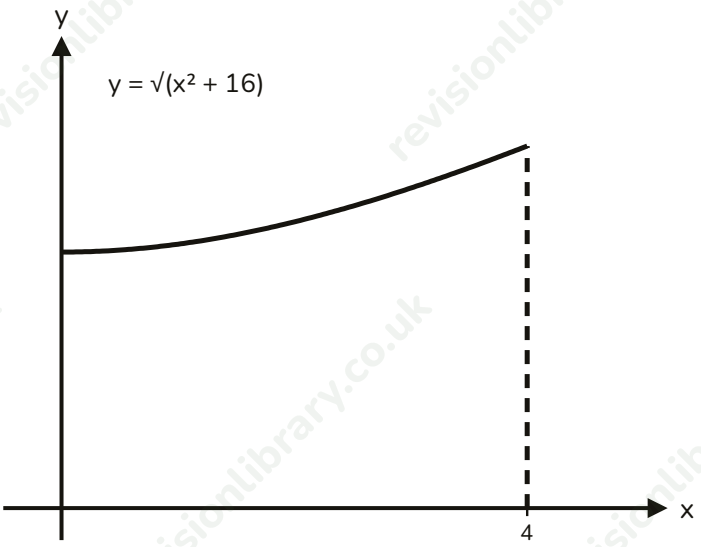
(c) The point S has position vector $18\mathbf{i} - 2\mathbf{j}$. Show that O, R and S are collinear. **(2)**

(Total for Question 11 is 7 marks)

12 Two lines, l1 and l2, have direction vectors 2i + 3j - 6k and 3i - 6j + 2k respectively. Find the angle between the two lines, giving your answer to 1 decimal place.

(Total for Question 12 is 5 marks)

13 The curve C has equation $y = \sqrt{x^2 + 16}$, for $0 \leq x \leq 4$.



(a) Using the trapezium rule with 4 strips of equal width, find an estimate for the integral from x=0 to x=4 of $\sqrt{x^2+16}$ dx, giving your answer to 3 decimal places. You should show the value of y at each of the 5 values of x used. **(5)**

(b) State, with a reason, whether your estimate in part (a) is an overestimate or an underestimate of the true value of the integral. **(2)**

(Total for Question 13 is 7 marks)

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14 $g(x) = e^x - 3x - 2$. The equation $g(x) = 0$ has a root α in the interval $[2,3]$.

(a) Show that α lies between $x = 2.1$ and $x = 2.2$. **(2)**

(b) By further systematic decimal search (or otherwise), show that $\alpha = 2.13$, correct to 2 decimal places. **(3)**

(Total for Question 14 is 5 marks)

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15 $a = 3i - 4j + 12k$. Find $|a|$.

(Total for Question 15 is 1 mark)

16 The trapezium rule is to be used with 2 strips of equal width to estimate the integral from x = 0 to x = 4. State the width of each strip.

(Total for Question 16 is 1 mark)

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17 Relative to a fixed origin O, OP = 2i + 3j - k and OQ = 5i - j + 2k. Find the vector PQ.

(Total for Question 17 is 1 mark)

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18 State the formula for the scalar (dot) product of vectors a = a1 i + a2 j + a3 k and b = b1 i + b2 j + b3 k.

(Total for Question 18 is 1 mark)

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19 a = 2i + j - 3k, b = i - 2j + 4k. Find a . b.

(Total for Question 19 is 1 mark)

20 $f(x) = x^3 - 6x - 3$.

(a) Show that the equation $f(x) = 0$ has a root between $x = 2$ and $x = 3$. **(2)**

(b) Show that the equation $f(x) = 0$ can be rearranged into the form $x = (6x+3)^{1/3}$. **(2)**

(c) Using the iteration $x_{n+1} = (6x_n+3)^{1/3}$ with $x0 = 2$, find the values of x1, x2 and x3, giving your answers to 4 decimal places. **(3)**

(d) By considering the change of sign of $f(x)$ on a suitable interval, show that the root of $f(x) = 0$ in $(2,3)$ is $x = 2.669$, correct to 3 decimal places. **(3)**

(Total for Question 20 is 10 marks)

21 $f(x) = x^3 - 5x - 3$.

(a) Show that $f(x) = 0$ has a root α between $x = 2$ and $x = 3$. **(2)**

(b) Using $x0 = 2.5$, apply the Newton-Raphson method once to $f(x)$ to find $x1$, giving your answer to 6 decimal places. **(3)**

(c) Apply the Newton-Raphson method a second time to find $x2$, giving your answer to 6 decimal places. **(2)**

(d) Given that $x1$ and $x2$ already agree to 4 decimal places (both round to 2.4909), comment on the rate of convergence of the Newton-Raphson method here compared with the fixed-point iteration used in Question 20. **(2)**

(Total for Question 21 is 9 marks)

22 $f(x) = x^3 - 2x - 5$. Given that $f(2) = -1$ and $f(3) = 16$, state whether $f(x) = 0$ has a root between $x = 2$ and $x = 3$, with a reason.

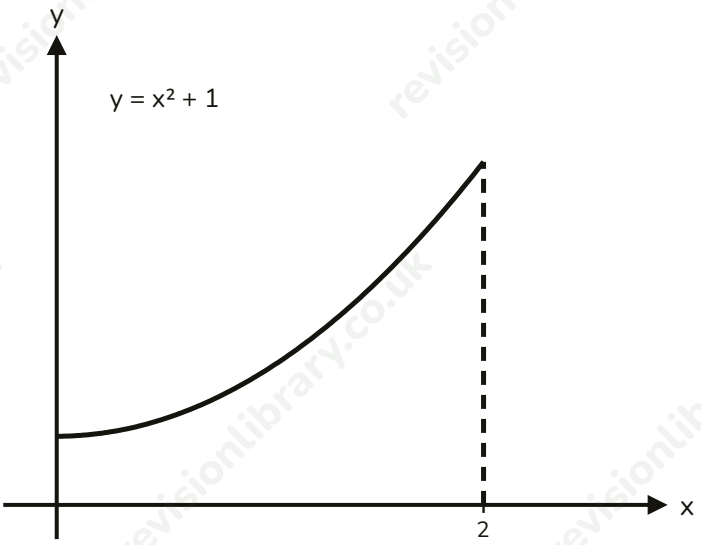
(Total for Question 22 is 1 mark)

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23 Relative to a fixed origin O, points P and Q have position vectors $\overrightarrow{OP} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\overrightarrow{OQ} = 4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$. Find the vector $\overrightarrow{PQ}$, and hence find $|\overrightarrow{PQ}|$, giving your answer as an exact simplified surd.

(Total for Question 23 is 2 marks)

24 Using the trapezium rule with 4 strips of equal width, find an estimate for the integral from x = 0 to x = 2 of (x^2+1) dx, giving your answer to 2 decimal places.



(Total for Question 24 is 2 marks)

25 $f(x) = x^3 - 4x^2 + 2$.

(a) Show that the equation $f(x) = 0$ has a root between $x = 3$ and $x = 4$. **(2)**

(b) Show that the equation $f(x) = 0$ can be rearranged into the form $x = 4 - 2/x^2$. **(2)**

(Total for Question 25 is 4 marks)

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26 Using the iteration formula $x_{n+1} = \sqrt{3x_n + 1}$, with $x0 = 2$, find $x1$, giving your answer to 4 decimal places.

(Total for Question 26 is 1 mark)

27 The line l has vector equation $r = t(i + 2j + 2k)$, where t is a scalar parameter. The point P has position vector $8i + j + 4k$.

(a) Find the position vector of N, the foot of the perpendicular from P to l. **(6)**

(b) Find the distance PN, giving your answer in the form $k*\sqrt{5}$, where k is an integer. **(3)**

(c) The point P' is the reflection of P in the line l. Find the position vector of P'. **(3)**

(Total for Question 27 is 12 marks)

28 $g(x) = x^3 + 2x - 6$. The equation $g(x) = 0$ has a root α in the interval $[1,2]$.

(a) Using $x0 = 1.5$, apply the Newton-Raphson method once to $g(x)$ to find $x1$, giving your answer to 4 decimal places. **(2)**

(b) Apply the Newton-Raphson method a second time to find $x2$, giving your answer to 4 decimal places. **(2)**

(Total for Question 28 is 4 marks)

29 Relative to a fixed origin O, the line l has vector equation $\mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + t(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$, where t is a scalar parameter. The point A has position vector $9\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$.

(a) Show that the point C with position vector $4\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ lies on l, stating the corresponding value of t. **(2)**

(b) Find the vector AC, and hence find |AC|, giving your answer as an exact simplified surd. **(2)**

(Total for Question 29 is 4 marks)

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30 J = the definite integral, from x = 0 to x = 1, of $1/(1+x)\,dx$. The exact value of J is ln(2).

(a) Using the trapezium rule with 2 strips of equal width, find an estimate for J, giving your answer to 3 decimal places. **(2)**

(b) Given that the exact value of J is ln(2), find the percentage error in your estimate from part (a), giving your answer to 2 significant figures. **(1)**

(Total for Question 30 is 3 marks)

31 A drone follows a path such that its position vector, in km, relative to a fixed point O, at time t hours ($t \geq 0$) is given by $r(t) = t\,i + t^2\,j + t\,k$. A ground station is located at the point P with position vector $4i + 2j + 5k$.

(a) Show that the square of the distance between the drone and the ground station at time t is given by $D(t) = t^4 - 2t^2 - 18t + 45$. **(4)**

(b) Show that the value of t at which the distance is a minimum satisfies $2t^3 - 2t - 9 = 0$. **(3)**

(c) Show that this equation has a root between $t = 1$ and $t = 2$. **(2)**

(d) Taking $t0 = 1.85$ as a first approximation, use the Newton-Raphson method once to find an improved estimate t1 for the root, giving your answer to 4 decimal places. **(4)**

(Total for Question 31 is 13 marks)

Mark scheme · 1.16 Pure: Vectors and Numerical Methods Depth

Question 1

(a) AB = b - a **B1**

(a) AP = (1/3) AB and OP = OA + AP **M1**

(a) OP = (2/3)a + (1/3)b **A1**

(a) Answer: AB = b - a, OP = (2/3)a + (1/3)b

(b) OQ = (4/6) OP **M1**

(b) (4/9)a + (2/9)b **A1**

(b) Answer: (4/9)a + (2/9)b

(c) AQ = AO + OQ = -a + OQ **M1**

(c) -(5/9)a + (2/9)b **A1**

(c) Answer: -(5/9)a + (2/9)b

Question 2

(a) AB = OB - OA **M1**

(a) AB = 7i - 10j + 8k **A1**

(a) uses Pythagoras in three dimensions **M1**

(a) $|AB| = \sqrt{213}$ **A1**

(a) Answer: AB = 7i - 10j + 8k, $|AB| = \sqrt{213}$

(b) (1/2)(OA + OB) **M1**

(b) -(3/2)i + 2j + 4k **A1**

(b) Answer: -(3/2)i + 2j + 4k

(c) OC = OA + 1/2 AB **M1**

(c) -(3/2)i + 2j + 4k **A1**

(c) Answer: -(3/2)i + 2j + 4k

(d) finds AD = OD - OA **M1**

(d) AD = -7i + 10j - 8k = -1 AB, a multiple of AB, so D lies on the line AB **A1**

(d) AD : AB = 1 : 1 (D is on the opposite side of A from B) **B1**

(d) Answer: AD = -1 AB, so AD : AB = 1 : 1

Question 3

(a) finds AB, AC and BC by subtracting position vectors **M1**

(a) $|AB| = \sqrt{34}$ **A1**

(a) $|AC| = 3\sqrt{2}$, $|BC| = \sqrt{42}$ **A1**

(a) Answer: $|AB| = \sqrt{34}$, $|AC| = 3\sqrt{2}$, $|BC| = \sqrt{42}$

(b) cosine rule: $\cos A = (AB^2 + AC^2 - BC^2)/(2\,|AB||AC|)$ **M1**

(b) cos A = (34 + 18 - 42)/(2 x 5.831 x 4.243) = 0.2021 **A1**

(b) angle BAC = 78.3 degrees **A1**

(b) Answer: 78.3 degrees

(c) (1/2)|AB||AC| sin A **M1**

(c) 12.1 **A1**

(c) Answer: 12.1

Question 4

(a) differentiates each term **M1**

(a) $f'(x) = 3x^2 + 5$ **A1**

(a) Answer: $f'(x) = 3x^2 + 5$

(b) $x_{n+1} = x_n - f(x_n)/f'(x_n)$ **M1**

(b) $x_1 = 1.3750$ **A1**

(b) $x_2 = 1.3305$ **A1**

(b) $x_3 = 1.3297$ **A1**

(b) Answer: $x_1 = 1.3750$, $x_2 = 1.3305$, $x_3 = 1.3297$

(c) α = 1.330 **B1**

(c) evaluates f(1.3295) and f(1.3305) **M1**

(c) values -0.0025 and 0.0078 have opposite signs **A1**

(c) Answer: α = 1.330

Question 5

(a) adds the i components and the j components: a + 3b **M1**

(a) -15i - 28j **A1**

(a) uses $|v| = \sqrt{x^2 + y^2}$ **M1**

(a) 31.8 **A1**

(a) Answer: -15i - 28j, magnitude 31.8

(b) tan θ = y/x with a quadrant check **M1**

(b) 241.8 degrees **A1**

(b) Answer: 241.8 degrees

(c) sets the i component of a + λ b to zero **M1**

(c) 3 + λ(-6) = 0, so λ = 1/2 **A1**

(c) Answer: λ = 1/2

Question 6

(a) uses AB = OB - OA and BC = OC - OB **M1**

(a) AB = 6i + 2j - 6k **A1**

(a) BC = 6i + 2j + (p + 8)k **A1**

(a) Answer: AB = 6i + 2j - 6k, BC = 6i + 2j + (p + 8)k

(b) BC = λ AB, so compares the known components to find λ **M1**

(b) λ = 1 **A1**

(b) p = -14 **A1**

(b) Answer: p = -14

(c) AB : BC = 1 : 1 **B1**

(c) Answer: AB : BC = 1 : 1

Question 7

(a) AB = OB - OA **M1**

(a) AB = -8i + 8j + 3k **A1**

(a) uses Pythagoras in three dimensions **M1**

(a) $|AB| = \sqrt{137}$ **A1**

(a) Answer: AB = -8i + 8j + 3k, $|AB| = \sqrt{137}$

(b) (1/2)(OA + OB) **M1**

(b) 3j + (7/2)k **A1**

(b) Answer: 3j + (7/2)k

(c) OC = OA + 3/2 AB **M1**

(c) -8i + 11j + (13/2)k **A1**

(c) Answer: -8i + 11j + (13/2)k

(d) finds AD = OD - OA **M1**

(d) AD = -16i + 16j + 6k = 2 AB, a multiple of AB, so D lies on the line AB **A1**

(d) AD : AB = 2 : 1 **B1**

(d) Answer: AD = 2 AB, so AD : AB = 2 : 1

Question 8

(a) AB = b - a **B1**

(a) AP = (1/2) AB and OP = OA + AP **M1**

(a) OP = (1/2)a + (1/2)b **A1**

(a) Answer: AB = b - a, OP = (1/2)a + (1/2)b

(b) OQ = (2/4) OP **M1**

(b) (1/4)a + (1/4)b **A1**

(b) Answer: (1/4)a + (1/4)b

(c) AQ = AO + OQ = -a + OQ **M1**

(c) -(3/4)a + (1/4)b **A1**

(c) Answer: -(3/4)a + (1/4)b

Question 9

(a) h = 4/4 = 1 **B1**

(a) trapezium rule: $(h/2)[y_0 + y_n + 2(\text{sum of the middle values})]$ **M1**

(a) sum of middle values = 4.7875 **A1**

(a) 6.3098 **A1**

(a) Answer: 6.3098

(b) underestimate **B1**

(b) the curve is concave (concave down), so the chords lie below the curve **B1**

(b) Answer: underestimate

(c) |estimate - exact|/exact x 100 **M1**

(c) 0.25% **A1**

(c) Answer: 0.25%

Question 10

(a) finds AB, AC and BC by subtracting position vectors **M1**

(a) $|AB| = \sqrt{17}$ **A1**

(a) $|AC| = \sqrt{53}$, $|BC| = 3\sqrt{2}$ **A1**

(a) Answer: $|AB| = \sqrt{17}$, $|AC| = \sqrt{53}$, $|BC| = 3\sqrt{2}$

(b) cosine rule: $\cos A = (AB^2 + AC^2 - BC^2)/(2\,|AB||AC|)$ **M1**

(b) cos A = (17 + 53 - 18)/(2 x 4.123 x 7.280) = 0.8662 **A1**

(b) angle BAC = 30.0 degrees **A1**

(b) Answer: 30.0 degrees

(c) (1/2)|AB||AC| sin A **M1**

(c) 7.50 **A1**

(c) Answer: 7.50

Question 11

(a) attempts OQ - OP **M1**

(a) PQ = 8i - 8j oe **A1**

(a) Answer: PQ = 8i - 8j

(b) correct method OR = OP + (3/4)PQ, or equivalent ratio approach **M1**

(b) correct substitution (3i+5j) + (3/4)(8i-8j) **A1**

(b) OR = 9i - j oe **A1**

(b) Answer: OR = 9i - j

(c) shows OS = 2 OR, i.e. 18i - 2j = 2(9i - j) **M1**

(c) valid conclusion: since OS is a scalar multiple of OR, OR and OS are parallel and share the common point O, so O, R, S are collinear cso **A1**

(c) Answer: OS = 2 OR, so O, R and S are collinear

Question 12

attempts dot product (2)(3) + (3)(-6) + (-6)(2) **M1**

dot product = -24 **A1**

attempts |d1| and |d2| (both equal 7) **M1**

attempts $\cos(\theta) = |d1.d2|/(|d1||d2|)$, or finds the angle between the direction vectors and takes its supplement **M1**

θ = 60.7 degrees awrt **A1**

Answer: θ = awrt 60.7 degrees

Question 13

(a) correct strip width h = 1 **M1**

(a) attempts at least 3 correct y-values (y0=4, y1=4.1231, y2=4.4721, y3=5, y4=5.6569, or ft) **M1**

(a) all 5 y-values correct awrt 4dp **A1**

(a) correct trapezium structure (h/2)[y0+y4+2(y1+y2+y3)] **M1**

(a) estimate = 18.424 awrt **A1**

(a) Answer: Integral (approx) = 18.424

(b) states overestimate **B1**

(b) valid reason referencing the shape of the curve, e.g. $y = \sqrt{x^2+16}$ is concave up (convex) on [0,4], so the chords used by the trapezium rule lie above the curve **B1**

(b) Answer: Overestimate, because the curve is concave up (convex) over [0,4], so the trapezium chords lie above the curve.

Question 14

(a) evaluates g(2.1) and g(2.2) **M1**

(a) g(2.1) = -0.1338 (<0), g(2.2) = 0.4250 (>0); sign change confirms root in (2.1,2.2) cso **A1**

(a) Answer: g(2.1) = -0.1338, g(2.2) = 0.4250; sign change confirms root in (2.1,2.2)

(b) evaluates g(2.125) and g(2.135), or an equivalent bounding interval for 2.13 **M1**

(b) g(2.125) = -0.0021 (awrt, negative) and g(2.135) = 0.0520 (awrt, positive), both correct **A1**

(b) sign change on (2.125,2.135), g continuous, hence α = 2.13 correct to 2dp cso **A1**

(b) Answer: α = 2.13 (2dp)

Question 15

13 cao **B1**

Answer: 13

Question 16

2 cao **B1**

Answer: 2

Question 17

3i - 4j + 3k cao **B1**

Answer: PQ = 3i - 4j + 3k

Question 18

a.b = a1 b1 + a2 b2 + a3 b3 **B1**

Answer: a.b = a1b1 + a2b2 + a3b3

Question 19

-12 cao **B1**

Answer: -12

Question 20

(a) evaluates f(2) and f(3): f(2) = -7, f(3) = 6 **M1**

(a) sign change (f(2) < 0, f(3) > 0) and f continuous, so a root lies in (2,3) cso **A1**

(a) Answer: f(2) = -7, f(3) = 6; sign change confirms a root in (2,3)

(b) rearranges $x^3 - 6x - 3 = 0$ to $x^3 = 6x + 3$ **M1**

(b) takes cube roots of both sides to obtain $x = (6x+3)^{1/3}$ cso **A1**

(b) Answer: $x = (6x+3)^{1/3}$

(c) correct method, $x1 = (6(2)+3)^{1/3} = 15^{1/3}$, x1 = 2.4662 awrt **M1**

(c) x2 = 2.6109 awrt **A1**

(c) x3 = 2.6526 awrt **A1**

(c) Answer: x1 = 2.4662, x2 = 2.6109, x3 = 2.6526

(d) evaluates f(2.6685) and f(2.6695), or an equivalent bounding interval for 2.669 **M1**

(d) f(2.6685) = -0.0089 (awrt, negative) and f(2.6695) = 0.0065 (awrt, positive), both correct **A1**

(d) sign change on (2.6685, 2.6695), f continuous, hence root = 2.669 correct to 3dp cso **A1**

(d) Answer: Root = 2.669 (3dp)

Question 21

(a) evaluates f(2) and f(3): f(2) = -5, f(3) = 9 **M1**

(a) sign change (f(2) < 0, f(3) > 0), f continuous, so a root lies in (2,3) cso **A1**

(a) Answer: f(2) = -5, f(3) = 9; sign change confirms a root in (2,3)

(b) correct derivative $f'(x) = 3x^2 - 5$ **M1**

(b) correct Newton-Raphson substitution using f(2.5) = 0.125 and f'(2.5) = 13.75 **M1**

(b) x1 = 2.490909 awrt **A1**

(b) Answer: x1 = 2.490909

(c) correct (ft) substitution using x1: f(x1) = 0.000619, f'(x1) = 13.6139 **M1**

(c) x2 = 2.490864 awrt **A1**

(c) Answer: x2 = 2.490864

(d) recognises Newton-Raphson converges much faster here; only one iteration was needed to reach 4dp accuracy **B1**

(d) valid comparison: the fixed-point iteration in Question 20 was still changing in the 2nd decimal place after three iterations (x3 = 2.6526 against the true root 2.669), so it converges much more slowly (linearly) than Newton-Raphson (which converges quadratically) **B1**

(d) Answer: Newton-Raphson converges much faster (quadratic convergence) than the fixed-point iteration of Question 20 (linear convergence): here x1 and x2 already agree to 4dp, whereas the Question 20 iteration was still 0.016 away from its root after 3 iterations.

Question 22

yes, since f changes sign (from negative to positive) and f is continuous **B1**

Answer: Yes, by the change of sign

Question 23

PQ = OQ - OP = 3i - 4j + 6k **M1**

$|PQ| = \sqrt{9+16+36} = \sqrt{61}$ cao (exact) **A1**

Answer: PQ = 3i - 4j + 6k, $|PQ| = \sqrt{61}$

Question 24

uses the five correct ordinates 1, 1.25, 2, 3.25, 5 with strip width 0.5 **M1**

estimate = 4.75 cao **A1**

Answer: 4.75

Question 25

(a) evaluates f(3) = -7 and f(4) = 2 **M1**

(a) cso: sign change confirms a root in (3,4) **A1**

(a) Answer: f(3) = -7, f(4) = 2; sign change confirms a root

(b) rearranges $x^3-4x^2+2=0$ to $x^2(x-4) = -2$ **M1**

(b) cso: divides by x^2 to obtain $x = 4 - 2/x^2$ **A1**

(b) Answer: $x = 4 - 2/x^2$

Question 26

2.6458 cao **B1**

Answer: 2.6458

Question 27

(a) writes a general point on l as (t, 2t, 2t)	**M1**
(a) forms the vector from P to this general point (or vice versa) in terms of t	**M1**
(a) sets the dot product of this vector with the direction vector (1,2,2) equal to zero	**M1**
(a) correct equation 9t - 18 = 0 oe	**A1**
(a) t = 2	**A1**
(a) N = 2i + 4j + 4k	**A1**
(a) Answer: N = 2i + 4j + 4k	
(b) forms vector PN = N - P	**M1**
(b) attempts magnitude $\sqrt{(-6)^2+3^2+0^2}$	**M1**
(b) PN = $3\sqrt{5}$ cso, exact form required	**A1**
(b) Answer: PN = $3\sqrt{5}$	
(c) recognises that N is the midpoint of P and P'	**M1**
(c) correct method P' = 2N - P, or equivalent	**M1**
(c) P' = -4i + 7j + 4k	**A1**
(c) Answer: P' = -4i + 7j + 4k	

Question 28

(a) uses g'(x)=$3x^2$+2 and substitutes x0=1.5 into the Newton-Raphson formula	**M1**
(a) x1 = 1.4571 cao	**A1**
(a) Answer: x1 = 1.4571	
(b) applies the formula again using x1 from (a) (ft)	**M1**
(b) x2 = 1.4562 cao (ft)	**A1**
(b) Answer: x2 = 1.4562	

Question 29

(a) sets 2+t=4, -1+2t=3 and 1-2t=-3, and solves for t	**M1**
(a) cso: t=2 satisfies all three equations, so C lies on l	**A1**
(a) Answer: t = 2, C lies on l	
(b) AC = OC - OA = -5i + 0j + 2k	**M1**
(b) |AC| = $\sqrt{25+0+4}$ = $\sqrt{29}$ cao	**A1**
(b) Answer: AC = -5i + 2k, |AC| = $\sqrt{29}$	

Question 30

(a) uses the three correct ordinates 1, 2/3, 0.5 with strip width 0.5	**M1**
(a) estimate = 0.708 (3dp)	**A1**
(a) Answer: 0.708	
(b) awrt 2.2% (ft from (a))	**B1**
(b) Answer: awrt 2.2%	

Question 31

(a) forms the vector from P to the drone (or vice versa): (t-4, t^2-2, t-5)	**M1**
(a) squares and sums the three components	**M1**
(a) correct unsimplified expression $(t-4)^2+(t^2-2)^2+(t-5)^2$	**A1**
(a) simplifies to D(t) = t^4-$2t^2$-18t+45 cso	**A1**
(a) Answer: D(t) = t^4 - $2t^2$ - 18t + 45	
(b) differentiates D(t): D'(t) = $4t^3$-4t-18	**M1**
(b) sets D'(t) = 0	**M1**
(b) divides by 2 to obtain $2t^3$-2t-9 = 0 cso	**A1**
(b) Answer: $2t^3$ - 2t - 9 = 0	
(c) evaluates g(1) = 2(1)-2(1)-9 and g(2) = 2(8)-2(2)-9	**M1**
(c) sign change (g(1) = -9 < 0, g(2) = 3 > 0), g continuous, hence root in (1,2) cso	**A1**
(c) Answer: g(1) = -9, g(2) = 3; sign change confirms a root between t=1 and t=2	
(d) correct derivative g'(t) = $6t^2$-2	**M1**
(d) correct Newton-Raphson substitution using g(1.85) = -0.0368 and g'(1.85) = 18.535	**M1**
(d) correct unrounded value t1 = 1.85198...	**A1**
(d) t1 = 1.8520 awrt	**A1**
(d) Answer: t1 = 1.8520	