1.17 Pure: Functions, Graphs and Transformations: 1-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The function f is defined by $f(x) = (5x + 7)/(x - 2)$, x in R, $x \neq 2$. The function g is defined by $g(x) = x^2 + 4$, x in R.

(a) Find $f^{-1}(x)$, stating its domain. **(4)**

(b) Write down the range of f. **(1)**

(c) Find fg(x) and hence solve $fg(x) = 107/18$. **(4)**

(Total for Question 1 is 9 marks)

---

2 The graph of $y = e^x$ is transformed to the graph of $y = e^{x - 1} + 4$.

(a) Describe the single translation that maps $y = e^x$ onto $y = e^{x - 1} + 4$. **(2)**

(b) Write down the equation of the horizontal asymptote of the new curve and the exact coordinates of its intersection with the y axis. **(2)**

(Total for Question 2 is 4 marks)

---

3 The curve C has equation $y = 6/(x + 2) - 1$, $x \neq -2$.

(a) Write down the equations of the asymptotes of C. **(2)**

(b) Find the coordinates of the points where C meets the coordinate axes. **(4)**

(c) State whether the gradient of C is positive or negative for every x in its domain, giving a reason. **(2)**

(Total for Question 3 is 8 marks)

© Revision Library | revisionlibrary.co.uk

4 The graph of $y = \sin x$, in degrees, is transformed to the graph of $y = 4 \sin(x - 20) + 6$.

(a) Describe the sequence of three transformations that maps $y = \sin x$ onto $y = 4 \sin(x - 20) + 6$. **(3)**

(b) Write down the maximum value of y and the smallest positive value of x at which it occurs. **(2)**

(Total for Question 4 is 5 marks)

---

5 The function f is defined by $f(x) = 2(x - 2)^2 + 5$, for $x \geq 2$.

(a) State the range of f. **(1)**

(b) Find an expression for $f^{-1}(x)$, and state its domain. **(4)**

(c) Find the value of $f^{-1}(37)$. **(2)**

(Total for Question 5 is 7 marks)

© Revision Library | revisionlibrary.co.uk

6 The curve C with equation $y = f(x)$ has a minimum point at $(-3, -2)$ and passes through the point $(5, 0)$.

(a) Write down the coordinates of the minimum point on the curve with equation $y = f(-x)$, and the coordinates of the image of the point $(5, 0)$. **(2)**

(b) Write down the coordinates of the minimum point on the curve with equation $y = f(2x)$, and the coordinates of the image of the point $(5, 0)$. **(2)**

(c) Write down the coordinates of the minimum point on the curve with equation $y = 2f(x)$, and the coordinates of the image of the point $(5, 0)$. **(2)**

(Total for Question 6 is 6 marks)

---

7 A campsite hire company found that the cost, C pounds, of hiring a bell tent for a weekend is directly proportional to the number of guests, n, that the tent must sleep. A tent that sleeps 4 guests costs £60 to hire.

(a) Write down an equation connecting C and n, in terms of a constant k, and find the value of k. **(2)**

(b) State the cost of hiring a tent that sleeps 7 guests. **(1)**

(c) Sketch the graph of C against n for $n > 0$, labelling the gradient of the line and the coordinates of one other point on the line. **(3)**

(d) A different company hires the same style of tent for a fee, F pounds per weekend, that is inversely proportional to the number of weekends, w, that the customer books, for $w > 0$. A customer who books 3 weekends is charged a fee of £160 per weekend. Find an equation connecting F and w, and sketch the graph of F against w for $w > 0$, labelling the equations of the asymptotes and the coordinates of one point on the curve. **(5)**

(Total for Question 7 is 11 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 1.17 Pure: Functions, Graphs and Transformations: 1-hour Pack

Question 1

(a) sets $y = (5x + 7)/(x - 2)$ and rearranges: $y(x - 2) = 5x + 7$ **M1**

(a) collects x terms: $x(y - 5) = 2y + 7$ **M1**

(a) $f^{-1}(x) = (2x + 7)/(x - 5)$ **A1**

(a) domain x in R, $x \neq 5$ **B1**

(a) Answer: $f^{-1}(x) = (2x + 7)/(x - 5)$, $x \neq 5$

(b) $f(x) \neq 5$, i.e. y in R, $y \neq 5$ **B1**

(b) Answer: $y \neq 5$

(c) $fg(x) = f(x^2 + 4) = (5(x^2 + 4) + 7)/(x^2 + 4 - 2)$ **M1**

(c) sets equal to 107/18 and rearranges to a quadratic in x^2 (or x) **M1**

(c) $x^2 = 16$ **A1**

(c) $x = 4$ or $x = -4$ **A1**

(c) Answer: $x = \pm 4$

Question 2

(a) translation **B1**

(a) by the vector (1, 4) **B1**

(a) Answer: translation by (1, 4)

(b) $y = 4$ **B1**

(b) $(0, e^{-1} + 4)$ **B1**

(b) Answer: $y = 4$; $(0, e^{-1} + 4)$

Question 3

(a) $x = -2$ **B1**

(a) $y = -1$ **B1**

(a) Answer: $x = -2$ and $y = -1$

(b) sets $x = 0$ to find the y intercept **M1**

(b) (0, 2) **A1**

(b) sets $y = 0$ to find the x intercept **M1**

(b) (4, 0) **A1**

(b) Answer: (0, 2) and (4, 0)

(c) $dy/dx = -6/(x + 2)^2$ **M1**

(c) the sign is negative for all $x \neq -2$ since the square is positive **A1**

(c) Answer: negative

Question 4

(a) translation by (20, 0), i.e. 20 degrees to the right **B1**

(a) stretch scale factor 4 parallel to the y axis **B1**

(a) translation by (0, 6) **B1**

(a) Answer: translate (20, 0); stretch y by 4; translate (0, 6)

(b) maximum y = 10 **B1**

(b) x - 20 = 90, so x = 110 **B1**

(b) Answer: 10 at x = 110

Question 5

(a) $f(x) \geq 5$ **B1**

(a) Answer: $f(x) \geq 5$

(b) sets $y = 2(x - 2)^2 + 5$ and rearranges to $(x - 2)^2 = (y - 5)/2$ **M1**

(b) $x = 2 + \sqrt{(y - 5)/2}$ choosing the positive root since $x \geq 2$ **A1**

(b) $f^{-1}(x) = 2 + \sqrt{(x - 5)/2}$ **B1**

(b) domain $x \geq 5$ **B1**

(b) Answer: $f^{-1}(x) = 2 + \sqrt{(x - 5)/2}$, $x \geq 5$

(c) substitutes into the inverse function (or solves $f(x) = 37$) **M1**

(c) 6 **A1**

(c) Answer: 6

Question 6

(a) stationary point (3, -2) **B1**

(a) image of the intercept (-5, 0) **B1**

(a) Answer: (3, -2); (-5, 0)

(b) stationary point (-3/2, -2) **B1**

(b) image of the intercept (5/2, 0) **B1**

(b) Answer: (-3/2, -2); (5/2, 0)

(c) stationary point (-3, -4) **B1**

(c) image of the intercept (5, 0) **B1**

(c) Answer: (-3, -4); (5, 0)

Question 7

(a) correctly states the proportional relationship $C = kn$ (oe) **M1**

(a) k = 15 cao **A1**

(a) Answer: $C = 15n$, $k = 15$

(b) £105, ft using k from (a) **B1**

(b) Answer: £105

(c) straight line through the origin, drawn only for $n > 0$ **B1**

(c) gradient labelled as 15, ft from (a) **B1**

(c) coordinates of one further point on the line correct, e.g. (4, 60), ft **B1**

(c) Answer: Straight line through the origin, gradient 15, e.g. passing through (4, 60)

(d) correctly states the proportional relationship $F = k/w$ (oe) **M1**

(d) k = 480, so $F = 480/w$, cao **A1**

(d) correct branch shape: single decreasing curve lying entirely in the region $w > 0$, $F > 0$, never crossing either axis **B1**

(d) asymptotes labelled $w = 0$ and $F = 0$ **B1**

(d) one correct point on the curve labelled, e.g. (3, 160), ft **B1**

(d) Answer: $F = 480/w$; asymptotes $w = 0$ and $F = 0$; e.g. passes through (3, 160)

© Revision Library | revisionlibrary.co.uk