1.17 Pure: Functions, Graphs and Transformations: 2-hour Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The curve C has equation $y = (x + 3)(x - 1)^2$.

(a) Write down the coordinates of the points where C meets the x-axis, and state which is a point where the x-axis is a tangent. **(2)**

(b) Find the coordinates of the point where C meets the y-axis. **(2)**

(c) Show that C has a stationary point at $x = -5/3$ and find its y coordinate. **(3)**

(Total for Question 1 is 7 marks)

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2 The curve C with equation $y = f(x)$ has a maximum point at (-2, -3) and passes through the point (6, 0).

(a) Write down the coordinates of the maximum point on the curve with equation $y = f(-x)$, and the coordinates of the image of the point (6, 0). **(2)**

(b) Write down the coordinates of the minimum point on the curve with equation $y = -f(x)$, and the coordinates of the image of the point (6, 0). **(2)**

(c) Write down the coordinates of the maximum point on the curve with equation $y = f(x + 1)$, and the coordinates of the image of the point (6, 0). **(2)**

(Total for Question 2 is 6 marks)

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3 The function f is defined by $f(x) = 2(x + 3)^2 - 6$, for $x \geq -3$.

(a) State the range of f. **(1)**

(b) Find an expression for $f^{-1}(x)$, and state its domain. **(4)**

(c) Find the value of $f^{-1}(26)$. **(2)**

(Total for Question 3 is 7 marks)

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4 Solve the following involving the modulus function.

(a) Solve $|2x| = 4x - 4$. **(4)**

(b) Solve $|2x| < 4$. **(2)**

(Total for Question 4 is 6 marks)

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5 The curve C has equation $y = -4/(x - 4) + 3$, $x \neq 4$.

(a) Write down the equations of the asymptotes of C. **(2)**

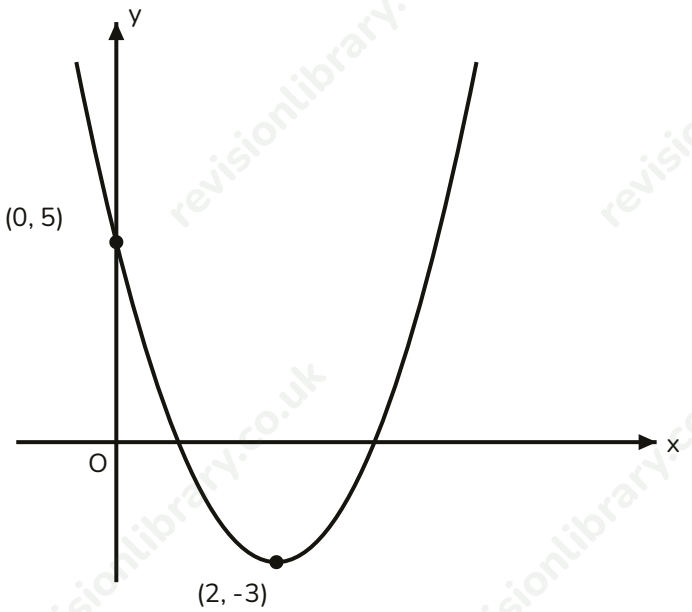
(b) Find the coordinates of the points where C meets the coordinate axes. **(4)**

(c) State whether the gradient of C is positive or negative for every x in its domain, giving a reason. **(2)**

(Total for Question 5 is 8 marks)

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6 The diagram shows a sketch of the curve $y = f(x)$.



(a) State the coordinates of the minimum turning point of the curve $y = f(x) + 4$. **(2)**

(b) State the coordinates of the minimum turning point of the curve $y = f(x - 3)$. **(2)**

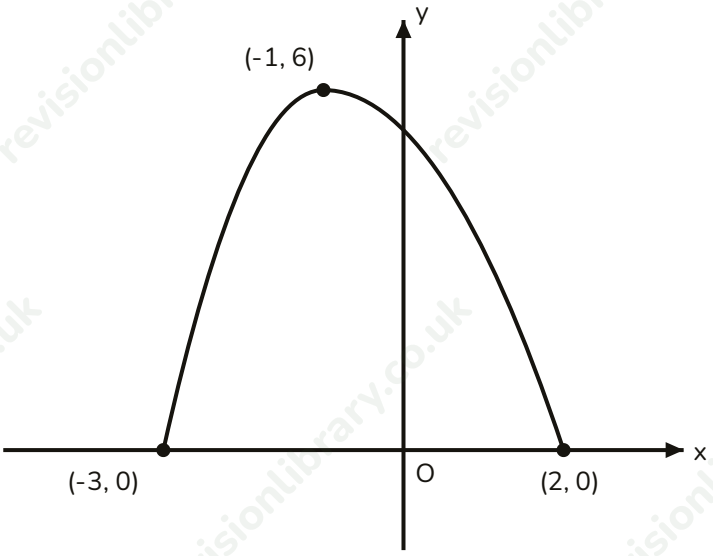
(c) The point A(0, 5) lies on the curve $y = f(x)$. State the coordinates of the image of A under the transformation that produces the curve $y = f(x + 1) - 2$. **(2)**

(d) Given further that the curve $y = f(x)$ crosses the x-axis only at $x = a$ and $x = b$, where $a < b$, state, in terms of a and b, the x-intercepts of the curve $y = f(x - 3)$. **(3)**

(Total for Question 6 is 9 marks)

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7 The diagram shows a sketch of the curve $y = g(x)$.



(a) State the coordinates of the maximum turning point of the curve $y = 3g(x)$. **(2)**

(b) State the coordinates of the maximum turning point of the curve $y = g(2x)$. **(2)**

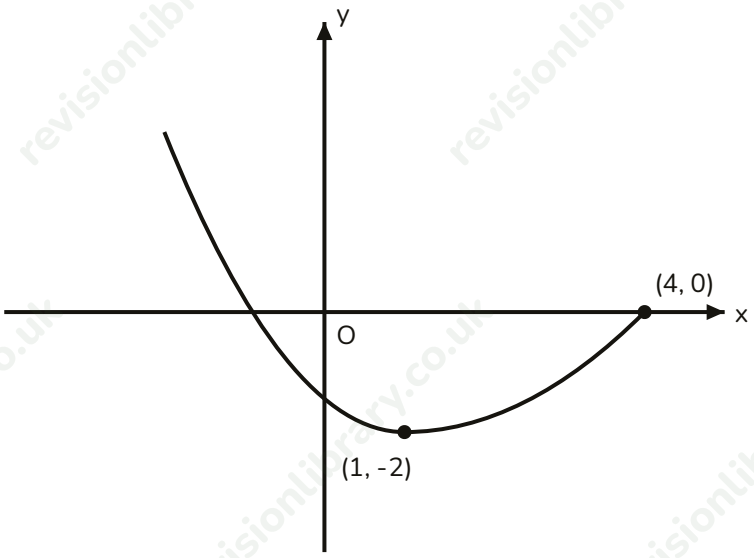
(c) State the coordinates of the two points where the curve $y = g(x/2)$ crosses the x-axis. **(3)**

(d) Given that the maximum value of $g(x)$ is 6, state the maximum value of the function $h(x) = 2g(x) - 5$, and the value of x at which this maximum occurs. **(2)**

(Total for Question 7 is 9 marks)

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8 The diagram shows a sketch of the curve $y = h(x)$.



(a) State the coordinates of the turning point of the curve $y = -h(x)$, and state whether it is a maximum or a minimum. **(2)**

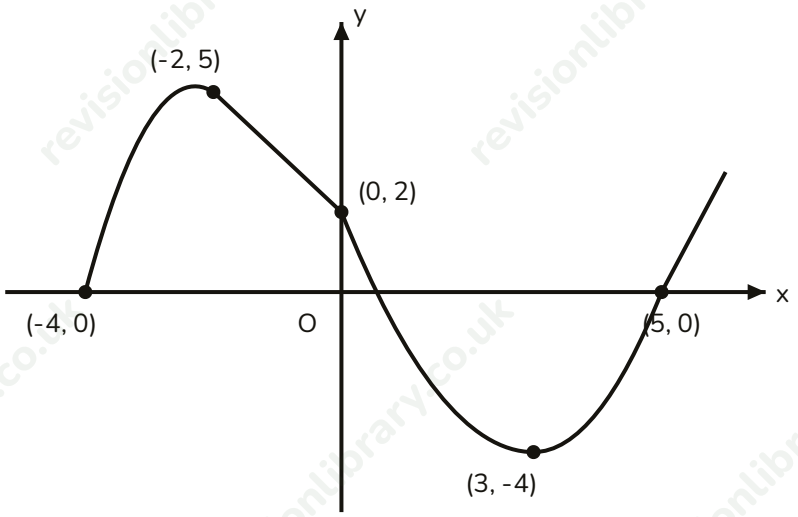
(b) State the root of the curve $y = h(-x)$. **(2)**

(c) State the coordinates of the turning point of the curve $y = -h(-x)$, and state whether it is a maximum or a minimum. **(3)**

(Total for Question 8 is 7 marks)

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9 The diagram shows a sketch of the curve $y = g(x)$ for $-4 \leq x \leq 6$.



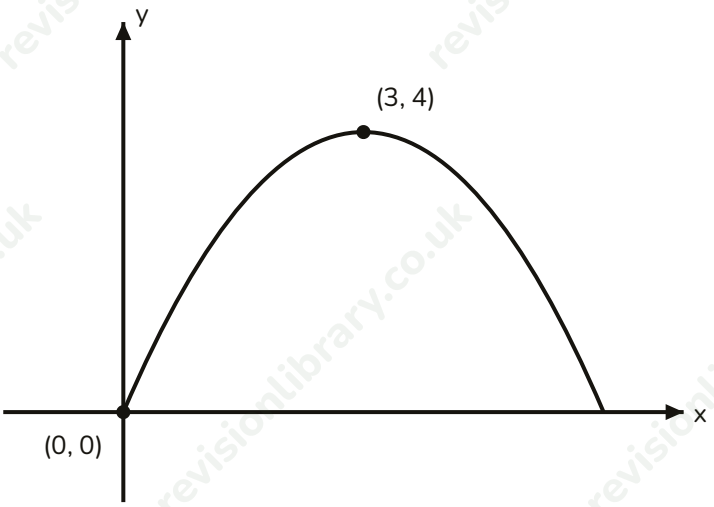
(a) On a sketch of $y = g(|x|)$, show the curve for $-6 \leq x \leq 6$. State the coordinates of all labelled points on your sketch. **(4)**

(b) Explain why the curve $y = g(|x|)$ is always symmetric about the y-axis, whatever the original function g. **(2)**

(Total for Question 9 is 6 marks)

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10 The diagram shows a sketch of the curve y = p(x). The curve y = q(x) is obtained from the curve y = p(x) by a stretch, scale factor 1/2, parallel to the x-axis, followed by a translation of 3 units in the positive y-direction.



(a) Write down q(x) in terms of p. **(2)**

(b) Find the coordinates of the image of the maximum turning point of y = p(x) under this combined transformation. **(3)**

(c) Find the coordinates of the image of the point (0, 0) on y = p(x) under the same combined transformation. **(2)**

(d) A different combined transformation is applied to y = p(x): a translation of 2 units in the negative x-direction, followed by a stretch, scale factor 3, parallel to the y-axis. State the resulting equation in terms of p, and find the image of the maximum turning point. **(3)**

(Total for Question 10 is 10 marks)

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11 The curve C has equation $y = 1/(x - 2)$, for x not equal to 2.

(a) Sketch the curve C, labelling the equations of both asymptotes. **(3)**

(b) The curve D has equation $y = |1/(x - 2)|$. Sketch the curve D for $-2 \leq x \leq 6$, x not equal to 2, and state the equations of its asymptotes. **(3)**

(c) The curve E is obtained from C by a stretch, scale factor 1/3, parallel to the y-axis, followed by a translation of 1 unit in the negative x-direction. Find the equation of E in the form $y = 1/(ax + b)$, where a and b are integers to be found, and state the equation of its vertical asymptote. **(4)**

(Total for Question 11 is 10 marks)

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12 The point P(2, 5) lies on the curve $y = k(x)$. The curve $y = m(x)$ is defined by $m(x) = -2*k(x - 1) + 3$.

(a) Describe fully the sequence of three geometric transformations that maps the curve $y = k(x)$ onto the curve $y = m(x)$, stating the order in which they should be applied. **(4)**

(b) Find the coordinates of the image of the point P under this combined transformation. **(3)**

(c) Given also that the curve $y = k(x)$ passes through the point Q(-3, 0), find the coordinates of the image of Q under the same combined transformation. **(3)**

(Total for Question 12 is 10 marks)

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Mark scheme · 1.17 Pure: Functions, Graphs and Transformations: 2-hour Pack

Question 1

(a) (-3, 0) and (1, 0) **B1**

(a) the x-axis touches C at (1, 0) because of the repeated factor **B1**

(a) Answer: (-3, 0), (1, 0); tangent at x = 1

(b) sets x = 0 **M1**

(b) (0, 3) **A1**

(b) Answer: (0, 3)

(c) $dy/dx = (x - 1)^2 + 2(x + 3)(x - 1)$ **M1**

(c) factorises as $(x - 1)(3x + 5) = 0$ **A1**

(c) x = -5/3 and y = 256/27 **A1**

(c) Answer: x = -5/3, y = 256/27

Question 2

(a) stationary point (2, -3) **B1**

(a) image of the intercept (-6, 0) **B1**

(a) Answer: (2, -3); (-6, 0)

(b) stationary point (-2, 3) **B1**

(b) image of the intercept (6, 0) **B1**

(b) Answer: (-2, 3); (6, 0)

(c) stationary point (-3, -3) **B1**

(c) image of the intercept (5, 0) **B1**

(c) Answer: (-3, -3); (5, 0)

Question 3

(a) $f(x) \geq -6$ **B1**

(a) Answer: $f(x) \geq -6$

(b) sets $y = 2(x + 3)^2 - 6$ and rearranges to $(x + 3)^2 = (y + 6)/2$ **M1**

(b) $x = -3 + \sqrt{(y + 6)/2}$ choosing the positive root since $x \geq -3$ **A1**

(b) $f^{-1}(x) = -3 + \sqrt{(x + 6)/2}$ **B1**

(b) domain $x \geq -6$ **B1**

(b) Answer: $f^{-1}(x) = -3 + \sqrt{(x + 6)/2}$, $x \geq -6$

(c) substitutes into the inverse function (or solves f(x) = 26) **M1**

(c) 1 **A1**

(c) Answer: 1

Question 4

(a) squares both sides, or considers 2x = 4x - 4 and 2x = -(4x - 4) **M1**

(a) obtains the candidate solutions 2, 2/3 **A1**

(a) checks each candidate in the original equation (the right side must be non-negative) **M1**

(a) x = 2 **A1**

(a) Answer: x = 2

(b) -4 < 2x < 4 **M1**

(b) -2 < x < 2 **A1**

(b) Answer: -2 < x < 2

Question 5

(a) x = 4 **B1**

(a) y = 3 **B1**

(a) Answer: x = 4 and y = 3

(b) sets x = 0 to find the y intercept **M1**

(b) (0, 4) **A1**

(b) sets y = 0 to find the x intercept **M1**

(b) (16/3, 0) **A1**

(b) Answer: (0, 4) and (16/3, 0)

(c) $dy/dx = 4/(x - 4)^2$ **M1**

(c) the sign is positive for all $x \neq 4$ since the square is positive **A1**

(c) Answer: positive

Question 6

(a) recognises the vertical translation by 4 units: y-coordinate of the minimum increases by 4, x-coordinate unchanged **M1**

(a) (2, 1) cao **A1**

(a) Answer: (2, 1)

(b) recognises the horizontal translation by 3 units in the positive x-direction: x-coordinate of the minimum increases by 3, y-coordinate unchanged **M1**

(b) (5, -3) cao **A1**

(b) Answer: (5, -3)

(c) recognises a translation of 1 unit in the negative x-direction combined with a translation of 2 units in the negative y-direction **M1**

(c) (-1, 3) cao **A1**

(c) Answer: (-1, 3)

(d) recognises the horizontal translation by 3 units in the positive x-direction applies to every point on the curve, including the x-intercepts **M1**

(d) x = a + 3 **A1**

(d) x = b + 3, both cao **A1**

(d) Answer: x = a + 3 and x = b + 3

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Question 7

(a) recognises a vertical stretch, scale factor 3: y-coordinate is multiplied by 3, x-coordinate unchanged **M1**

(a) (-1, 18) cao **A1**

(a) Answer: (-1, 18)

(b) recognises a horizontal stretch, scale factor 1/2: x-coordinate is multiplied by 1/2, y-coordinate unchanged **M1**

(b) (-0.5, 6) cao **A1**

(b) Answer: (-0.5, 6)

(c) recognises a horizontal stretch, scale factor 2: x-coordinate is multiplied by 2, y-coordinate unchanged **M1**

(c) (-6, 0) **A1**

(c) (4, 0), both cao **A1**

(c) Answer: (-6, 0) and (4, 0)

(d) maximum value = 2(6) - 5 = 7 **M1**

(d) occurs at x = -1 cao (the location of the maximiser is unchanged, since 2g(x) - 5 is a strictly increasing function of g(x)) **B1**

(d) Answer: Maximum value 7, occurring at x = -1

Question 8

(a) recognises a reflection in the x-axis: y-coordinate is negated, x-coordinate unchanged, and a minimum becomes a maximum **M1**

(a) (1, 2), a maximum, both needed, cao **A1**

(a) Answer: (1, 2), a maximum

(b) recognises a reflection in the y-axis: x-coordinate is negated, y-coordinate unchanged **M1**

(b) x = -4 cao **A1**

(b) Answer: x = -4

(c) recognises the combined effect: reflecting in the y-axis (x -> -x) and reflecting in the x-axis (y -> -y), applied to the point (1, -2) **M1**

(c) (-1, 2) cao **A1**

(c) a maximum, cao (reflecting a minimum in the x-axis produces a maximum; reflecting in the y-axis does not change whether a point is a maximum or minimum) **A1**

(c) Answer: (-1, 2), a maximum

Question 9

(a) recognises that the part of the given curve for x < 0 (through (-2, 5) and (-4, 0)) plays no part in y = g(|x|) **B1**

(a) the curve for x ≥ 0 is retained exactly as given, through (0, 2), minimum (3, -4), and (5, 0) **B1**

(a) the curve for x < 0 is the mirror image, in the y-axis, of the retained x ≥ 0 branch, giving a second minimum at (-3, -4) and a second x-intercept at (-5, 0) **B1**

(a) correct overall shape: symmetric about the y-axis, with minima at (3, -4) and (-3, -4), x-intercepts at (5, 0) and (-5, 0), and passing through (0, 2) **B1**

(a) Answer: y = g(|x|) is symmetric about the y-axis: it retains the x ≥ 0 branch through (0, 2), minimum (3, -4), (5, 0), and mirrors it for x < 0, giving a second minimum at (-3, -4) and x-intercept at (-5, 0); the given information about g for x < 0 (points (-2, 5) and (-4, 0)) plays no part

(b) states that replacing x with -x leaves |x| unchanged, since |-x| = |x| for every x **B1**

(b) correctly links this to g(|x|) being an even function, and every even function having a graph symmetric about the y-axis **B1**

(b) Answer: g(|x|) is an even function, because replacing x with -x leaves |x| unchanged (|-x| = |x|), so g(|-x|) = g(|x|) for every x; every even function has a graph symmetric about the y-axis

Question 10

(a) correctly applies the horizontal stretch, scale factor 1/2, first: replaces x with 2x to give p(2x) **M1**

(a) q(x) = p(2x) + 3 cao **A1**

(a) Answer: q(x) = p(2x) + 3

(b) applies the horizontal stretch to the x-coordinate: 3 * (1/2) = 1.5 **M1**

(b) applies the translation to the y-coordinate: 4 + 3 = 7 **A1**

(b) (1.5, 7) cao **A1**

(b) Answer: (1.5, 7)

(c) applies the horizontal stretch to the x-coordinate: 0 * (1/2) = 0, and the translation to the y-coordinate: 0 + 3 = 3 **M1**

(c) (0, 3) cao **A1**

(c) Answer: (0, 3)

(d) forms the combined function correctly in the given order: translate first to get p(x + 2), then stretch to get r(x) = 3p(x + 2) **M1**

(d) correctly translates the x-coordinate of the maximum: 3 - 2 = 1 **A1**

(d) correctly stretches the y-coordinate: 4 * 3 = 12, giving image (1, 12), cao **A1**

(d) Answer: y = 3p(x + 2); image of the maximum turning point = (1, 12)

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Question 11

(a) correct two-branch shape, the same shape as y = 1/x translated horizontally **B1**

(a) vertical asymptote x = 2 labelled correctly **B1**

(a) horizontal asymptote y = 0 labelled correctly **B1**

(a) Answer: Two-branch curve, the shape of y = 1/x translated 2 units in the positive x-direction; vertical asymptote x = 2, horizontal asymptote y = 0

(b) correct shape: both branches lie above the x-axis, since the branch for x > 2 (already positive) is unchanged and the branch for x < 2 (negative) is reflected in the x-axis **B1**

(b) vertical asymptote x = 2, unchanged from C **B1**

(b) horizontal asymptote y = 0, unchanged from C **B1**

(b) Answer: Both branches of D lie above the x-axis: unchanged for x > 2, reflected in the x-axis for x < 2; asymptotes x = 2 and y = 0, unchanged from C

(c) applies the vertical stretch, scale factor 1/3, to C: y = (1/3) * (1/(x - 2)) **M1**

(c) applies the horizontal translation of 1 unit in the negative x-direction: replaces x with x + 1, giving y = (1/3) * (1/((x + 1) - 2)) = (1/3) * (1/(x - 1)) **M1**

(c) simplifies to y = 1/(3x - 3), i.e. a = 3, b = -3, cao **A1**

(c) vertical asymptote x = 1, ft from the translated denominator **B1**

(c) Answer: y = 1/(3x - 3) (a = 3, b = -3); vertical asymptote x = 1

Question 12

(a) a translation of 1 unit in the positive x-direction (applied first, giving k(x - 1)) **B1**

(a) a stretch, scale factor -2, parallel to the y-axis (applied second, giving -2k(x - 1)); accept described as a stretch scale factor 2 combined with a reflection in the x-axis, provided both the factor and the reflection are stated **B1**

(a) a translation of 3 units in the positive y-direction (applied third and last, giving -2k(x - 1) + 3) **B1**

(a) correct order stated: translation in x, then stretch in y, then translation in y, matching this build-up **B1**

(a) Answer: Translation of 1 unit in the positive x-direction, then a stretch scale factor -2 parallel to the y-axis, then a translation of 3 units in the positive y-direction, applied in that order

(b) applies the horizontal translation to the x-coordinate: 2 + 1 = 3 **M1**

(b) applies the stretch, reflection and vertical translation to the y-coordinate: 5*(-2) + 3 = -7 **M1**

(b) (3, -7) cao **A1**

(b) Answer: (3, -7)

(c) applies the horizontal translation to the x-coordinate: -3 + 1 = -2 **M1**

(c) applies the stretch, reflection and vertical translation to the y-coordinate: 0*(-2) + 3 = 3 **M1**

(c) (-2, 3) cao **A1**

(c) Answer: (-2, 3)

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