1.17 Pure: Functions, Graphs and Transformations: Test Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The curve C has equation $y = (x + 3)(x - 1)(x - 2)$.

(a) Write down the coordinates of the three points where C crosses the x-axis. **(2)**

(b) Write down the coordinate where C crosses the y-axis. **(1)**

(c) State the behaviour of y as x -> +infinity and as x -> -infinity. **(1)**

(d) Hence sketch the graph of C for $-4 \leq x \leq 3$, labelling the coordinates of the points where the curve meets the coordinate axes. **(3)**

(Total for Question 1 is 7 marks)

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2 The function f is defined by $f(x) = (6x + 5)/(x - 1)$, x in R, $x \neq 1$. The function g is defined by $g(x) = x^2 + 5$, x in R.

(a) Find $f^{-1}(x)$, stating its domain. **(4)**

(b) Write down the range of f. **(1)**

(c) Find fg(x) and hence solve $fg(x) = 131/20$. **(4)**

(Total for Question 2 is 9 marks)

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3 The curve C has equation $y = -2/(x + 3) + 2$, $x \neq -3$.

(a) Write down the equations of the asymptotes of C. **(2)**

(b) Find the coordinates of the points where C meets the coordinate axes. **(4)**

(c) State whether the gradient of C is positive or negative for every x in its domain, giving a reason. **(2)**

(Total for Question 3 is 8 marks)

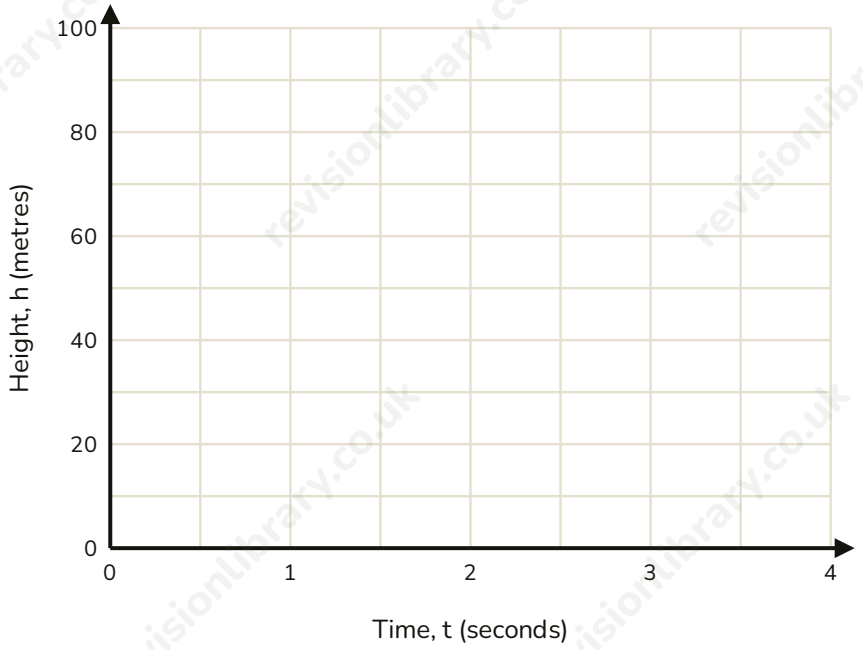
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4 A model rocket is launched from rest. For the first 4 seconds of flight, the height, h metres, reached by the rocket is proportional to the square of the time, t seconds, since launch. After 2 seconds, the rocket has reached a height of 20 metres.

(a) Find an equation connecting h and t, and state the value of the constant of proportionality. **(2)**

(b) Find the height reached after 3.5 seconds. **(2)**

(c) Sketch the graph of h against t for $0 \leq t \leq 4$, labelling the coordinates of the endpoints of the curve. **(3)**



(d) The intensity, I watts per square metre, of light received from a lamp is inversely proportional to the square of the distance, x metres, from the lamp. At a distance of 2 m the intensity is 45 W/m^2. Find an equation connecting I and x, and hence find the distance at which the intensity is 5 W/m^2. **(4)**

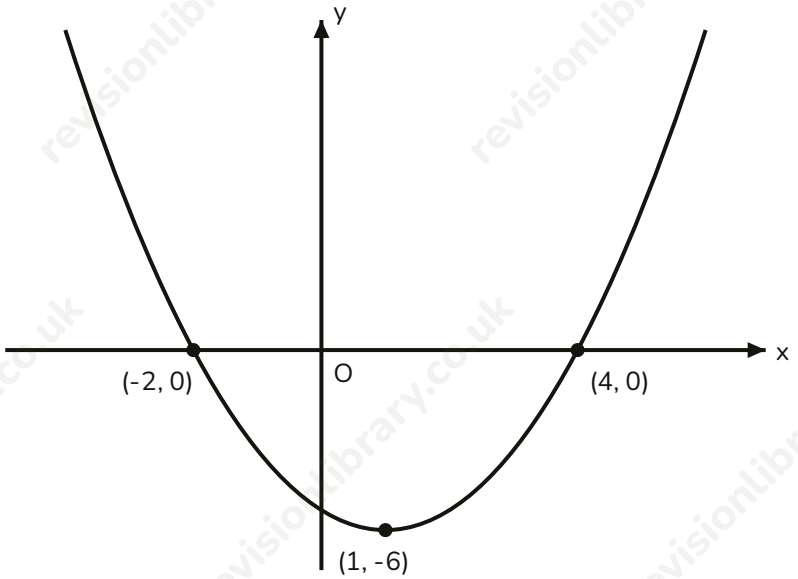
(e) For a relationship of the form $y = kx^2$, state the effect on y of doubling x. For a relationship of the form $y = k*\sqrt{x}$, state the effect on y of doubling x, giving your answer as an exact multiplier. **(2)**

(f) The time, T seconds, for one complete swing of a simple pendulum is directly proportional to the square root of its length, L metres. A pendulum of length 4 m has a swing time of 4 seconds. Find an equation connecting T and L, and find the time for one swing of a pendulum of length 9 m. **(3)**

(Total for Question 4 is 16 marks)

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5 The diagram shows a sketch of the curve $y = f(x)$ for $-4 \leq x \leq 6$.



(a) On a sketch of $y = |f(x)|$, show the effect of the modulus transformation. State the coordinates of the images of the minimum turning point and the two x-intercepts. **(4)**

(b) By considering the interval $-2 \leq x \leq 4$ only, state the number of solutions to the equation $|f(x)| = 3$ in this interval, giving a reason for your answer. **(2)**

(c) State the number of solutions to the equation $|f(x)| = -2$, and justify your answer briefly. **(2)**

(Total for Question 5 is 8 marks)

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Mark scheme · 1.17 Pure: Functions, Graphs and Transformations: Test Pack

Question 1

(a) at least two of (-3, 0), (1, 0), (2, 0) stated correctly **B1**

(a) all three x-intercepts correct: (-3, 0), (1, 0), (2, 0), cao **B1**

(a) Answer: (-3, 0), (1, 0), (2, 0)

(b) (0, 6) cao **B1**

(b) Answer: (0, 6)

(c) y -> +infinity as x -> +infinity, and y -> -infinity as x -> -infinity (since the highest power of x, x^3, has a positive coefficient), both directions needed **B1**

(c) Answer: y -> +infinity as x -> +infinity; y -> -infinity as x -> -infinity

(d) correct general shape: single curve with a local maximum between x = -3 and x = 1 and a local minimum between x = 1 and x = 2, consistent with (c), ft **B1**

(d) all three x-intercepts labelled (-3, 0), (1, 0), (2, 0), ft from (a) **B1**

(d) y-intercept labelled (0, 6), ft from (b) **B1**

(d) Answer: Cubic curve rising from bottom-left, local maximum between x=-3 and x=1, local minimum between x=1 and x=2, then rising to top-right; passes through (-3,0), (1,0), (2,0) and (0,6)

Question 2

(a) sets $y = (6x + 5)/(x - 1)$ and rearranges: $y(x - 1) = 6x + 5$ **M1**

(a) collects x terms: $x(y - 6) = y + 5$ **M1**

(a) $f^{-1}(x) = (x + 5)/(x - 6)$ **A1**

(a) domain x in R, $x \neq 6$ **B1**

(a) Answer: $f^{-1}(x) = (x + 5)/(x - 6)$, $x \neq 6$

(b) $f(x) \neq 6$, i.e. y in R, $y \neq 6$ **B1**

(b) Answer: $y \neq 6$

(c) $fg(x) = f(x^2 + 5) = (6(x^2 + 5) + 5)/(x^2 + 5 - 1)$ **M1**

(c) sets equal to 131/20 and rearranges to a quadratic in x^2 (or x) **M1**

(c) $x^2 = 16$ **A1**

(c) x = 4 or x = -4 **A1**

(c) Answer: $x = \pm 4$

Question 3

(a) x = -3 **B1**

(a) y = 2 **B1**

(a) Answer: x = -3 and y = 2

(b) sets x = 0 to find the y intercept **M1**

(b) (0, 4/3) **A1**

(b) sets y = 0 to find the x intercept **M1**

(b) (-2, 0) **A1**

(b) Answer: (0, 4/3) and (-2, 0)

(c) $dy/dx = 2/(x + 3)^2$ **M1**

(c) the sign is positive for all $x \neq -3$ since the square is positive **A1**

(c) Answer: positive

Question 4

(a) correctly states the proportional relationship $h = kt^2$ (oe) **M1**

(a) k = 5 cao **A1**

(a) Answer: $h = 5t^2$, k = 5

(b) substitutes t = 3.5 into $h = 5t^2$ **M1**

(b) h = 61.25 m cao **A1**

(b) Answer: 61.25 m

(c) correct shape: single increasing, concave-up curve (positive quadratic through the origin), no other turning points **B1**

(c) endpoint (0, 0) labelled **B1**

(c) endpoint (4, 80) labelled, ft from (a) **B1**

(c) Answer: Increasing, concave-up curve from (0, 0) to (4, 80)

(d) correctly states the proportional relationship $I = k/x^2$ (oe) **M1**

(d) k = 180, so $I = 180/x^2$, cao **A1**

(d) sets $5 = 180/x^2$ and rearranges to $x^2 = 36$ **M1**

(d) x = 6 m cao (rejects x = -6 since distance must be positive) **A1**

(d) Answer: $I = 180/x^2$; x = 6 m

(e) y is multiplied by 4, oe **B1**

(e) y is multiplied by $\sqrt{2}$, oe **B1**

(e) Answer: y proportional to x^2: multiplied by 4. y proportional to $\sqrt{x}$: multiplied by $\sqrt{2}$

(f) correctly states the proportional relationship $T = k*\sqrt{L}$ (oe) **M1**

(f) k = 2, so $T = 2*\sqrt{L}$, cao **A1**

(f) T = 6 seconds, ft using k from this part **B1**

(f) Answer: $T = 2*\sqrt{L}$; T = 6 seconds when L = 9

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Question 5

(a) the section of the curve that lies below the x-axis (for $-2 < x < 4$) is reflected in the x-axis; the sections that already lie above the x-axis ($x < -2$ and $x > 4$) are unchanged **B1**

(a) the minimum turning point (1, -6) becomes a maximum turning point at (1, 6) **B1**

(a) the x-intercepts are unchanged at (-2, 0) and (4, 0), since points on the x-axis map to themselves **B1**

(a) correct overall shape: the entire curve lies on or above the x-axis, touching (not crossing) the x-axis at $x = -2$ and $x = 4$ **B1**

(a) Answer: $y = |f(x)|$ matches $y = f(x)$ for $x \leq -2$ and $x \geq 4$; between $x = -2$ and $x = 4$ the curve is reflected in the x-axis, giving a maximum turning point at (1, 6); x-intercepts unchanged at (-2, 0) and (4, 0); the entire curve lies on or above the x-axis

(b) 2 solutions **B1**

(b) correct reason: $|f(x)|$ rises continuously from 0 (at $x = -2$) to 6 (at $x = 1$) then falls continuously back to 0 (at $x = 4$); since $0 < 3 < 6$, the horizontal line $y = 3$ meets this single hump exactly once on each side of $x = 1$ **B1**

(b) Answer: 2 solutions

(c) 0 solutions (no solutions) **B1**

(c) correct justification: $|f(x)| \geq 0$ for all x, so it can never equal a negative number **B1**

(c) Answer: 0 solutions, since $|f(x)| \geq 0$ for every x

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