1.17 Pure: Functions, Graphs and Transformations

EDEXCEL 9MA0 · Calculator allowed · about 230 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The curves C1: $y = 1/x$ and C2: $y = 1/x^2$ are both defined for x not equal to 0.

(a) State the equations of the asymptotes of C1. **(2)**

(b) Sketch the graph of C1 for $-3 \leq x \leq 3$, x not equal to 0, showing the shape of the curve in each of the regions $x > 0$ and $x < 0$. **(3)**

(c) Explain how the graph of C2: $y = 1/x^2$ differs from the graph of C1 in the regions $x > 0$ and $x < 0$. **(2)**

(d) Write down the equations of the asymptotes of C2, and state the number of points at which C2 crosses either coordinate axis. **(2)**

(Total for Question 1 is 9 marks)

2 The function f is defined by $f(x) = 2(x + 4)^2 - 1$, for $x \geq -4$.

(a) State the range of f. **(1)**

(b) Find an expression for $f^{-1}(x)$, and state its domain. **(4)**

(c) Find the value of $f^{-1}(17)$. **(2)**

(Total for Question 2 is 7 marks)

3 Solve the following involving the modulus function.

(a) Solve $|2x - 3| = x - 8$. **(4)**

(b) Solve $|2x - 3| < 4$. **(2)**

(Total for Question 3 is 6 marks)

4 The function f is defined by $f(x) = (5x + 7)/(x - 2)$, x in R, $x \neq 2$. The function g is defined by $g(x) = x^2 + 4$, x in R.

(a) Find $f^{-1}(x)$, stating its domain. **(4)**

(b) Write down the range of f. **(1)**

(c) Find fg(x) and hence solve $fg(x) = 107/18$. **(4)**

(Total for Question 4 is 9 marks)

5 The graph of $y = f(x)$ is transformed in each of the ways described below.

(a) Describe fully the single transformation that maps the graph of $y = f(x)$ onto the graph of $y = f(x - 3)$. **(2)**

(b) Describe fully the single transformation that maps $y = f(x)$ onto $y = f(3x)$. **(2)**

(c) The graph $y = f(x)$ is mapped onto $y = 3f(x - 3) + 4$ by a sequence of transformations. State the three transformations in a suitable order. **(3)**

(Total for Question 5 is 7 marks)

6 The graph of $y = e^x$ is transformed to the graph of $y = e^{x - 1} + 4$.

(a) Describe the single translation that maps $y = e^x$ onto $y = e^{x - 1} + 4$. **(2)**

(b) Write down the equation of the horizontal asymptote of the new curve and the exact coordinates of its intersection with the y axis. **(2)**

(Total for Question 6 is 4 marks)

7 The curve C has equation $y = (x + 4)(x - 2)^2$.

(a) Write down the coordinates of the points where C meets the x-axis, and state which is a point where the x-axis is a tangent. **(2)**

(b) Find the coordinates of the point where C meets the y-axis. **(2)**

(c) Show that C has a stationary point at $x = -2$ and find its y coordinate. **(3)**

(Total for Question 7 is 7 marks)

8 The curve C has equation $y = 6/(x + 2) - 1$, $x \neq -2$.

(a) Write down the equations of the asymptotes of C. **(2)**

(b) Find the coordinates of the points where C meets the coordinate axes. **(4)**

(c) State whether the gradient of C is positive or negative for every x in its domain, giving a reason. **(2)**

(Total for Question 8 is 8 marks)

9 The graph of $y = \sin x$, in degrees, is transformed to the graph of $y = 4 \sin(x - 20) + 6$.

(a) Describe the sequence of three transformations that maps $y = \sin x$ onto $y = 4 \sin(x - 20) + 6$. **(3)**

(b) Write down the maximum value of y and the smallest positive value of x at which it occurs. **(2)**

(Total for Question 9 is 5 marks)

10 The function f is defined by $f(x) = 2(x - 2)^2 + 5$, for $x \geq 2$.

(a) State the range of f. **(1)**

(b) Find an expression for $f^{-1}(x)$, and state its domain. **(4)**

(c) Find the value of $f^{-1}(37)$. **(2)**

(Total for Question 10 is 7 marks)

11 The curve C with equation $y = f(x)$ has a minimum point at $(-3, -2)$ and passes through the point $(5, 0)$.

(a) Write down the coordinates of the minimum point on the curve with equation $y = f(-x)$, and the coordinates of the image of the point $(5, 0)$. **(2)**

.....

(b) Write down the coordinates of the minimum point on the curve with equation $y = f(2x)$, and the coordinates of the image of the point $(5, 0)$. **(2)**

.....

(c) Write down the coordinates of the minimum point on the curve with equation $y = 2f(x)$, and the coordinates of the image of the point $(5, 0)$. **(2)**

.....

(Total for Question 11 is 6 marks)

12 The curve C has equation $y = (x + 3)(x - 1)^2$.

(a) Write down the coordinates of the points where C meets the x-axis, and state which is a point where the x-axis is a tangent. **(2)**

(b) Find the coordinates of the point where C meets the y-axis. **(2)**

(c) Show that C has a stationary point at $x = -5/3$ and find its y coordinate. **(3)**

(Total for Question 12 is 7 marks)

13 The curve C with equation $y = f(x)$ has a maximum point at $(-2, -3)$ and passes through the point $(6, 0)$.

(a) Write down the coordinates of the maximum point on the curve with equation $y = f(-x)$, and the coordinates of the image of the point $(6, 0)$. **(2)**

.....

(b) Write down the coordinates of the minimum point on the curve with equation $y = -f(x)$, and the coordinates of the image of the point $(6, 0)$. **(2)**

.....

(c) Write down the coordinates of the maximum point on the curve with equation $y = f(x + 1)$, and the coordinates of the image of the point $(6, 0)$. **(2)**

.....

(Total for Question 13 is 6 marks)

14 The function f is defined by $f(x) = 2(x + 3)^2 - 6$, for $x \geq -3$.

(a) State the range of f. **(1)**

(b) Find an expression for $f^{-1}(x)$, and state its domain. **(4)**

(c) Find the value of $f^{-1}(26)$. **(2)**

(Total for Question 14 is 7 marks)

15 Solve the following involving the modulus function.

(a) Solve $|2x| = 4x - 4$. **(4)**

(b) Solve $|2x| < 4$. **(2)**

(Total for Question 15 is 6 marks)

16 The curve C has equation $y = -4/(x - 4) + 3$, $x \neq 4$.

(a) Write down the equations of the asymptotes of C. **(2)**

(b) Find the coordinates of the points where C meets the coordinate axes. **(4)**

(c) State whether the gradient of C is positive or negative for every x in its domain, giving a reason. **(2)**

(Total for Question 16 is 8 marks)

17 A campsite hire company found that the cost, C pounds, of hiring a bell tent for a weekend is directly proportional to the number of guests, n, that the tent must sleep. A tent that sleeps 4 guests costs £60 to hire.

(a) Write down an equation connecting C and n, in terms of a constant k, and find the value of k. **(2)**

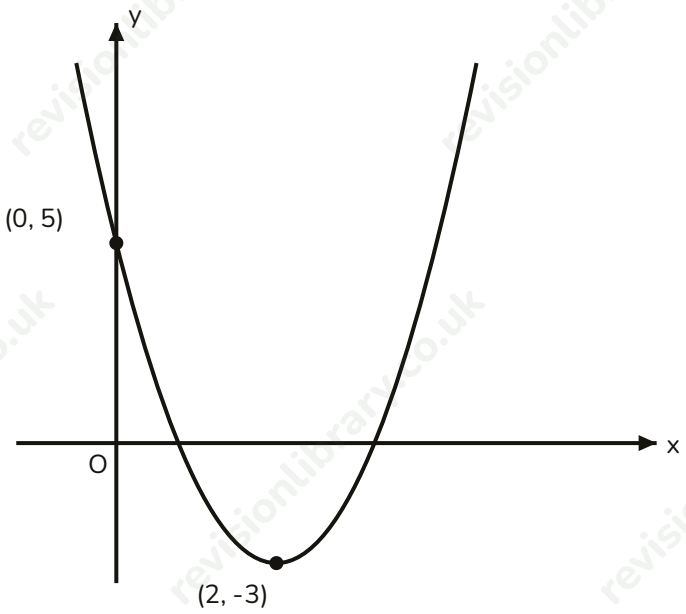
(b) State the cost of hiring a tent that sleeps 7 guests. **(1)**

(c) Sketch the graph of C against n for $n > 0$, labelling the gradient of the line and the coordinates of one other point on the line. **(3)**

(d) A different company hires the same style of tent for a fee, F pounds per weekend, that is inversely proportional to the number of weekends, w, that the customer books, for $w > 0$. A customer who books 3 weekends is charged a fee of £160 per weekend. Find an equation connecting F and w, and sketch the graph of F against w for $w > 0$, labelling the equations of the asymptotes and the coordinates of one point on the curve. **(5)**

(Total for Question 17 is 11 marks)

18 The diagram shows a sketch of the curve y = f(x).



(a) State the coordinates of the minimum turning point of the curve y = f(x) + 4. **(2)**

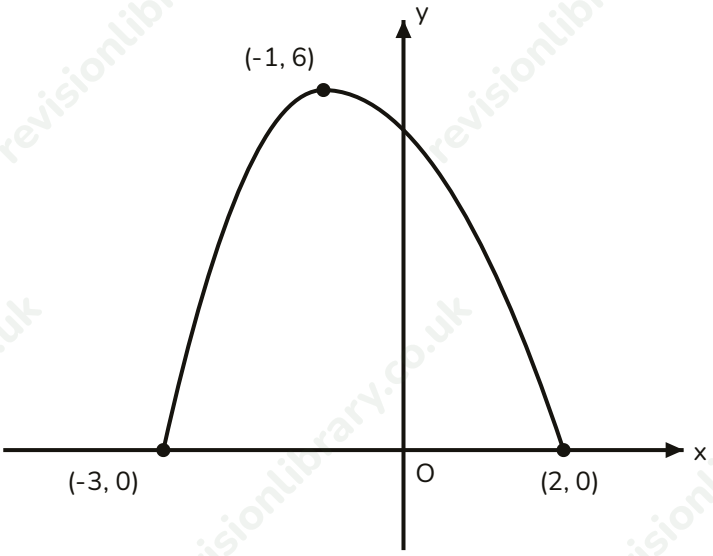
(b) State the coordinates of the minimum turning point of the curve y = f(x - 3). **(2)**

(c) The point A(0, 5) lies on the curve y = f(x). State the coordinates of the image of A under the transformation that produces the curve y = f(x + 1) - 2. **(2)**

(d) Given further that the curve $y = f(x)$ crosses the x-axis only at $x = a$ and $x = b$, where $a < b$, state, in terms of a and b, the x-intercepts of the curve $y = f(x - 3)$. **(3)**

(Total for Question 18 is 9 marks)

19 The diagram shows a sketch of the curve $y = g(x)$.



(a) State the coordinates of the maximum turning point of the curve $y = 3g(x)$. **(2)**

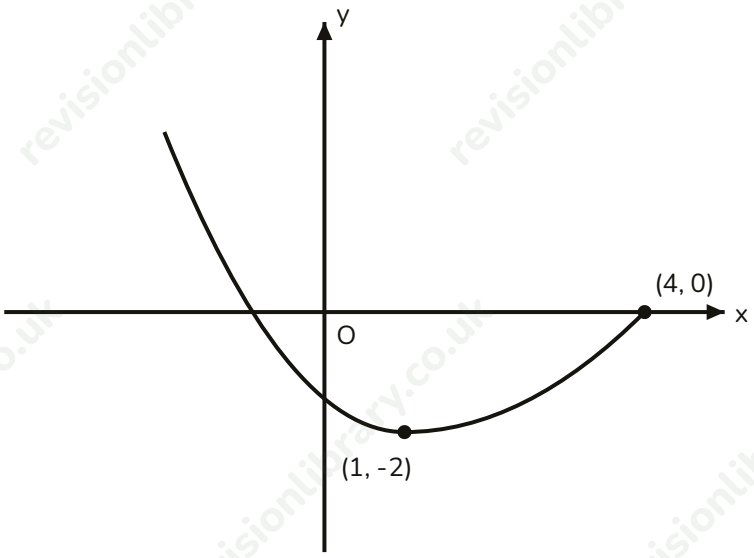
(b) State the coordinates of the maximum turning point of the curve $y = g(2x)$. **(2)**

(c) State the coordinates of the two points where the curve $y = g(x/2)$ crosses the x-axis. **(3)**

(d) Given that the maximum value of g(x) is 6, state the maximum value of the function h(x) = 2g(x) - 5, and the value of x at which this maximum occurs. **(2)**

(Total for Question 19 is 9 marks)

20 The diagram shows a sketch of the curve y = h(x).



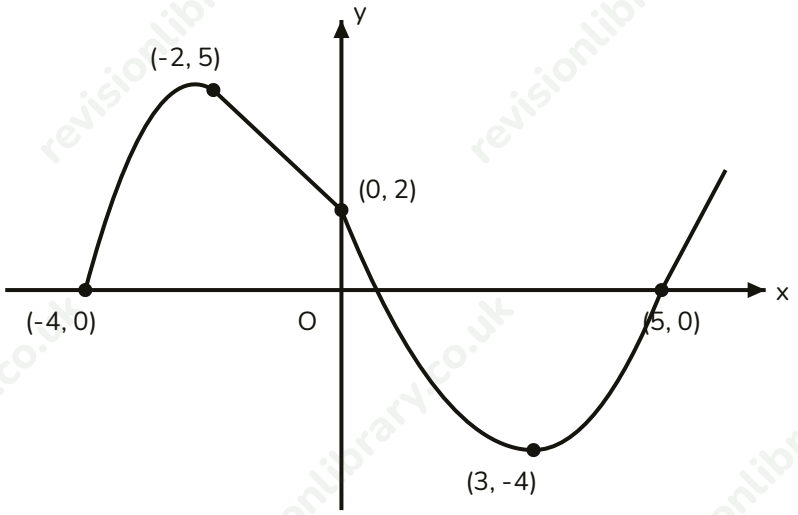
(a) State the coordinates of the turning point of the curve y = -h(x), and state whether it is a maximum or a minimum. **(2)**

(b) State the root of the curve y = h(-x). **(2)**

(c) State the coordinates of the turning point of the curve y = -h(-x), and state whether it is a maximum or a minimum. **(3)**

(Total for Question 20 is 7 marks)

21 The diagram shows a sketch of the curve $y = g(x)$ for $-4 \leq x \leq 6$.

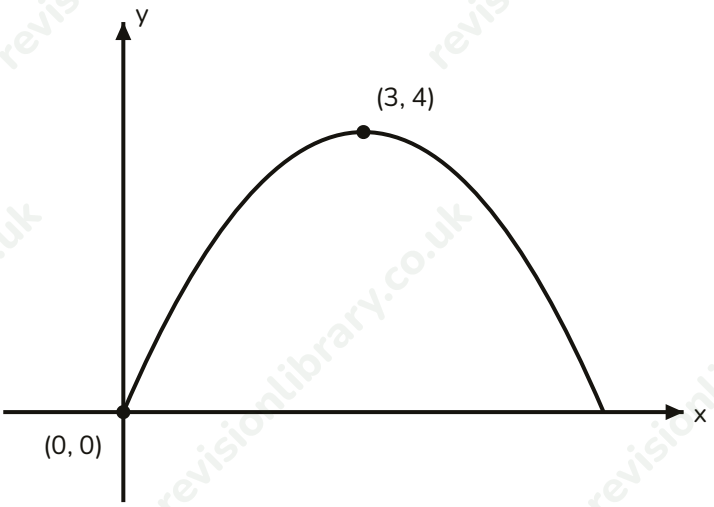


(a) On a sketch of $y = g(|x|)$, show the curve for $-6 \leq x \leq 6$. State the coordinates of all labelled points on your sketch. **(4)**

(b) Explain why the curve $y = g(|x|)$ is always symmetric about the y-axis, whatever the original function g. **(2)**

(Total for Question 21 is 6 marks)

22 The diagram shows a sketch of the curve y = p(x). The curve y = q(x) is obtained from the curve y = p(x) by a stretch, scale factor 1/2, parallel to the x-axis, followed by a translation of 3 units in the positive y-direction.



(a) Write down q(x) in terms of p. **(2)**

(b) Find the coordinates of the image of the maximum turning point of y = p(x) under this combined transformation. **(3)**

(c) Find the coordinates of the image of the point (0, 0) on y = p(x) under the same combined transformation. **(2)**

(d) A different combined transformation is applied to $y = p(x)$: a translation of 2 units in the negative x-direction, followed by a stretch, scale factor 3, parallel to the y-axis. State the resulting equation in terms of p, and find the image of the maximum turning point. **(3)**

(Total for Question 22 is 10 marks)

23 The curve C has equation $y = 1/(x - 2)$, for x not equal to 2.

(a) Sketch the curve C, labelling the equations of both asymptotes. **(3)**

(b) The curve D has equation $y = |1/(x - 2)|$. Sketch the curve D for $-2 \leq x \leq 6$, x not equal to 2, and state the equations of its asymptotes. **(3)**

(c) The curve E is obtained from C by a stretch, scale factor 1/3, parallel to the y-axis, followed by a translation of 1 unit in the negative x-direction. Find the equation of E in the form $y = 1/(ax + b)$, where a and b are integers to be found, and state the equation of its vertical asymptote. **(4)**

(Total for Question 23 is 10 marks)

24 The point P(2, 5) lies on the curve y = k(x). The curve y = m(x) is defined by m(x) = -2*k(x - 1) + 3.

(a) Describe fully the sequence of three geometric transformations that maps the curve y = k(x) onto the curve y = m(x), stating the order in which they should be applied. **(4)**

(b) Find the coordinates of the image of the point P under this combined transformation. **(3)**

(c) Given also that the curve y = k(x) passes through the point Q(-3, 0), find the coordinates of the image of Q under the same combined transformation. **(3)**

(Total for Question 24 is 10 marks)

Mark scheme · 1.17 Pure: Functions, Graphs and Transformations

Question 1

(a) x = 0 cao **B1**

(a) y = 0 cao **B1**

(a) Answer: x = 0 and y = 0

(b) correct branch for x > 0: positive, decreasing towards y=0 as x increases, increasing without bound as x -> 0 from the right **B1**

(b) correct branch for x < 0: negative, increasing towards y=0 as x decreases, decreasing without bound as x -> 0 from the left **B1**

(b) both asymptotes x=0 and y=0 drawn/labelled **B1**

(b) Answer: Two-branch curve: one branch in the region x>0, y>0 approaching y=0 as x increases and approaching the y-axis as x->0+; one branch in the region x<0, y<0 approaching y=0 as x decreases and approaching the y-axis as x->0-; asymptotes x=0 and y=0

(c) both branches of C2 lie above the x-axis (y > 0 for all x not equal to 0), since squaring removes negative values, unlike C1 which is negative for x < 0 **B1**

(c) C2 is symmetric about the y-axis (an even function), whereas C1 has rotational symmetry about the origin (an odd function) **B1**

(c) Answer: C2 lies entirely above the x-axis and is symmetric about the y-axis; C1 has one branch above the axis (x>0) and one below (x<0), with rotational symmetry about the origin

(d) asymptotes x = 0 and y = 0, both needed **B1**

(d) C2 crosses neither axis: 0 intercepts, cao **B1**

(d) Answer: Asymptotes x = 0 and y = 0; C2 crosses neither axis (0 intercepts)

Question 2

(a) f(x) ≥ -1 **B1**

(a) Answer: f(x) ≥ -1

(b) sets $y = 2(x + 4)^2 - 1$ and rearranges to $(x + 4)^2 = (y + 1)/2$ **M1**

(b) $x = -4 + \sqrt{(y + 1)/2}$ choosing the positive root since x ≥ -4 **A1**

(b) $f^{-1}(x) = -4 + \sqrt{(x + 1)/2}$ **B1**

(b) domain x ≥ -1 **B1**

(b) Answer: $f^{-1}(x) = -4 + \sqrt{(x + 1)/2}$, x ≥ -1

(c) substitutes into the inverse function (or solves f(x) = 17) **M1**

(c) -1 **A1**

(c) Answer: -1

Question 3

(a) squares both sides, or considers 2x - 3 = x - 8 and 2x - 3 = -(x - 8) **M1**

(a) obtains the candidate solutions -5, 11/3 **A1**

(a) checks each candidate in the original equation (the right side must be non-negative) **M1**

(a) no solutions **A1**

(a) Answer: no solutions

(b) -4 < 2x - 3 < 4 **M1**

(b) -1/2 < x < 7/2 **A1**

(b) Answer: -1/2 < x < 7/2

Question 4

(a) sets y = (5x + 7)/(x - 2) and rearranges: y(x - 2) = 5x + 7 **M1**

(a) collects x terms: x(y - 5) = 2y + 7 **M1**

(a) $f^{-1}(x) = (2x + 7)/(x - 5)$ **A1**

(a) domain x in R, x ≠ 5 **B1**

(a) Answer: $f^{-1}(x) = (2x + 7)/(x - 5)$, x ≠ 5

(b) f(x) ≠ 5, i.e. y in R, y ≠ 5 **B1**

(b) Answer: y ≠ 5

(c) $fg(x) = f(x^2 + 4) = (5(x^2 + 4) + 7)/(x^2 + 4 - 2)$ **M1**

(c) sets equal to 107/18 and rearranges to a quadratic in x^2 (or x) **M1**

(c) $x^2 = 16$ **A1**

(c) x = 4 or x = -4 **A1**

(c) Answer: x = ±4

Question 5

(a) translation **B1**

(a) by the vector (3, 0), i.e. 3 units in the positive x direction **B1**

(a) Answer: translation by (3, 0)

(b) stretch **B1**

(b) scale factor 1/3 parallel to the x axis **B1**

(b) Answer: stretch, scale factor 1/3, x direction

(c) translation by (3, 0) **B1**

(c) stretch scale factor 3 parallel to the y axis **B1**

(c) translation by (0, 4) **B1**

(c) Answer: translate (3, 0); stretch y by 3; translate (0, 4)

Question 6

(a) translation **B1**

(a) by the vector (1, 4) **B1**

(a) Answer: translation by (1, 4)

(b) y = 4 **B1**

(b) $(0, e^{-1} + 4)$ **B1**

(b) Answer: y = 4; $(0, e^{-1} + 4)$

Question 7

(a) (-4, 0) and (2, 0) **B1**

(a) the x-axis touches C at (2, 0) because of the repeated factor **B1**

(a) Answer: (-4, 0), (2, 0); tangent at x = 2

(b) sets x = 0 **M1**

(b) (0, 16) **A1**

(b) Answer: (0, 16)

(c) $dy/dx = (x - 2)^2 + 2(x + 4)(x - 2)$ **M1**

(c) factorises as (x - 2)(3x + 6) = 0 **A1**

(c) x = -2 and y = 32 **A1**

(c) Answer: x = -2, y = 32

Question 8

(a) x = -2 **B1**

(a) y = -1 **B1**

(a) Answer: x = -2 and y = -1

(b) sets x = 0 to find the y intercept **M1**

(b) (0, 2) **A1**

(b) sets y = 0 to find the x intercept **M1**

(b) (4, 0) **A1**

(b) Answer: (0, 2) and (4, 0)

(c) $dy/dx = -6/(x + 2)^2$ **M1**

(c) the sign is negative for all $x \neq -2$ since the square is positive **A1**

(c) Answer: negative

Question 9

(a) translation by (20, 0), i.e. 20 degrees to the right **B1**

(a) stretch scale factor 4 parallel to the y axis **B1**

(a) translation by (0, 6) **B1**

(a) Answer: translate (20, 0); stretch y by 4; translate (0, 6)

(b) maximum y = 10 **B1**

(b) x - 20 = 90, so x = 110 **B1**

(b) Answer: 10 at x = 110

Question 10

(a) $f(x) \geq 5$ **B1**

(a) Answer: $f(x) \geq 5$

(b) sets $y = 2(x - 2)^2 + 5$ and rearranges to $(x - 2)^2 = (y - 5)/2$ **M1**

(b) $x = 2 + \sqrt{(y - 5)/2}$ choosing the positive root since $x \geq 2$ **A1**

(b) $f^{-1}(x) = 2 + \sqrt{(x - 5)/2}$ **B1**

(b) domain $x \geq 5$ **B1**

(b) Answer: $f^{-1}(x) = 2 + \sqrt{(x - 5)/2}$, $x \geq 5$

(c) substitutes into the inverse function (or solves f(x) = 37) **M1**

(c) 6 **A1**

(c) Answer: 6

Question 11

(a) stationary point (3, -2) **B1**

(a) image of the intercept (-5, 0) **B1**

(a) Answer: (3, -2); (-5, 0)

(b) stationary point (-3/2, -2) **B1**

(b) image of the intercept (5/2, 0) **B1**

(b) Answer: (-3/2, -2); (5/2, 0)

(c) stationary point (-3, -4) **B1**

(c) image of the intercept (5, 0) **B1**

(c) Answer: (-3, -4); (5, 0)

Question 12

(a) (-3, 0) and (1, 0) **B1**

(a) the x-axis touches C at (1, 0) because of the repeated factor **B1**

(a) Answer: (-3, 0), (1, 0); tangent at x = 1

(b) sets x = 0 **M1**

(b) (0, 3) **A1**

(b) Answer: (0, 3)

(c) $dy/dx = (x - 1)^2 + 2(x + 3)(x - 1)$ **M1**

(c) factorises as $(x - 1)(3x + 5) = 0$ **A1**

(c) x = -5/3 and y = 256/27 **A1**

(c) Answer: x = -5/3, y = 256/27

Question 13

(a) stationary point (2, -3) **B1**

(a) image of the intercept (-6, 0) **B1**

(a) Answer: (2, -3); (-6, 0)

(b) stationary point (-2, 3) **B1**

(b) image of the intercept (6, 0) **B1**

(b) Answer: (-2, 3); (6, 0)

(c) stationary point (-3, -3) **B1**

(c) image of the intercept (5, 0) **B1**

(c) Answer: (-3, -3); (5, 0)

Question 14

(a) $f(x) \geq -6$ **B1**

(a) Answer: $f(x) \geq -6$

(b) sets $y = 2(x + 3)^2 - 6$ and rearranges to $(x + 3)^2 = (y + 6)/2$ **M1**

(b) $x = -3 + \sqrt{(y + 6)/2}$ choosing the positive root since $x \geq -3$ **A1**

(b) $f^{-1}(x) = -3 + \sqrt{(x + 6)/2}$ **B1**

(b) domain $x \geq -6$ **B1**

(b) Answer: $f^{-1}(x) = -3 + \sqrt{(x + 6)/2}$, $x \geq -6$

(c) substitutes into the inverse function (or solves f(x) = 26) **M1**

(c) 1 **A1**

(c) Answer: 1

Question 15

(a) squares both sides, or considers 2x = 4x - 4 and 2x = -(4x - 4) **M1**

(a) obtains the candidate solutions 2, 2/3 **A1**

(a) checks each candidate in the original equation (the right side must be non-negative) **M1**

(a) x = 2 **A1**

(a) Answer: x = 2

(b) -4 < 2x < 4 **M1**

(b) -2 < x < 2 **A1**

(b) Answer: -2 < x < 2

Question 16

(a) x = 4 **B1**

(a) y = 3 **B1**

(a) Answer: x = 4 and y = 3

(b) sets x = 0 to find the y intercept **M1**

(b) (0, 4) **A1**

(b) sets y = 0 to find the x intercept **M1**

(b) (16/3, 0) **A1**

(b) Answer: (0, 4) and (16/3, 0)

(c) $dy/dx = 4/(x - 4)^2$ **M1**

(c) the sign is positive for all $x \neq 4$ since the square is positive **A1**

(c) Answer: positive

Question 17

(a) correctly states the proportional relationship C = kn (oe) **M1**

(a) k = 15 cao **A1**

(a) Answer: C = 15n, k = 15

(b) £105, ft using k from (a) **B1**

(b) Answer: £105

(c) straight line through the origin, drawn only for n > 0 **B1**

(c) gradient labelled as 15, ft from (a) **B1**

(c) coordinates of one further point on the line correct, e.g. (4, 60), ft **B1**

(c) Answer: Straight line through the origin, gradient 15, e.g. passing through (4, 60)

(d) correctly states the proportional relationship F = k/w (oe) **M1**

(d) k = 480, so F = 480/w, cao **A1**

(d) correct branch shape: single decreasing curve lying entirely in the region w > 0, F > 0, never crossing either axis **B1**

(d) asymptotes labelled w = 0 and F = 0 **B1**

(d) one correct point on the curve labelled, e.g. (3, 160), ft **B1**

(d) Answer: F = 480/w; asymptotes w = 0 and F = 0; e.g. passes through (3, 160)

Question 18

(a) recognises the vertical translation by 4 units: y-coordinate of the minimum increases by 4, x-coordinate unchanged **M1**

(a) (2, 1) cao **A1**

(a) Answer: (2, 1)

(b) recognises the horizontal translation by 3 units in the positive x-direction: x-coordinate of the minimum increases by 3, y-coordinate unchanged **M1**

(b) (5, -3) cao **A1**

(b) Answer: (5, -3)

(c) recognises a translation of 1 unit in the negative x-direction combined with a translation of 2 units in the negative y-direction **M1**

(c) (-1, 3) cao **A1**

(c) Answer: (-1, 3)

(d) recognises the horizontal translation by 3 units in the positive x-direction applies to every point on the curve, including the x-intercepts **M1**

(d) x = a + 3 **A1**

(d) x = b + 3, both cao **A1**

(d) Answer: x = a + 3 and x = b + 3

Question 19

(a) recognises a vertical stretch, scale factor 3: y-coordinate is multiplied by 3, x-coordinate unchanged **M1**

(a) (-1, 18) cao **A1**

(a) Answer: (-1, 18)

(b) recognises a horizontal stretch, scale factor 1/2: x-coordinate is multiplied by 1/2, y-coordinate unchanged **M1**

(b) (-0.5, 6) cao **A1**

(b) Answer: (-0.5, 6)

(c) recognises a horizontal stretch, scale factor 2: x-coordinate is multiplied by 2, y-coordinate unchanged **M1**

(c) (-6, 0) **A1**

(c) (4, 0), both cao **A1**

(c) Answer: (-6, 0) and (4, 0)

(d) maximum value = 2(6) - 5 = 7 **M1**

(d) occurs at x = -1 cao (the location of the maximiser is unchanged, since 2g(x) - 5 is a strictly increasing function of g(x)) **B1**

(d) Answer: Maximum value 7, occurring at x = -1

Question 20

(a) recognises a reflection in the x-axis: y-coordinate is negated, x-coordinate unchanged, and a minimum becomes a maximum **M1**

(a) (1, 2), a maximum, both needed, cao **A1**

(a) Answer: (1, 2), a maximum

(b) recognises a reflection in the y-axis: x-coordinate is negated, y-coordinate unchanged **M1**

(b) x = -4 cao **A1**

(b) Answer: x = -4

(c) recognises the combined effect: reflecting in the y-axis (x -> -x) and reflecting in the x-axis (y -> -y), applied to the point (1, -2) **M1**

(c) (-1, 2) cao **A1**

(c) a maximum, cao (reflecting a minimum in the x-axis produces a maximum; reflecting in the y-axis does not change whether a point is a maximum or minimum) **A1**

(c) Answer: (-1, 2), a maximum

Question 21

(a) recognises that the part of the given curve for $x < 0$ (through (-2, 5) and (-4, 0)) plays no part in $y = g(|x|)$ **B1**

(a) the curve for $x \geq 0$ is retained exactly as given, through (0, 2), minimum (3, -4), and (5, 0) **B1**

(a) the curve for $x < 0$ is the mirror image, in the y-axis, of the retained $x \geq 0$ branch, giving a second minimum at (-3, -4) and a second x-intercept at (-5, 0) **B1**

(a) correct overall shape: symmetric about the y-axis, with minima at (3, -4) and (-3, -4), x-intercepts at (5, 0) and (-5, 0), and passing through (0, 2) **B1**

(a) Answer: $y = g(|x|)$ is symmetric about the y-axis: it retains the $x \geq 0$ branch through (0, 2), minimum (3, -4), (5, 0), and mirrors it for $x < 0$, giving a second minimum at (-3, -4) and x-intercept at (-5, 0); the given information about g for $x < 0$ (points (-2, 5) and (-4, 0)) plays no part

(b) states that replacing x with -x leaves |x| unchanged, since $|-x| = |x|$ for every x **B1**

(b) correctly links this to g(|x|) being an even function, and every even function having a graph symmetric about the y-axis **B1**

(b) Answer: g(|x|) is an even function, because replacing x with -x leaves |x| unchanged ($|-x| = |x|$), so $g(|-x|) = g(|x|)$ for every x; every even function has a graph symmetric about the y-axis

Question 22

(a) correctly applies the horizontal stretch, scale factor 1/2, first: replaces x with 2x to give p(2x) **M1**

(a) q(x) = p(2x) + 3 cao **A1**

(a) Answer: q(x) = p(2x) + 3

(b) applies the horizontal stretch to the x-coordinate: 3 * (1/2) = 1.5 **M1**

(b) applies the translation to the y-coordinate: 4 + 3 = 7 **A1**

(b) (1.5, 7) cao **A1**

(b) Answer: (1.5, 7)

(c) applies the horizontal stretch to the x-coordinate: 0 * (1/2) = 0, and the translation to the y-coordinate: 0 + 3 = 3 **M1**

(c) (0, 3) cao **A1**

(c) Answer: (0, 3)

(d) forms the combined function correctly in the given order: translate first to get p(x + 2), then stretch to get r(x) = 3p(x + 2) **M1**

(d) correctly translates the x-coordinate of the maximum: 3 - 2 = 1 **A1**

(d) correctly stretches the y-coordinate: 4 * 3 = 12, giving image (1, 12), cao **A1**

(d) Answer: y = 3p(x + 2); image of the maximum turning point = (1, 12)

Question 23

(a) correct two-branch shape, the same shape as $y = 1/x$ translated horizontally **B1**

(a) vertical asymptote x = 2 labelled correctly **B1**

(a) horizontal asymptote y = 0 labelled correctly **B1**

(a) Answer: Two-branch curve, the shape of $y = 1/x$ translated 2 units in the positive x-direction; vertical asymptote x = 2, horizontal asymptote y = 0

(b) correct shape: both branches lie above the x-axis, since the branch for $x > 2$ (already positive) is unchanged and the branch for $x < 2$ (negative) is reflected in the x-axis **B1**

(b) vertical asymptote x = 2, unchanged from C **B1**

(b) horizontal asymptote y = 0, unchanged from C **B1**

(b) Answer: Both branches of D lie above the x-axis: unchanged for $x > 2$, reflected in the x-axis for $x < 2$; asymptotes x = 2 and y = 0, unchanged from C

(c) applies the vertical stretch, scale factor 1/3, to C: $y = (1/3) * (1/(x - 2))$ **M1**

(c) applies the horizontal translation of 1 unit in the negative x-direction: replaces x with x + 1, giving $y = (1/3) * (1/((x + 1) - 2)) = (1/3) * (1/(x - 1))$ **M1**

(c) simplifies to $y = 1/(3x - 3)$, i.e. a = 3, b = -3, cao **A1**

(c) vertical asymptote x = 1, ft from the translated denominator **B1**

(c) Answer: $y = 1/(3x - 3)$ (a = 3, b = -3); vertical asymptote x = 1

Question 24

(a) a translation of 1 unit in the positive x-direction (applied first, giving k(x - 1)) **B1**

(a) a stretch, scale factor -2, parallel to the y-axis (applied second, giving -2k(x - 1)); accept described as a stretch scale factor 2 combined with a reflection in the x-axis, provided both the factor and the reflection are stated **B1**

(a) a translation of 3 units in the positive y-direction (applied third and last, giving -2k(x - 1) + 3) **B1**

(a) correct order stated: translation in x, then stretch in y, then translation in y, matching this build-up **B1**

(a) Answer: Translation of 1 unit in the positive x-direction, then a stretch scale factor -2 parallel to the y-axis, then a translation of 3 units in the positive y-direction, applied in that order

(b) applies the horizontal translation to the x-coordinate: 2 + 1 = 3 **M1**

(b) applies the stretch, reflection and vertical translation to the y-coordinate: 5*(-2) + 3 = -7 **M1**

(b) (3, -7) cao **A1**

(b) Answer: (3, -7)

(c) applies the horizontal translation to the x-coordinate: -3 + 1 = -2 **M1**

(c) applies the stretch, reflection and vertical translation to the y-coordinate: 0*(-2) + 3 = 3 **M1**

(c) (-2, 3) cao **A1**

(c) Answer: (-2, 3)