1.18 Pure: Trigonometric Graphs: 1-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Solve $\cos^2 x + \sin x - 1 = 0$ for $-180 < x \leq 180$, giving all solutions in degrees.

(a) Use $\cos^2 x = 1 - \sin^2 x$ to show that the equation can be written as a quadratic in $\sin x$, and state that quadratic. **(3)**

(b) Hence solve the equation. **(4)**

(Total for Question 1 is 7 marks)

---

2 The graph of $y = f(x)$ is transformed in each of the ways described below.

(a) Describe fully the single transformation that maps the graph of $y = f(x)$ onto the graph of $y = f(x - 3)$. **(2)**

(b) Describe fully the single transformation that maps $y = f(x)$ onto $y = f(3x)$. **(2)**

(c) The graph $y = f(x)$ is mapped onto $y = 3f(x - 3) + 1$ by a sequence of transformations. State the three transformations in a suitable order. **(3)**

(Total for Question 2 is 7 marks)

---

3 The acute angles A and B satisfy $\sin A = 4/5$ and $\cos B = 20/29$.

(a) Find the exact values of $\cos A$, $\sin B$ and $\tan A$. **(3)**

(b) Find the exact value of $\cos(A + B)$. **(3)**

(c) Find the exact value of $\sin(A + B)$. **(3)**

(d) Find the exact value of $\sin 2A$ and of $\cos 2A$. **(3)**

(Total for Question 3 is 12 marks)

© Revision Library | revisionlibrary.co.uk

4 $f(x) = 8 \sin x + 1 \cos x$, where x is in degrees.

(a) Express $f(x)$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < 90$ degrees. Give R as an exact value and α to 1 decimal place. **(4)**

(b) Write down the maximum value of $f(x)$, and the smallest positive value of x at which it occurs, to 1 decimal place. **(2)**

(c) Solve $f(x) = 2$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 4 is 10 marks)

---

5 This question concerns the graph of $y = \tan(x)$ for $-180 \text{ degrees} \leq x \leq 180$ degrees.

(a) State the period, in degrees, of $y = \tan(x)$. **(1)**

(b) Sketch $y = \tan(x)$ for $-180 \text{ degrees} \leq x \leq 180$ degrees, showing the vertical asymptotes and the points where the curve crosses the x-axis. **(4)**

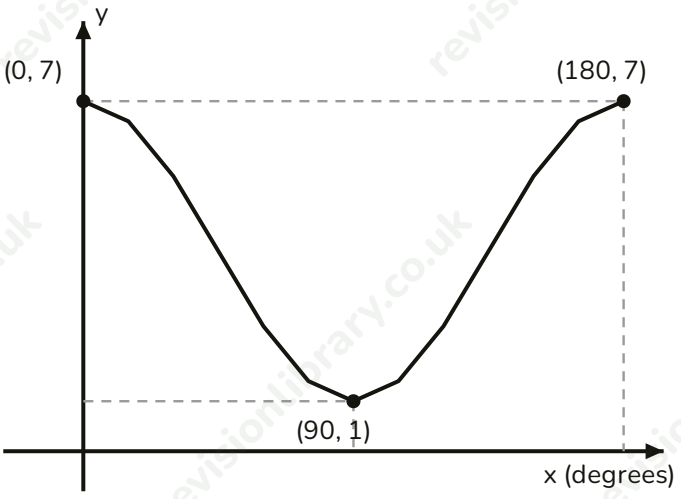
(c) State the equations of the vertical asymptotes of $y = \tan(x)$ for $-180 \text{ degrees} \leq x \leq 180$ degrees. **(2)**

(d) State the domain restriction that applies to $y = \tan(x)$ for all real x, using n to represent an integer. **(2)**

(Total for Question 5 is 9 marks)

---

6 The diagram shows part of the curve $y = a + b \cos(cx)$ for $0 \text{ degrees} \leq x \leq 180$ degrees, where a, b and c are positive constants.



(a) Using the information shown in the diagram, find the values of the constants a, b and c. **(6)**

(b) State the range of $y = a + b \cos(cx)$, using your values from part (a). **(1)**

(Total for Question 6 is 7 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 1.18 Pure: Trigonometric Graphs: 1-hour Pack

Question 1

(a) replaces $\cos^2 x$ by $1 - \sin^2 x$ **M1**

(a) collects terms **M1**

(a) $s^2 - s = 0$ where $s = \sin x$ **A1**

(a) Answer: $s^2 - s = 0$

(b) solves the quadratic **M1**

(b) sin x = 1 or sin x = 0 **A1**

(b) uses inverse sine and symmetry for each value **M1**

(b) 0, 90, 180 degrees **A1**

(b) Answer: 0, 90, 180 degrees

Question 2

(a) translation **B1**

(a) by the vector (3, 0), i.e. 3 units in the positive x direction **B1**

(a) Answer: translation by (3, 0)

(b) stretch **B1**

(b) scale factor 1/3 parallel to the x axis **B1**

(b) Answer: stretch, scale factor 1/3, x direction

(c) translation by (3, 0) **B1**

(c) stretch scale factor 3 parallel to the y axis **B1**

(c) translation by (0, 1) **B1**

(c) Answer: translate (3, 0); stretch y by 3; translate (0, 1)

Question 3

(a) uses $\sin^2 + \cos^2 = 1$ for each angle (or a right-angled triangle) **M1**

(a) cos A = 3/5, tan A = 4/3 **A1**

(a) sin B = 21/29 **A1**

(a) Answer: cos A = 3/5, sin B = 21/29, tan A = 4/3

(b) uses cos(A + B) = cos A cos B - sin A sin B **M1**

(b) substitutes the exact values **M1**

(b) -24/145 **A1**

(b) Answer: -24/145

(c) uses sin(A + B) = sin A cos B + cos A sin B **M1**

(c) substitutes the exact values **M1**

(c) 143/145 **A1**

(c) Answer: 143/145

(d) uses sin 2A = 2 sin A cos A and cos 2A = 1 - 2 $\sin^2$ A **M1**

(d) sin 2A = 24/25 **A1**

(d) cos 2A = -7/25 **A1**

(d) Answer: sin 2A = 24/25, cos 2A = -7/25

Question 4

(a) R cos α = 8 and R sin α = 1 **M1**

(a) $R = \sqrt{65}$ **A1**

(a) tan α = 1/8 **M1**

(a) α = 7.1 degrees **A1**

(a) Answer: $f(x) = \sqrt{65}\sin(x + 7.1)$

(b) maximum $\sqrt{65}$ **B1**

(b) x + 7.1 = 90, so x = 82.9 **B1**

(b) Answer: maximum $\sqrt{65}$ at x = 82.9

(c) $\sin(x + 7.1) = 2/\sqrt{65}$ **M1**

(c) x + 7.1 = 14.4 or 165.6 (and adds 360 if needed) **M1**

(c) x = 7.2 **A1**

(c) x = 158.5 **A1**

(c) Answer: x = 7.2, 158.5

Question 5

(a) 180 degrees cao **B1**

(a) Answer: 180 degrees

(b) three separate increasing branches drawn, each going from -infinity to +infinity **B1**

(b) two vertical asymptotes drawn as dashed lines at x = -90 and x = 90 **B1**

(b) x-intercepts correctly plotted at x = -180, 0, 180 **B1**

(b) curve shows correct symmetry about the origin (odd function), passing through (0, 0) **B1**

(b) Answer: Three increasing branches, asymptotes at x = -90 and x = 90, crossing the x-axis at x = -180, 0, 180

(c) x = -90 **B1**

(c) x = 90 **B1**

(c) Answer: x = -90, x = 90

(d) recognises the asymptote pattern repeats with period 180 degrees from a base value of 90 degrees **M1**

(d) x is not equal to 90 + 180n, n an integer, oe cao **A1**

(d) Answer: x is not equal to 90 + 180n, for n an integer

Question 6

(a) uses a = (maximum + minimum) / 2, e.g. a = (7 + 1) / 2 **M1**

(a) a = 4 **A1**

(a) uses b = (maximum - minimum) / 2, e.g. b = (7 - 1) / 2 **M1**

(a) b = 3 **A1**

(a) uses the period shown (180 degrees, from maximum to maximum) with period = 360 / c **M1**

(a) c = 2 cao **A1**

(a) Answer: a = 4, b = 3, c = 2

(b) $1 \leq y \leq 7$ oe, ft from values of a and b in (a) **B1**

(b) Answer: $1 \leq y \leq 7$

© Revision Library | revisionlibrary.co.uk