1.18 Pure: Trigonometric Graphs: 2-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question is about trigonometric identities and equations.

(a) Show that $(1 + \sin x)/\cos x + \cos x/(1 + \sin x) = 2 \sec x$. **(3)**

(b) Hence solve $(1 + \sin x)/\cos x + \cos x/(1 + \sin x) = 4$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(3)**

(Total for Question 1 is 6 marks)

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2 Solve $2\cos^2 x + \sin x - 1 = 0$ for $-180 < x \leq 180$, giving all solutions in degrees.

(a) Use $\cos^2 x = 1 - \sin^2 x$ to show that the equation can be written as a quadratic in $\sin x$, and state that quadratic. **(3)**

(b) Hence solve the equation. **(4)**

(Total for Question 2 is 7 marks)

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3 The curve C has equation $y = 4 \sin x + 2$, for $0 \leq x \leq 360$, where x is in degrees.

(a) Write down the amplitude of the curve, its period, and its maximum and minimum values. **(4)**

(b) Write down the number of complete cycles of C in the given range, and the coordinates of the point where C crosses the y-axis. **(2)**

(c) Solve $4 \sin x + 2 = 4$ for $0 \leq x < 360$. **(4)**

(Total for Question 3 is 10 marks)

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4 The curve C has equation $y = 3 \sin(3x) - 2$, for $0 \leq x \leq 360$, where x is in degrees.

(a) Write down the amplitude of the curve, its period, and its maximum and minimum values. **(4)**

(b) Write down the number of complete cycles of C in the given range, and the coordinates of the point where C crosses the y-axis. **(2)**

(c) Solve $3 \sin(3x) - 2 = -1/2$ for $0 \leq x < 360$. **(4)**

(Total for Question 4 is 10 marks)

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5 This question concerns the graph of $y = \sin(x) + 1$ for 0 degrees $\leq x \leq 360$ degrees.

(a) Sketch $y = \sin(x) + 1$ for 0 degrees $\leq x \leq 360$ degrees. **(3)**

(b) State the period of $y = \sin(x) + 1$, and the coordinates of its maximum and minimum points for 0 degrees $\leq x \leq 360$ degrees. **(3)**

(c) Find the coordinates of the point where the curve $y = \sin(x) + 1$ touches the x-axis, and explain why the curve touches the x-axis at this point rather than crossing it. **(2)**

(Total for Question 5 is 8 marks)

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6 This question concerns the graph of $y = \cos(2x)$ for 0 degrees $\leq x \leq 360$ degrees.

(a) State the period, in degrees, of $y = \cos(2x)$. **(1)**

(b) Sketch $y = \cos(2x)$ for 0 degrees $\leq x \leq 360$ degrees. **(3)**

(c) Find the exact coordinates of every point where $y = \cos(2x)$ crosses the x-axis for 0 degrees $\leq x \leq 360$ degrees. **(3)**

(Total for Question 6 is 7 marks)

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7 This question concerns the graph of $y = \text{cosec}(x)$ for 0 degrees $\leq x \leq 360$ degrees.

(a) Write $\text{cosec}(x)$ in terms of $\sin(x)$. **(1)**

(b) State the values of x in the range 0 degrees $\leq x \leq 360$ degrees for which $\text{cosec}(x)$ is undefined, explaining why $\text{cosec}(x)$ is undefined at these values. **(3)**

(c) Sketch $y = \text{cosec}(x)$ for 0 degrees $< x < 360$ degrees, x not equal to 180 degrees, showing the vertical asymptotes and the coordinates of the two turning points. **(4)**

(d) State the range of $y = \text{cosec}(x)$. **(1)**

(Total for Question 7 is 9 marks)

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8 This question concerns the graph of $y = \sec(x)$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$.

(a) Write $\sec(x)$ in terms of $\cos(x)$. **(1)**

(b) State the values of x in the range $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$ for which $\sec(x)$ is undefined, giving a reason. **(3)**

(c) Sketch $y = \sec(x)$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$, x not equal to 90 degrees or 270 degrees, showing the vertical asymptotes and the coordinates of the turning points. **(4)**

(d) State the range of $y = \sec(x)$. **(1)**

(Total for Question 8 is 9 marks)

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9 This question uses the graphs of $y = \sec(x)$ and $y = \cot(x)$.

(a) Explain, using the definition $\sec(x) = 1 / \cos(x)$, why the range of $y = \sec(x)$ cannot include any value of y satisfying $-1 < y < 1$. **(3)**

(b) State, with a reason, the number of values of x in the interval $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$ for which $\sec(x) = 0.5$. **(2)**

(c) By considering the graph of $y = \cot(x)$ for $0 \text{ degrees} < x < 360 \text{ degrees}$, determine the number of solutions of the equation $\cot(x) = k$ in this range, where k is any real number. **(4)**

(Total for Question 9 is 9 marks)

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10 This question concerns the graph of $y = \cot(x)$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$.

(a) Write $\cot(x)$ in terms of $\tan(x)$, and also in terms of $\sin(x)$ and $\cos(x)$. **(2)**

(b) State the period, in degrees, of $y = \cot(x)$. **(1)**

(c) State the values of x in the range $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$ for which $\cot(x)$ is undefined. **(2)**

(d) State the x-intercepts of $y = \cot(x)$ for $0 \text{ degrees} < x < 360 \text{ degrees}$. **(2)**

(e) Determine, with justification, whether $y = \cot(x)$ is an odd function or an even function. **(3)**

(Total for Question 10 is 10 marks)

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11 The function g is defined by $g(x) = 3 \operatorname{cosec}(x) + 2$, for $0 \text{ degrees} < x < 360 \text{ degrees}$, x not equal to 180 degrees.

(a) State the range of $g(x)$. **(2)**

(b) Sketch $y = g(x)$ for $0 \text{ degrees} < x < 360 \text{ degrees}$, x not equal to 180 degrees, showing the equations of the vertical asymptotes and the coordinates of the turning points. **(4)**

(c) Show that the equation $g(x) = 8$ can be written in the form $\sin(x) = 1/2$. **(2)**

(d) Hence solve $g(x) = 8$ for $0 \text{ degrees} < x < 360 \text{ degrees}$, giving your answers in degrees. **(2)**

(Total for Question 11 is 10 marks)

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Mark scheme · 1.18 Pure: Trigonometric Graphs: 2-hour Pack

Question 1

(a) writes over a common denominator cos x (1 + sin x) **M1**

(a) numerator $(1 + \sin x)^2 + \cos^2 x = 2 + 2 \sin x$ **M1**

(a) factorises 2(1 + sin x) and cancels, giving 2/cos x = 2 sec x (cso) **A1**

(a) Answer: expression = 2/cos x = 2 sec x

(b) uses the identity: 2 sec x = 4, so cos x = 1/2 **M1**

(b) x = 60.0 **A1**

(b) x = 300.0 **A1**

(b) Answer: x = 60.0, 300.0

Question 2

(a) replaces $\cos^2 x$ by $1 - \sin^2 x$ **M1**

(a) collects terms **M1**

(a) $2s^2 - s - 1 = 0$ where s = sin x **A1**

(a) Answer: $2s^2 - s - 1 = 0$

(b) solves the quadratic **M1**

(b) sin x = 1 or sin x = -1/2 **A1**

(b) uses inverse sine and symmetry for each value **M1**

(b) -150, -30, 90 degrees **A1**

(b) Answer: -150, -30, 90 degrees

Question 3

(a) amplitude 4 **B1**

(a) period 360 degrees **B1**

(a) maximum value 6 **B1**

(a) minimum value -2 **B1**

(a) Answer: amplitude 4, period 360, max 6, min -2

(b) 1 cycles **B1**

(b) (0, 2) **B1**

(b) Answer: 1 cycles; (0, 2)

(c) sin(x) = 1/2 **M1**

(c) principal value 30 degrees and the symmetric value **M1**

(c) adds multiples of 360 to the angle and divides by 1 **A1**

(c) all solutions: 30, 150 **A1**

(c) Answer: x = 30, 150

Question 4

(a) amplitude 3 **B1**

(a) period 120 degrees **B1**

(a) maximum value 1 **B1**

(a) minimum value -5 **B1**

(a) Answer: amplitude 3, period 120, max 1, min -5

(b) 3 cycles **B1**

(b) (0, -2) **B1**

(b) Answer: 3 cycles; (0, -2)

(c) sin(3x) = 1/2 **M1**

(c) principal value 30 degrees and the symmetric value **M1**

(c) adds multiples of 360 to the angle and divides by 3 **A1**

(c) all solutions: 10, 50, 130, 170, 250, 290 **A1**

(c) Answer: x = 10, 50, 130, 170, 250, 290

Question 5

(a) single smooth wave with the same period as y = sin(x) **B1**

(a) curve lies entirely on or above the x-axis, touching it at exactly one point **B1**

(a) maximum value of 2 correctly shown on the curve **B1**

(a) Answer: Sine wave translated up by 1, from (0,1) up to (90,2), down to touch (270,0), back up to (360,1)

(b) period = 360 degrees (unchanged by a vertical translation) **B1**

(b) maximum point (90, 2) **B1**

(b) minimum point (270, 0) **B1**

(b) Answer: Period 360 degrees; maximum (90, 2); minimum (270, 0)

(c) coordinates (270, 0) stated **B1**

(c) correct explanation: $\sin(x) \geq -1$ for all x, with equality only at x = 270 in this range, so $\sin(x) + 1 \geq 0$ with equality only there, meaning the curve cannot go below the x-axis and only just reaches it **B1**

(c) Answer: (270, 0); the curve touches rather than crosses because $\sin(x) + 1 \geq 0$ always, with equality only at x = 270

Question 6

(a) 180 degrees cao **B1**

(a) Answer: 180 degrees

(b) correct shape showing two complete cosine cycles across the given range **B1**

(b) maximum points at (0, 1), (180, 1) and (360, 1) all correctly shown **B1**

(b) minimum points at (90, -1) and (270, -1) correctly shown **B1**

(b) Answer: Two full cosine cycles: max at x = 0, 180, 360; min at x = 90, 270

(c) sets 2x = 90 + 180k and finds at least one correct value of 2x or x **M1**

(c) at least three of the four values x = 45, 135, 225, 315 correct **A1**

(c) all four x-intercepts correct cao, no extra values: (45, 0), (135, 0), (225, 0), (315, 0) **A1**

(c) Answer: x = 45, 135, 225, 315

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Question 7

(a) cosec(x) = 1 / sin(x) **B1**

(a) Answer: cosec(x) = 1 / sin(x)

(b) recognises cosec(x) = 1/sin(x) is undefined where sin(x) = 0 **M1**

(b) x = 0, 180, 360 all correctly identified, no extra values **A1**

(b) correct explanation: division by zero is undefined, so cosec(x) has no value wherever sin(x) = 0 **B1**

(b) Answer: x = 0, 180, 360; undefined because sin(x) = 0 there, and division by zero is not defined

(c) correct branch shapes: a U-shaped branch opening upwards on (0, 180) and an inverted branch opening downwards on (180, 360) **B1**

(c) vertical asymptotes drawn as dashed lines at x = 0, 180, 360 **B1**

(c) turning point (90, 1) correctly labelled **B1**

(c) turning point (270, -1) correctly labelled **B1**

(c) Answer: Two branches with asymptotes at x = 0, 180, 360; turning points (90, 1) and (270, -1)

(d) y ≤ -1 or y ≥ 1 oe **B1**

(d) Answer: y ≤ -1 or y ≥ 1

Question 8

(a) sec(x) = 1 / cos(x) **B1**

(a) Answer: sec(x) = 1 / cos(x)

(b) recognises sec(x) = 1/cos(x) is undefined where cos(x) = 0 **M1**

(b) x = 90, 270 both correctly identified, no extra values **A1**

(b) correct reason: division by zero is undefined, so sec(x) has no value wherever cos(x) = 0 **B1**

(b) Answer: x = 90, 270; undefined because cos(x) = 0 there, and division by zero is not defined

(c) correct branch shapes: an upward U-shaped branch on each side of x = 180, and a downward branch between x = 90 and x = 270 **B1**

(c) vertical asymptotes drawn as dashed lines at x = 90 and x = 270 **B1**

(c) turning points (0, 1) and (360, 1) both correctly labelled **B1**

(c) turning point (180, -1) correctly labelled **B1**

(c) Answer: Branches with asymptotes at x = 90, 270; turning points (0, 1), (180, -1), (360, 1)

(d) y ≤ -1 or y ≥ 1 oe **B1**

(d) Answer: y ≤ -1 or y ≥ 1

Question 9

(a) states -1 ≤ cos(x) ≤ 1 for all x, with cos(x) not equal to 0 where sec(x) is defined **M1**

(a) deduces |sec(x)| = 1/|cos(x)| ≥ 1 for all x where sec(x) is defined **A1**

(a) correctly concludes sec(x) ≤ -1 or sec(x) ≥ 1, so no value of sec(x) can lie strictly between -1 and 1 **A1**

(a) Answer: Since |cos(x)| ≤ 1 (and nonzero where sec is defined), |sec(x)| = 1/|cos(x)| ≥ 1, so sec(x) is never strictly between -1 and 1

(b) 0 solutions **B1**

(b) correct reason: 0.5 lies strictly between -1 and 1, and the range of sec(x) is y ≤ -1 or y ≥ 1, so sec(x) = 0.5 has no solutions **B1**

(b) Answer: 0 solutions, since 0.5 is not in the range of sec(x)

(c) identifies that y = cot(x) consists of two separate branches on (0, 180) and (180, 360) **M1**

(c) recognises each branch is continuous and strictly decreasing, taking every real value exactly once (unbounded above near the lower end, unbounded below near the upper end) **A1**

(c) deduces the horizontal line y = k meets each branch exactly once, for any real value of k **A1**

(c) correct conclusion: cot(x) = k has exactly 2 solutions in 0 < x < 360, for every real value of k (unlike sec(x) = k or cosec(x) = k, whose number of solutions depends on k) **B1**

(c) Answer: Exactly 2 solutions, for every real value of k

Question 10

(a) cot(x) = 1 / tan(x) **B1**

(a) cot(x) = cos(x) / sin(x) **B1**

(a) Answer: cot(x) = 1/tan(x) = cos(x)/sin(x)

(b) 180 degrees cao **B1**

(b) Answer: 180 degrees

(c) recognises cot(x) is undefined where sin(x) = 0 **B1**

(c) x = 0, 180, 360 all correctly identified, no extra values **B1**

(c) Answer: x = 0, 180, 360

(d) recognises cot(x) = 0 where cos(x) = 0 **B1**

(d) x = 90, 270 both correctly identified, no extra values **B1**

(d) Answer: x = 90, 270

(e) considers cot(-x) = cos(-x) / sin(-x) **M1**

(e) uses cos(-x) = cos(x) and sin(-x) = -sin(x) to simplify to cot(-x) = -cot(x) **A1**

(e) correct conclusion: cot(x) is an odd function, since cot(-x) = -cot(x) for all valid x **A1**

(e) Answer: y = cot(x) is an odd function, since cot(-x) = -cot(x)

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Question 11

(a) correct method: applies cosec(x) ≤ -1 or cosec(x) ≥ 1 to the transformation 3cosec(x) + 2, e.g. 3cosec(x) ≤ -3 or 3cosec(x) ≥ 3 **B1**

(a) g(x) ≤ -1 or g(x) ≥ 5, cao **B1**

(a) Answer: g(x) ≤ -1 or g(x) ≥ 5

(b) correct branch shapes: a U-shaped branch opening upwards on (0, 180) and an inverted branch opening downwards on (180, 360), matching the shape of cosec(x) **B1**

(b) vertical asymptotes drawn as dashed lines at x = 0, 180, 360 **B1**

(b) turning point (90, 5) correctly labelled **B1**

(b) turning point (270, -1) correctly labelled **B1**

(b) Answer: Branches with asymptotes at x = 0, 180, 360; turning points (90, 5) and (270, -1)

(c) forms 3cosec(x) + 2 = 8 and rearranges to cosec(x) = 2 **M1**

(c) cso uses cosec(x) = 1/sin(x) to complete to sin(x) = 1/2 **A1**

(c) Answer: sin(x) = 1/2 (shown)

(d) finds the principal value x = 30 from arcsin(1/2) **M1**

(d) both x = 30 and x = 150 correct cao, no extra values **A1**

(d) Answer: x = 30, 150

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