1.18 Pure: Trigonometric Graphs: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 65 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question concerns the graph of $y = \sin(x)$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$.

(a) State the period, in degrees, of $y = \sin(x)$. **(1)**

(b) Write down the coordinates of the maximum point and the minimum point of $y = \sin(x)$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$. **(2)**

(c) State the x-intercepts of $y = \sin(x)$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$. **(2)**

(d) Sketch $y = \sin(x)$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$, labelling the coordinates of the maximum point, the minimum point and every x-intercept found in parts (b) and (c). **(3)**

(Total for Question 1 is 8 marks)

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2 The acute angles A and B satisfy $\sin A = 21/29$ and $\cos B = 5/13$.

(a) Find the exact values of $\cos A$, $\sin B$ and $\tan A$. **(3)**

(b) Find the exact value of $\cos(A + B)$. **(3)**

(c) Find the exact value of $\tan(A + B)$. **(3)**

(d) Find the exact value of $\sin 2A$ and of $\cos 2A$. **(3)**

(Total for Question 2 is 12 marks)

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3 This question is about trigonometric identities and equations.

(a) Show that $(\sin x + \cos x)^2 = 1 + \sin 2x$. **(2)**

(b) Hence solve $(\sin x + \cos x)^2 = 5/3$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 3 is 6 marks)

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4 The graph of $y = f(x)$ is transformed in each of the ways described below.

(a) Describe fully the single transformation that maps the graph of $y = f(x)$ onto the graph of $y = f(x - 1)$. **(2)**

(b) Describe fully the single transformation that maps $y = f(x)$ onto $y = f(3x)$. **(2)**

(c) The graph $y = f(x)$ is mapped onto $y = 3f(x - 1) + 2$ by a sequence of transformations. State the three transformations in a suitable order. **(3)**

(Total for Question 4 is 7 marks)

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5 $f(x) = 3 \sin x + 1 \cos x$, where x is in degrees.

(a) Express $f(x)$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < 90$ degrees. Give R as an exact value and α to 1 decimal place. **(4)**

(b) Write down the maximum value of $f(x)$, and the smallest positive value of x at which it occurs, to 1 decimal place. **(2)**

(c) Solve $f(x) = 2$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 5 is 10 marks)

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6 This question concerns the graph of $y = \text{cosec}(2x)$.

(a) State the period, in degrees, of $y = \text{cosec}(2x)$. **(1)**

(b) Find the values of x in the range 0 degrees $\leq x \leq 360$ degrees for which $y = \text{cosec}(2x)$ is undefined. **(4)**

(c) Sketch $y = \text{cosec}(2x)$ for 0 degrees $< x < 90$ degrees, showing the equations of the vertical asymptotes and the coordinates of the turning point. **(3)**

(Total for Question 6 is 8 marks)

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Mark scheme · 1.18 Pure: Trigonometric Graphs: Test Pack

Question 1

(a) 360 degrees cao **B1**

(a) Answer: 360 degrees

(b) maximum point (90, 1) **B1**

(b) minimum point (270, -1) **B1**

(b) Answer: Maximum (90, 1); minimum (270, -1)

(c) at least two of x = 0, 180, 360 correctly identified **B1**

(c) all three values x = 0, 180, 360 cao, no extra values **B1**

(c) Answer: x = 0, 180, 360

(d) single smooth wave, correct overall shape, starting at the origin **B1**

(d) maximum (90, 1) and minimum (270, -1) both correctly labelled on the curve **B1**

(d) x-intercepts at (0, 0), (180, 0) and (360, 0) all correctly labelled **B1**

(d) Answer: Sine wave from (0,0) up to (90,1), down through (180,0) to (270,-1), back up to (360,0)

Question 2

(a) uses $\sin^2 + \cos^2 = 1$ for each angle (or a right-angled triangle) **M1**

(a) cos A = 20/29, tan A = 21/20 **A1**

(a) sin B = 12/13 **A1**

(a) Answer: cos A = 20/29, sin B = 12/13, tan A = 21/20

(b) uses cos(A + B) = cos A cos B - sin A sin B **M1**

(b) substitutes the exact values **M1**

(b) -152/377 **A1**

(b) Answer: -152/377

(c) uses tan(A + B) = (tan A + tan B)/(1 - tan A tan B) **M1**

(c) substitutes the exact values **M1**

(c) -345/152 **A1**

(c) Answer: -345/152

(d) uses sin 2A = 2 sin A cos A and cos 2A = 1 - 2 $\sin^2$ A **M1**

(d) sin 2A = 840/841 **A1**

(d) cos 2A = -41/841 **A1**

(d) Answer: sin 2A = 840/841, cos 2A = -41/841

Question 3

(a) expands to $\sin^2 x + 2 \sin x \cos x + \cos^2 x$ **M1**

(a) uses $\sin^2 x + \cos^2 x = 1$ and 2 sin x cos x = sin 2x (cso) **A1**

(a) Answer: $(\sin x + \cos x)^2 = 1 + \sin 2x$

(b) forms sin 2x = 2/3 **M1**

(b) finds 2x = 41.8 and the symmetric value 180 - 41.8, extended by 360 **M1**

(b) any two correct values of x **A1**

(b) all four values: 20.9, 69.1, 200.9, 249.1 **A1**

(b) Answer: x = 20.9, 69.1, 200.9, 249.1

Question 4

(a) translation **B1**

(a) by the vector (1, 0), i.e. 1 unit in the positive x direction **B1**

(a) Answer: translation by (1, 0)

(b) stretch **B1**

(b) scale factor 1/3 parallel to the x axis **B1**

(b) Answer: stretch, scale factor 1/3, x direction

(c) translation by (1, 0) **B1**

(c) stretch scale factor 3 parallel to the y axis **B1**

(c) translation by (0, 2) **B1**

(c) Answer: translate (1, 0); stretch y by 3; translate (0, 2)

Question 5

(a) R cos α = 3 and R sin α = 1 **M1**

(a) R = $\sqrt{10}$ **A1**

(a) tan α = 1/3 **M1**

(a) α = 18.4 degrees **A1**

(a) Answer: f(x) = $\sqrt{10}$ sin(x + 18.4)

(b) maximum $\sqrt{10}$ **B1**

(b) x + 18.4 = 90, so x = 71.6 **B1**

(b) Answer: maximum $\sqrt{10}$ at x = 71.6

(c) sin(x + 18.4) = 2/$\sqrt{10}$ **M1**

(c) x + 18.4 = 39.2 or 140.8 (and adds 360 if needed) **M1**

(c) x = 20.8 **A1**

(c) x = 122.3 **A1**

(c) Answer: x = 20.8, 122.3

Question 6

(a) 180 degrees cao **B1**

(a) Answer: 180 degrees

(b) recognises cosec(2x) undefined where sin(2x) = 0 **M1**

(b) sets 2x = 0, 180, 360, 540, 720 (using the correct range $0 \leq 2x \leq 720$) **M1**

(b) at least three of the five values x = 0, 90, 180, 270, 360 correct **A1**

(b) all five values correct cao, no extra values: x = 0, 90, 180, 270, 360 **A1**

(b) Answer: x = 0, 90, 180, 270, 360

(c) single U-shaped branch opening upwards, unbounded near both ends of the interval **B1**

(c) vertical asymptotes drawn as dashed lines at x = 0 and x = 90 **B1**

(c) turning point (45, 1) correctly labelled **B1**

(c) Answer: One U-shaped branch, asymptotes at x = 0 and x = 90, turning point (45, 1)

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