1.18 Pure: Trigonometric Graphs

EDEXCEL 9MA0 · Calculator allowed · about 235 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question concerns the graph of $y = \cos(x)$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$.

(a) State the period and the range of $y = \cos(x)$. **(2)**

(b) Write down the coordinates of the maximum point(s) and the minimum point of $y = \cos(x)$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$. **(2)**

(c) State the x-intercepts of $y = \cos(x)$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$. **(2)**

(d) Given that $\cos(x) = \sin(x + 90)$, describe fully the single transformation that maps the graph of $y = \sin(x)$ onto the graph of $y = \cos(x)$. **(2)**

(Total for Question 1 is 8 marks)

2 The graph of $y = f(x)$ is transformed in each of the ways described below.

(a) Describe fully the single transformation that maps the graph of $y = f(x)$ onto the graph of $y = f(x - 3)$. **(2)**

(b) Describe fully the single transformation that maps $y = f(x)$ onto $y = f(4x)$. **(2)**

(c) The graph $y = f(x)$ is mapped onto $y = 4f(x - 3) + 5$ by a sequence of transformations. State the three transformations in a suitable order. **(3)**

(Total for Question 2 is 7 marks)

3 This question is about trigonometric identities and equations.

(a) Show that $(1 - \cos 2x)/\sin 2x = \tan x$, for $\sin 2x \neq 0$. **(3)**

(b) Hence solve $(1 - \cos 2x)/\sin 2x = 3$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(3)**

(Total for Question 3 is 6 marks)

4 $f(x) = 8 \sin x + 6 \cos x$, where x is in degrees.

(a) Express $f(x)$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < 90$ degrees. Give R as an exact value and α to 1 decimal place. **(4)**

(b) Write down the maximum value of $f(x)$, and the smallest positive value of x at which it occurs, to 1 decimal place. **(2)**

(c) Solve $f(x) = 1$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 4 is 10 marks)

5 Solve $\cos^2 x + \sin x - 1 = 0$ for $-180 < x \leq 180$, giving all solutions in degrees.

(a) Use $\cos^2 x = 1 - \sin^2 x$ to show that the equation can be written as a quadratic in $\sin x$, and state that quadratic. **(3)**

(b) Hence solve the equation. **(4)**

(Total for Question 5 is 7 marks)

6 This question is about trigonometric identities and equations.

(a) Show that $(1 + \sin x)/\cos x + \cos x/(1 + \sin x) = 2 \sec x$. **(3)**

(b) Hence solve $(1 + \sin x)/\cos x + \cos x/(1 + \sin x) = 4$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(3)**

(Total for Question 6 is 6 marks)

7 Solve $2\cos^2 x + \sin x - 1 = 0$ for $-180 < x \leq 180$, giving all solutions in degrees.

(a) Use $\cos^2 x = 1 - \sin^2 x$ to show that the equation can be written as a quadratic in $\sin x$, and state that quadratic. **(3)**

(b) Hence solve the equation. **(4)**

(Total for Question 7 is 7 marks)

8 The graph of $y = f(x)$ is transformed in each of the ways described below.

(a) Describe fully the single transformation that maps the graph of $y = f(x)$ onto the graph of $y = f(x - 3)$. **(2)**

(b) Describe fully the single transformation that maps $y = f(x)$ onto $y = f(3x)$. **(2)**

(c) The graph $y = f(x)$ is mapped onto $y = 3f(x - 3) + 1$ by a sequence of transformations. State the three transformations in a suitable order. **(3)**

(Total for Question 8 is 7 marks)

9 The acute angles A and B satisfy $\sin A = \frac{4}{5}$ and $\cos B = \frac{20}{29}$.

(a) Find the exact values of cos A, sin B and tan A. **(3)**

(b) Find the exact value of cos(A + B). **(3)**

(c) Find the exact value of sin(A + B). **(3)**

(d) Find the exact value of sin 2A and of cos 2A. **(3)**

(Total for Question 9 is 12 marks)

10 In this question x is measured in degrees and $0 \leq x < 360$.

(a) Solve the quadratic equation $2y^2 - y = 0$ and hence state the possible values of $\cos x$. **(2)**

(b) Hence solve $2\cos^2 x - \cos x = 0$ for $0 \leq x < 360$, in degrees. **(4)**

(Total for Question 10 is 6 marks)

11 The curve C has equation $y = 4 \sin x + 2$, for $0 \leq x \leq 360$, where x is in degrees.

(a) Write down the amplitude of the curve, its period, and its maximum and minimum values. **(4)**

(b) Write down the number of complete cycles of C in the given range, and the coordinates of the point where C crosses the y-axis. **(2)**

(c) Solve $4 \sin x + 2 = 4$ for $0 \leq x < 360$. **(4)**

(Total for Question 11 is 10 marks)

12 The curve C has equation $y = 3\sin(3x) - 2$, for $0 \leq x \leq 360$, where x is in degrees.

(a) Write down the amplitude of the curve, its period, and its maximum and minimum values. **(4)**

(b) Write down the number of complete cycles of C in the given range, and the coordinates of the point where C crosses the y-axis. **(2)**

(c) Solve $3\sin(3x) - 2 = -1/2$ for $0 \leq x < 360$. **(4)**

(Total for Question 12 is 10 marks)

13 $f(x) = 8 \sin x + 1 \cos x$, where x is in degrees.

(a) Express $f(x)$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < 90$ degrees. Give R as an exact value and α to 1 decimal place. **(4)**

(b) Write down the maximum value of $f(x)$, and the smallest positive value of x at which it occurs, to 1 decimal place. **(2)**

(c) Solve $f(x) = 2$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 13 is 10 marks)

14 This question concerns the graph of $y = \tan(x)$ for $-180 \text{ degrees} \leq x \leq 180 \text{ degrees}$.

(a) State the period, in degrees, of $y = \tan(x)$. **(1)**

(b) Sketch $y = \tan(x)$ for $-180 \text{ degrees} \leq x \leq 180 \text{ degrees}$, showing the vertical asymptotes and the points where the curve crosses the x-axis. **(4)**

(c) State the equations of the vertical asymptotes of $y = \tan(x)$ for $-180 \text{ degrees} \leq x \leq 180 \text{ degrees}$. **(2)**

(d) State the domain restriction that applies to $y = \tan(x)$ for all real x, using n to represent an integer. **(2)**

(Total for Question 14 is 9 marks)

15 This question concerns the graph of $y = \sin(x) + 1$ for 0 degrees $\leq x \leq$ 360 degrees.

(a) Sketch $y = \sin(x) + 1$ for 0 degrees $\leq x \leq$ 360 degrees. **(3)**

(b) State the period of $y = \sin(x) + 1$, and the coordinates of its maximum and minimum points for 0 degrees $\leq x \leq$ 360 degrees. **(3)**

(c) Find the coordinates of the point where the curve $y = \sin(x) + 1$ touches the x-axis, and explain why the curve touches the x-axis at this point rather than crossing it. **(2)**

(Total for Question 15 is 8 marks)

16 This question concerns the graph of $y = \cos(2x)$ for 0 degrees $\leq x \leq$ 360 degrees.

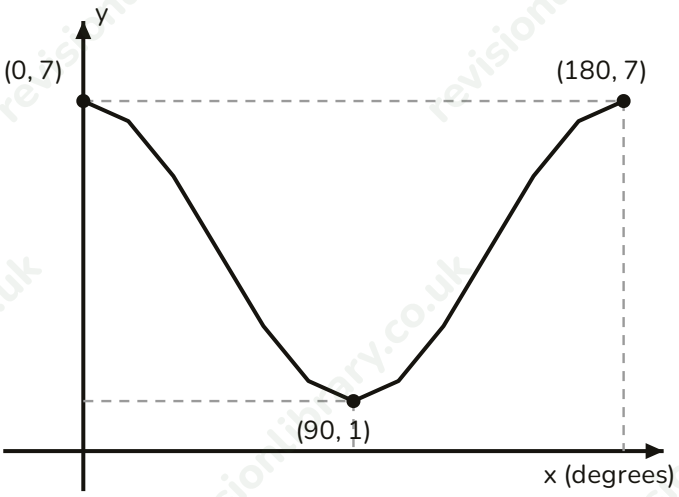
(a) State the period, in degrees, of $y = \cos(2x)$. **(1)**

(b) Sketch $y = \cos(2x)$ for 0 degrees $\leq x \leq$ 360 degrees. **(3)**

(c) Find the exact coordinates of every point where $y = \cos(2x)$ crosses the x-axis for 0 degrees $\leq x \leq$ 360 degrees. **(3)**

(Total for Question 16 is 7 marks)

17 The diagram shows part of the curve $y = a + b\cos(cx)$ for 0 degrees $\leq x \leq 180$ degrees, where a, b and c are positive constants.



(a) Using the information shown in the diagram, find the values of the constants a, b and c. **(6)**

(b) State the range of $y = a + b\cos(cx)$, using your values from part (a). **(1)**

(Total for Question 17 is 7 marks)

18 This question concerns the graph of y = cosec(x) for 0 degrees ≤ x ≤ 360 degrees.

(a) Write cosec(x) in terms of sin(x). **(1)**

(b) State the values of x in the range 0 degrees ≤ x ≤ 360 degrees for which cosec(x) is undefined, explaining why cosec(x) is undefined at these values. **(3)**

(c) Sketch y = cosec(x) for 0 degrees < x < 360 degrees, x not equal to 180 degrees, showing the vertical asymptotes and the coordinates of the two turning points. **(4)**

(d) State the range of y = cosec(x). **(1)**

(Total for Question 18 is 9 marks)

19 This question concerns the graph of $y = \sec(x)$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$.

(a) Write $\sec(x)$ in terms of $\cos(x)$. **(1)**

(b) State the values of x in the range $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$ for which $\sec(x)$ is undefined, giving a reason. **(3)**

(c) Sketch $y = \sec(x)$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$, x not equal to 90 degrees or 270 degrees, showing the vertical asymptotes and the coordinates of the turning points. **(4)**

(d) State the range of $y = \sec(x)$. **(1)**

(Total for Question 19 is 9 marks)

20 This question uses the graphs of $y = \sec(x)$ and $y = \cot(x)$.

(a) Explain, using the definition $\sec(x) = 1 / \cos(x)$, why the range of $y = \sec(x)$ cannot include any value of y satisfying $-1 < y < 1$. **(3)**

(b) State, with a reason, the number of values of x in the interval 0 degrees $\leq x \leq 360$ degrees for which $\sec(x) = 0.5$. **(2)**

(c) By considering the graph of $y = \cot(x)$ for 0 degrees $< x < 360$ degrees, determine the number of solutions of the equation $\cot(x) = k$ in this range, where k is any real number. **(4)**

(Total for Question 20 is 9 marks)

21 This question concerns the graph of $y = \cot(x)$ for 0 degrees $\leq x \leq 360$ degrees.

(a) Write $\cot(x)$ in terms of $\tan(x)$, and also in terms of $\sin(x)$ and $\cos(x)$. **(2)**

(b) State the period, in degrees, of $y = \cot(x)$. **(1)**

(c) State the values of x in the range 0 degrees $\leq x \leq 360$ degrees for which $\cot(x)$ is undefined. **(2)**

(d) State the x-intercepts of $y = \cot(x)$ for 0 degrees $< x < 360$ degrees. **(2)**

(e) Determine, with justification, whether $y = \cot(x)$ is an odd function or an even function. **(3)**

(Total for Question 21 is 10 marks)

22 The function g is defined by $g(x) = 3\,\text{cosec}(x) + 2$, for 0 degrees $< x <$ 360 degrees, x not equal to 180 degrees.

(a) State the range of g(x). **(2)**

(b) Sketch $y = g(x)$ for 0 degrees $< x <$ 360 degrees, x not equal to 180 degrees, showing the equations of the vertical asymptotes and the coordinates of the turning points. **(4)**

(c) Show that the equation $g(x) = 8$ can be written in the form $\sin(x) = 1/2$. **(2)**

(d) Hence solve $g(x) = 8$ for 0 degrees $< x <$ 360 degrees, giving your answers in degrees. **(2)**

(Total for Question 22 is 10 marks)

Mark scheme · 1.18 Pure: Trigonometric Graphs

Question 1

(a) period = 360 degrees **B1**

(a) range $-1 \leq y \leq 1$ oe **B1**

(a) Answer: Period 360 degrees; range $-1 \leq y \leq 1$

(b) maximum points (0, 1) and (360, 1), both stated **B1**

(b) minimum point (180, -1) **B1**

(b) Answer: Maxima (0, 1) and (360, 1); minimum (180, -1)

(c) one of x = 90, 270 correctly identified **B1**

(c) both values x = 90, 270 cao, no extra values **B1**

(c) Answer: x = 90, 270

(d) recognises that y = sin(x + 90) is a horizontal translation of y = sin(x) by (-90, 0) **M1**

(d) correctly states: translation by the vector (-90, 0), i.e. 90 degrees in the negative x-direction, cao **A1**

(d) Answer: Translation by the vector (-90, 0) (a shift of 90 degrees to the left)

Question 2

(a) translation **B1**

(a) by the vector (3, 0), i.e. 3 units in the positive x direction **B1**

(a) Answer: translation by (3, 0)

(b) stretch **B1**

(b) scale factor 1/4 parallel to the x axis **B1**

(b) Answer: stretch, scale factor 1/4, x direction

(c) translation by (3, 0) **B1**

(c) stretch scale factor 4 parallel to the y axis **B1**

(c) translation by (0, 5) **B1**

(c) Answer: translate (3, 0); stretch y by 4; translate (0, 5)

Question 3

(a) uses $\cos 2x = 1 - 2\sin^2 x$ for the numerator and $\sin 2x = 2 \sin x \cos x$ for the denominator **M1**

(a) numerator $2\sin^2 x$, denominator $2 \sin x \cos x$ **A1**

(a) cancels 2 sin x and states $\sin x/\cos x = \tan x$ (cso) **A1**

(a) Answer: $(1 - \cos 2x)/\sin 2x = 2\sin^2 x/(2 \sin x \cos x) = \tan x$

(b) uses the identity to write $\tan x = 3$ **M1**

(b) x = 71.6 **A1**

(b) x = 251.6 (180 degrees added) **A1**

(b) Answer: x = 71.6, 251.6

Question 4

(a) $R \cos \alpha = 8$ and $R \sin \alpha = 6$ **M1**

(a) R = 10 **A1**

(a) $\tan \alpha = 6/8$ **M1**

(a) $\alpha = 36.9$ degrees **A1**

(a) Answer: f(x) = 10 sin(x + 36.9)

(b) maximum 10 **B1**

(b) x + 36.9 = 90, so x = 53.1 **B1**

(b) Answer: maximum 10 at x = 53.1

(c) sin(x + 36.9) = 1/10 **M1**

(c) x + 36.9 = 5.7 or 174.3 (and adds 360 if needed) **M1**

(c) x = 137.4 **A1**

(c) x = 328.9 **A1**

(c) Answer: x = 137.4, 328.9

Question 5

(a) replaces $\cos^2 x$ by $1 - \sin^2 x$ **M1**

(a) collects terms **M1**

(a) $s^2 - s = 0$ where $s = \sin x$ **A1**

(a) Answer: $s^2 - s = 0$

(b) solves the quadratic **M1**

(b) sin x = 1 or sin x = 0 **A1**

(b) uses inverse sine and symmetry for each value **M1**

(b) 0, 90, 180 degrees **A1**

(b) Answer: 0, 90, 180 degrees

Question 6

(a) writes over a common denominator $\cos x (1 + \sin x)$ **M1**

(a) numerator $(1 + \sin x)^2 + \cos^2 x = 2 + 2 \sin x$ **M1**

(a) factorises $2(1 + \sin x)$ and cancels, giving $2/\cos x = 2 \sec x$ (cso) **A1**

(a) Answer: expression = $2/\cos x = 2 \sec x$

(b) uses the identity: $2 \sec x = 4$, so $\cos x = 1/2$ **M1**

(b) x = 60.0 **A1**

(b) x = 300.0 **A1**

(b) Answer: x = 60.0, 300.0

Question 7

(a) replaces $\cos^2 x$ by $1 - \sin^2 x$ **M1**

(a) collects terms **M1**

(a) $2s^2 - s - 1 = 0$ where $s = \sin x$ **A1**

(a) Answer: $2s^2 - s - 1 = 0$

(b) solves the quadratic **M1**

(b) sin x = 1 or sin x = -1/2 **A1**

(b) uses inverse sine and symmetry for each value **M1**

(b) -150, -30, 90 degrees **A1**

(b) Answer: -150, -30, 90 degrees

Question 8

(a) translation **B1**

(a) by the vector (3, 0), i.e. 3 units in the positive x direction **B1**

(a) Answer: translation by (3, 0)

(b) stretch **B1**

(b) scale factor 1/3 parallel to the x axis **B1**

(b) Answer: stretch, scale factor 1/3, x direction

(c) translation by (3, 0) **B1**

(c) stretch scale factor 3 parallel to the y axis **B1**

(c) translation by (0, 1) **B1**

(c) Answer: translate (3, 0); stretch y by 3; translate (0, 1)

Question 9

(a) uses $\sin^2 + \cos^2 = 1$ for each angle (or a right-angled triangle) **M1**

(a) cos A = 3/5, tan A = 4/3 **A1**

(a) sin B = 21/29 **A1**

(a) Answer: cos A = 3/5, sin B = 21/29, tan A = 4/3

(b) uses cos(A + B) = cos A cos B - sin A sin B **M1**

(b) substitutes the exact values **M1**

(b) -24/145 **A1**

(b) Answer: -24/145

(c) uses sin(A + B) = sin A cos B + cos A sin B **M1**

(c) substitutes the exact values **M1**

(c) 143/145 **A1**

(c) Answer: 143/145

(d) uses sin 2A = 2 sin A cos A and cos 2A = 1 - 2 $\sin^2$ A **M1**

(d) sin 2A = 24/25 **A1**

(d) cos 2A = -7/25 **A1**

(d) Answer: sin 2A = 24/25, cos 2A = -7/25

Question 10

(a) factorises or uses the quadratic formula **M1**

(a) cos x = 0 or cos x = 1/2 **A1**

(a) Answer: cos x = 0 or 1/2

(b) finds a solution of cos x = 0 using inverse cos **M1**

(b) solutions from cos x = 0: 90, 270 **A1**

(b) repeats for cos x = 1/2 **M1**

(b) all solutions: 60, 90, 270, 300 degrees **A1**

(b) Answer: 60, 90, 270, 300 degrees

Question 11

(a) amplitude 4 **B1**

(a) period 360 degrees **B1**

(a) maximum value 6 **B1**

(a) minimum value -2 **B1**

(a) Answer: amplitude 4, period 360, max 6, min -2

(b) 1 cycles **B1**

(b) (0, 2) **B1**

(b) Answer: 1 cycles; (0, 2)

(c) sin(x) = 1/2 **M1**

(c) principal value 30 degrees and the symmetric value **M1**

(c) adds multiples of 360 to the angle and divides by 1 **A1**

(c) all solutions: 30, 150 **A1**

(c) Answer: x = 30, 150

Question 12

(a) amplitude 3 **B1**

(a) period 120 degrees **B1**

(a) maximum value 1 **B1**

(a) minimum value -5 **B1**

(a) Answer: amplitude 3, period 120, max 1, min -5

(b) 3 cycles **B1**

(b) (0, -2) **B1**

(b) Answer: 3 cycles; (0, -2)

(c) sin(3x) = 1/2 **M1**

(c) principal value 30 degrees and the symmetric value **M1**

(c) adds multiples of 360 to the angle and divides by 3 **A1**

(c) all solutions: 10, 50, 130, 170, 250, 290 **A1**

(c) Answer: x = 10, 50, 130, 170, 250, 290

Question 13

(a) R cos α = 8 and R sin α = 1 **M1**

(a) R = $\sqrt{65}$ **A1**

(a) tan α = 1/8 **M1**

(a) α = 7.1 degrees **A1**

(a) Answer: f(x) = $\sqrt{65}$ sin(x + 7.1)

(b) maximum $\sqrt{65}$ **B1**

(b) x + 7.1 = 90, so x = 82.9 **B1**

(b) Answer: maximum $\sqrt{65}$ at x = 82.9

(c) sin(x + 7.1) = 2/$\sqrt{65}$ **M1**

(c) x + 7.1 = 14.4 or 165.6 (and adds 360 if needed) **M1**

(c) x = 7.2 **A1**

(c) x = 158.5 **A1**

(c) Answer: x = 7.2, 158.5

Question 14

(a) 180 degrees cao **B1**

(a) Answer: 180 degrees

(b) three separate increasing branches drawn, each going from -infinity to +infinity **B1**

(b) two vertical asymptotes drawn as dashed lines at x = -90 and x = 90 **B1**

(b) x-intercepts correctly plotted at x = -180, 0, 180 **B1**

(b) curve shows correct symmetry about the origin (odd function), passing through (0, 0) **B1**

(b) Answer: Three increasing branches, asymptotes at x = -90 and x = 90, crossing the x-axis at x = -180, 0, 180

(c) x = -90 **B1**

(c) x = 90 **B1**

(c) Answer: x = -90, x = 90

(d) recognises the asymptote pattern repeats with period 180 degrees from a base value of 90 degrees **M1**

(d) x is not equal to 90 + 180n, n an integer, oe cao **A1**

(d) Answer: x is not equal to 90 + 180n, for n an integer

Question 15

(a) single smooth wave with the same period as y = sin(x) **B1**

(a) curve lies entirely on or above the x-axis, touching it at exactly one point **B1**

(a) maximum value of 2 correctly shown on the curve **B1**

(a) Answer: Sine wave translated up by 1, from (0,1) up to (90,2), down to touch (270,0), back up to (360,1)

(b) period = 360 degrees (unchanged by a vertical translation) **B1**

(b) maximum point (90, 2) **B1**

(b) minimum point (270, 0) **B1**

(b) Answer: Period 360 degrees; maximum (90, 2); minimum (270, 0)

(c) coordinates (270, 0) stated **B1**

(c) correct explanation: sin(x) ≥ -1 for all x, with equality only at x = 270 in this range, so sin(x) + 1 ≥ 0 with equality only there, meaning the curve cannot go below the x-axis and only just reaches it **B1**

(c) Answer: (270, 0); the curve touches rather than crosses because sin(x) + 1 ≥ 0 always, with equality only at x = 270

Question 16

(a) 180 degrees cao **B1**

(a) Answer: 180 degrees

(b) correct shape showing two complete cosine cycles across the given range **B1**

(b) maximum points at (0, 1), (180, 1) and (360, 1) all correctly shown **B1**

(b) minimum points at (90, -1) and (270, -1) correctly shown **B1**

(b) Answer: Two full cosine cycles: max at x = 0, 180, 360; min at x = 90, 270

(c) sets 2x = 90 + 180k and finds at least one correct value of 2x or x **M1**

(c) at least three of the four values x = 45, 135, 225, 315 correct **A1**

(c) all four x-intercepts correct cao, no extra values: (45, 0), (135, 0), (225, 0), (315, 0) **A1**

(c) Answer: x = 45, 135, 225, 315

Question 17

(a) uses a = (maximum + minimum) / 2, e.g. a = (7 + 1) / 2 **M1**

(a) a = 4 **A1**

(a) uses b = (maximum - minimum) / 2, e.g. b = (7 - 1) / 2 **M1**

(a) b = 3 **A1**

(a) uses the period shown (180 degrees, from maximum to maximum) with period = 360 / c **M1**

(a) c = 2 cao **A1**

(a) Answer: a = 4, b = 3, c = 2

(b) 1 ≤ y ≤ 7 oe, ft from values of a and b in (a) **B1**

(b) Answer: 1 ≤ y ≤ 7

Question 18

(a) cosec(x) = 1 / sin(x) **B1**

(a) Answer: cosec(x) = 1 / sin(x)

(b) recognises cosec(x) = 1/sin(x) is undefined where sin(x) = 0 **M1**

(b) x = 0, 180, 360 all correctly identified, no extra values **A1**

(b) correct explanation: division by zero is undefined, so cosec(x) has no value wherever sin(x) = 0 **B1**

(b) Answer: x = 0, 180, 360; undefined because sin(x) = 0 there, and division by zero is not defined

(c) correct branch shapes: a U-shaped branch opening upwards on (0, 180) and an inverted branch opening downwards on (180, 360) **B1**

(c) vertical asymptotes drawn as dashed lines at x = 0, 180, 360 **B1**

(c) turning point (90, 1) correctly labelled **B1**

(c) turning point (270, -1) correctly labelled **B1**

(c) Answer: Two branches with asymptotes at x = 0, 180, 360; turning points (90, 1) and (270, -1)

(d) y ≤ -1 or y ≥ 1 oe **B1**

(d) Answer: y ≤ -1 or y ≥ 1

Question 19

(a) sec(x) = 1 / cos(x) **B1**

(a) Answer: sec(x) = 1 / cos(x)

(b) recognises sec(x) = 1/cos(x) is undefined where cos(x) = 0 **M1**

(b) x = 90, 270 both correctly identified, no extra values **A1**

(b) correct reason: division by zero is undefined, so sec(x) has no value wherever cos(x) = 0 **B1**

(b) Answer: x = 90, 270; undefined because cos(x) = 0 there, and division by zero is not defined

(c) correct branch shapes: an upward U-shaped branch on each side of x = 180, and a downward branch between x = 90 and x = 270 **B1**

(c) vertical asymptotes drawn as dashed lines at x = 90 and x = 270 **B1**

(c) turning points (0, 1) and (360, 1) both correctly labelled **B1**

(c) turning point (180, -1) correctly labelled **B1**

(c) Answer: Branches with asymptotes at x = 90, 270; turning points (0, 1), (180, -1), (360, 1)

(d) y ≤ -1 or y ≥ 1 oe **B1**

(d) Answer: y ≤ -1 or y ≥ 1

Question 20

(a) states $-1 \leq \cos(x) \leq 1$ for all x, with $\cos(x)$ not equal to 0 where $\sec(x)$ is defined **M1**

(a) deduces $|\sec(x)| = 1/|\cos(x)| \geq 1$ for all x where $\sec(x)$ is defined **A1**

(a) correctly concludes $\sec(x) \leq -1$ or $\sec(x) \geq 1$, so no value of $\sec(x)$ can lie strictly between -1 and 1 **A1**

(a) Answer: Since $|\cos(x)| \leq 1$ (and nonzero where sec is defined), $|\sec(x)| = 1/|\cos(x)| \geq 1$, so $\sec(x)$ is never strictly between -1 and 1

(b) 0 solutions **B1**

(b) correct reason: 0.5 lies strictly between -1 and 1, and the range of $\sec(x)$ is $y \leq -1$ or $y \geq 1$, so $\sec(x) = 0.5$ has no solutions **B1**

(b) Answer: 0 solutions, since 0.5 is not in the range of sec(x)

(c) identifies that $y = \cot(x)$ consists of two separate branches on (0, 180) and (180, 360) **M1**

(c) recognises each branch is continuous and strictly decreasing, taking every real value exactly once (unbounded above near the lower end, unbounded below near the upper end) **A1**

(c) deduces the horizontal line $y = k$ meets each branch exactly once, for any real value of k **A1**

(c) correct conclusion: $\cot(x) = k$ has exactly 2 solutions in $0 < x < 360$, for every real value of k (unlike $\sec(x) = k$ or $\operatorname{cosec}(x) = k$, whose number of solutions depends on k) **B1**

(c) Answer: Exactly 2 solutions, for every real value of k

Question 21

(a) $\cot(x) = 1 / \tan(x)$ **B1**

(a) $\cot(x) = \cos(x) / \sin(x)$ **B1**

(a) Answer: $\cot(x) = 1/\tan(x) = \cos(x)/\sin(x)$

(b) 180 degrees cao **B1**

(b) Answer: 180 degrees

(c) recognises $\cot(x)$ is undefined where $\sin(x) = 0$ **B1**

(c) x = 0, 180, 360 all correctly identified, no extra values **B1**

(c) Answer: x = 0, 180, 360

(d) recognises $\cot(x) = 0$ where $\cos(x) = 0$ **B1**

(d) x = 90, 270 both correctly identified, no extra values **B1**

(d) Answer: x = 90, 270

(e) considers $\cot(-x) = \cos(-x) / \sin(-x)$ **M1**

(e) uses $\cos(-x) = \cos(x)$ and $\sin(-x) = -\sin(x)$ to simplify to $\cot(-x) = -\cot(x)$ **A1**

(e) correct conclusion: $\cot(x)$ is an odd function, since $\cot(-x) = -\cot(x)$ for all valid x **A1**

(e) Answer: $y = \cot(x)$ is an odd function, since $\cot(-x) = -\cot(x)$

Question 22

(a) correct method: applies $\operatorname{cosec}(x) \leq -1$ or $\operatorname{cosec}(x) \geq 1$ to the transformation $3\operatorname{cosec}(x) + 2$, e.g. $3\operatorname{cosec}(x) \leq -3$ or $3\operatorname{cosec}(x) \geq 3$ **B1**

(a) $g(x) \leq -1$ or $g(x) \geq 5$, cao **B1**

(a) Answer: $g(x) \leq -1$ or $g(x) \geq 5$

(b) correct branch shapes: a U-shaped branch opening upwards on (0, 180) and an inverted branch opening downwards on (180, 360), matching the shape of $\operatorname{cosec}(x)$ **B1**

(b) vertical asymptotes drawn as dashed lines at x = 0, 180, 360 **B1**

(b) turning point (90, 5) correctly labelled **B1**

(b) turning point (270, -1) correctly labelled **B1**

(b) Answer: Branches with asymptotes at x = 0, 180, 360; turning points (90, 5) and (270, -1)

(c) forms $3\operatorname{cosec}(x) + 2 = 8$ and rearranges to $\operatorname{cosec}(x) = 2$ **M1**

(c) cso uses $\operatorname{cosec}(x) = 1/\sin(x)$ to complete to $\sin(x) = 1/2$ **A1**

(c) Answer: $\sin(x) = 1/2$ (shown)

(d) finds the principal value x = 30 from $\arcsin(1/2)$ **M1**

(d) both x = 30 and x = 150 correct cao, no extra values **A1**

(d) Answer: x = 30, 150