1.3 Pure: Coordinate Geometry (Lines and Circles)

EDEXCEL 9MA0 · Calculator allowed · about 220 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Find the midpoint of the points (4, -2) and (10, 6).

(Total for Question 1 is 1 mark)

2 Find the distance between the points (0, 0) and (6, 8).

(Total for Question 2 is 1 mark)

3 Write down the gradient of the line with equation $y = 4x - 7$.

(Total for Question 3 is 1 mark)

4 Write down the y-intercept of the line with equation $y = -2x + 9$.

(Total for Question 4 is 1 mark)

5 Find the midpoint of the points (-5, 3) and (1, -9).

(Total for Question 5 is 1 mark)

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6 The points A(-3, -1) and B(9, 7) are joined by a straight line. The point P lies on AB such that AP : PB = 1 : 3.

(a) Find the coordinates of P. **(3)**

(b) Find the exact distance AP. **(2)**

(Total for Question 6 is 5 marks)

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7 A circle has centre (0, 0) and passes through the point (5, 12). Write down the radius of the circle.

(Total for Question 7 is 1 mark)

8 The points A(1, 5) and B(9, -1) are the endpoints of a diameter of a circle. Find the centre of the circle.

(Total for Question 8 is 2 marks)

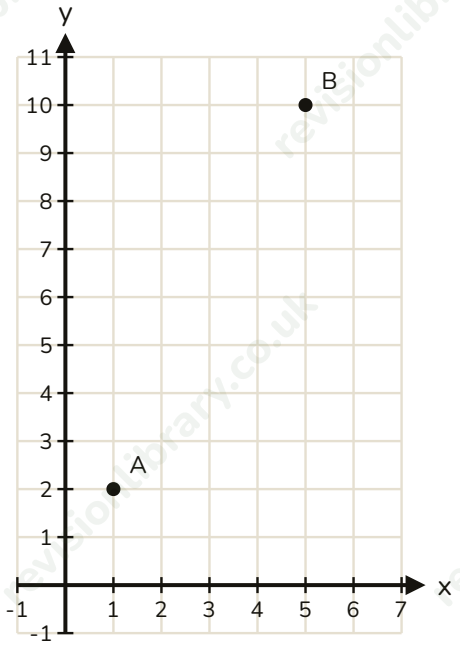
9 A circle has equation $x^2 + y^2 = 36$. Write down the radius of the circle.

(Total for Question 9 is 1 mark)

10 Find the gradient of the line joining P(-1, 4) and Q(3, -8).

(Total for Question 10 is 1 mark)

11 The points A(1, 2) and B(5, 10) are shown on a coordinate grid.



(a) Calculate the gradient of the line AB. **(2)**

(b) Find an equation of the line AB, giving your answer in the form $y = mx + c$. **(2)**

(c) The line l is parallel to AB and passes through the point (0, -3). Find an equation of l. **(2)**

(Total for Question 11 is 6 marks)

12 A circle has equation $x^2 + y^2 - 6x + 4y - 37 = 0$.

(a) Find the centre and the exact radius of the circle. **(4)**

(b) The line l has equation $y = x + k$, where k is a constant. Given that l is a tangent to the circle, find the two possible values of k. **(6)**

(Total for Question 12 is 10 marks)

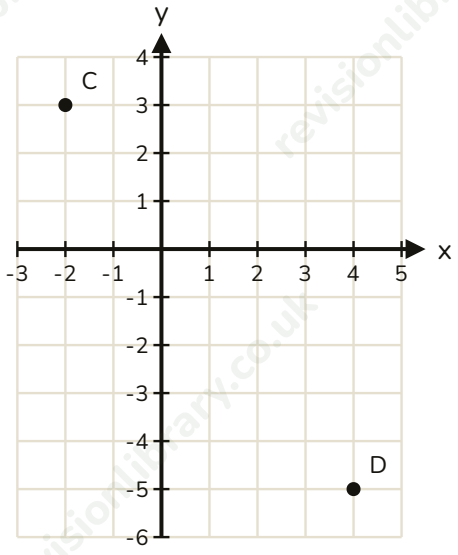
13 A circle C has equation $x^2 + y^2 = 20$. The point A has coordinates (6, 2), which lies outside C.

(a) Find the exact length of the tangent from A to the circle. **(3)**

(b) Find the equations of the two tangents from A to the circle, giving your answers in the form $y = mx + c$. **(7)**

(Total for Question 13 is 10 marks)

14 The points C(-2, 3) and D(4, -5) are shown on a coordinate grid.



(a) Find the midpoint M of CD. **(2)**

(b) Find the gradient of CD. **(2)**

(c) Find an equation of the perpendicular bisector of CD, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. **(3)**

(Total for Question 14 is 7 marks)

15 A(-1, 6) and B(7, -2) are the endpoints of a diameter of circle C.

(a) Find the centre of C. **(2)**

(b) Find the exact radius of C. **(3)**

(c) Write down an equation of circle C. **(2)**

(Total for Question 15 is 7 marks)

16 Find an equation of the line through the point (2, -3) with gradient 4, giving your answer in the form $y = mx + c$.

(Total for Question 16 is 2 marks)

17 The points C(-3, 4) and D(5, -2) are joined by a straight line. Find an equation of the line CD, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(Total for Question 17 is 3 marks)

18 A circle has centre (-1, 3) and passes through the point (4, 15). Find an equation of the circle.

(Total for Question 18 is 3 marks)

19 Determine, showing your working, whether the point (8, -5) lies inside, on, or outside the circle with equation $(x - 2)^2 + (y + 1)^2 = 50$.

(Total for Question 19 is 3 marks)

20 The points A(4, -2) and B(-4, 2) are the end points of a diameter of a circle.

(a) Find the coordinates of the centre of the circle and an equation of the circle. **(4)**

(b) Show that the point C(2, 4) lies on the circle. **(2)**

(c) Use gradients to show that angle ACB is a right angle, and state the circle theorem used. **(3)**

(Total for Question 20 is 9 marks)

21 The line l has equation $3x + 4y = 75$ and the circle C has equation $x^2 + y^2 = 225$.

(a) Show that l is a tangent to C. **(4)**

(b) Find the coordinates of the point where l touches C. **(3)**

(Total for Question 21 is 7 marks)

22 A curve C has parametric equations $x = -t - 4$, $y = 2t^2 - 32$, t in R.

(a) Find a Cartesian equation of C. **(3)**

(b) Find the coordinates of the points where C meets the x-axis. **(3)**

(c) Find the gradient of C at the point where $t = 2$. **(3)**

(Total for Question 22 is 9 marks)

23 A curve C has parametric equations $x = 4t - 3$, $y = -t^2 + 9$, t in R.

(a) Find a Cartesian equation of C. **(3)**

(b) Find the coordinates of the points where C meets the x-axis. **(3)**

(c) Find the gradient of C at the point where $t = 3$. **(3)**

(Total for Question 23 is 9 marks)

24 The circle C has equation $x^2 + y^2 + 8x + 10y - 128 = 0$.

(a) Find the coordinates of the centre of C and the exact radius of C. **(4)**

(b) Show that the point P(1, -17) lies on C. **(2)**

(c) Find an equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

(d) The tangent at P meets the x-axis at the point Q. Find the coordinates of Q. **(2)**

(Total for Question 24 is 11 marks)

25 The points A(5, -3) and B(-1, 5) are the end points of a diameter of a circle.

(a) Find the coordinates of the centre of the circle and an equation of the circle. **(4)**

(b) Show that the point C(6, 4) lies on the circle. **(2)**

(c) Use gradients to show that angle ACB is a right angle, and state the circle theorem used. **(3)**

(Total for Question 25 is 9 marks)

26 The circle C has equation $x^2 + y^2 + 8x + 2y - 83 = 0$.

(a) Find the coordinates of the centre of C and the exact radius of C. **(4)**

(b) Show that the point P(-10, 7) lies on C. **(2)**

(c) Find an equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

(d) The tangent at P meets the x-axis at the point Q. Find the coordinates of Q. **(2)**

(Total for Question 26 is 11 marks)

27 The circle C has equation $x^2 + y^2 + 6x - 2y - 15 = 0$.

(a) Find the coordinates of the centre of C and the exact radius of C. **(4)**

(b) Show that the point P(1, -2) lies on C. **(2)**

(c) Find an equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

(d) The tangent at P meets the x-axis at the point Q. Find the coordinates of Q. **(2)**

(Total for Question 27 is 11 marks)

28 The points A(-8, 8) and B(2, -4) are the end points of a diameter of a circle.

(a) Find the coordinates of the centre of the circle and an equation of the circle. **(4)**

(b) Show that the point C(-9, -3) lies on the circle. **(2)**

(c) Use gradients to show that angle ACB is a right angle, and state the circle theorem used. **(3)**

(Total for Question 28 is 9 marks)

29 The points A(1, 7) and B(9, 3) lie on a circle C. The centre of C also lies on the line with equation $y = x - 1$.

(a) Find an equation of the perpendicular bisector of AB. **(4)**

(b) Hence find an equation of circle C. **(4)**

(Total for Question 29 is 8 marks)

30 A circle C has equation $(x-1)^2 + (y-1)^2 = 50$. The line l has equation $y = x + 6$.

(a) State the coordinates of the centre and the exact radius of C. **(2)**

(b) Find the perpendicular distance from the centre of C to the line l. **(3)**

(c) Hence find the length of the chord cut from C by the line l, giving your answer in the form $k\sqrt{2}$ where k is an integer. **(3)**

(Total for Question 30 is 8 marks)

31 A circle has equation $x^2 + y^2 - 8x + 6y - 15 = 0$. Find the centre and the exact radius of the circle.

(Total for Question 31 is 4 marks)

32 A circle C has equation $x^2 + y^2 = 45$. The point A has coordinates (7, 4), which lies outside C. Find the exact length of the tangent from A to the circle.

(Total for Question 32 is 4 marks)

Mark scheme · 1.3 Pure: Coordinate Geometry (Lines and Circles)

Question 1

(7, 2) cao **B1**

Answer: (7, 2)

Question 2

10 cao **B1**

Answer: 10

Question 3

4 cao **B1**

Answer: 4

Question 4

(0, 9) oe cao **B1**

Answer: (0, 9)

Question 5

(-2, -3) cao **B1**

Answer: (-2, -3)

Question 6

(a) finds vector AB = (9 - (-3), 7 - (-1)) = (12, 8) **M1**

(a) uses P = A + (1/4)AB (since AP : PB = 1 : 3 means AP is 1/4 of AB) **M1**

(a) P = (0, 1) cao **A1**

(a) Answer: P = (0, 1)

(b) uses AP = $\sqrt{(0 - (-3))^2 + (1 - (-1))^2}$ **M1**

(b) AP = $\sqrt{13}$ exact, oe awrt 3.61 **A1**

(b) Answer: AP = $\sqrt{13}$ (awrt 3.61)

Question 7

13 cao **B1**

Answer: 13

Question 8

uses the midpoint formula on A and B **M1**

centre = (5, 2) cao **A1**

Answer: centre = (5, 2)

Question 9

6 cao **B1**

Answer: 6

Question 10

-3 cao **B1**

Answer: -3

Question 11

(a) uses (y2 - y1)/(x2 - x1) with correct values, e.g. (10 - 2)/(5 - 1) **M1**

(a) gradient = 2, cao **A1**

(a) Answer: Gradient = 2

(b) uses y - 2 = 2(x - 1) or substitutes a point into y = 2x + c **M1**

(b) y = 2x oe, cao **A1**

(b) Answer: y = 2x

(c) identifies gradient of l is 2 (parallel lines have equal gradient) **M1**

(c) y = 2x - 3 cao **A1**

(c) Answer: y = 2x - 3

Question 12

(a) completes the square on the x terms: $x^2 - 6x = (x-3)^2 - 9$ **M1**

(a) completes the square on the y terms: $y^2 + 4y = (y+2)^2 - 4$ **M1**

(a) centre = (3, -2) **A1**

(a) radius = $\sqrt{50} = 5\sqrt{2}$, exact **A1**

(a) Answer: Centre (3, -2), radius $5\sqrt{2}$

(b) substitutes y = x + k into $(x-3)^2 + (y+2)^2 = 50$ **M1**

(b) forms the correct quadratic in x: $2x^2 + (2k-2)x + (k^2+4k-37) = 0$ **A1**

(b) uses the tangency condition $b^2 - 4ac = 0$ **M1**

(b) simplifies to $k^2 + 10k - 75 = 0$ oe **A1**

(b) k = 5 **A1**

(b) k = -15 **A1**

(b) Answer: k = 5 or k = -15

Question 13

(a) finds $OA^2 = 6^2 + 2^2 = 40$, where O is the centre of C **M1**

(a) uses tangent length = $\sqrt{OA^2 - r^2}$ **M1**

(a) tangent length = $2\sqrt{5}$ exact, oe $\sqrt{20}$ **A1**

(a) Answer: Tangent length = $2\sqrt{5}$

(b) sets up a line through A with gradient m: y - 2 = m(x - 6), i.e. c = 2 - 6m **M1**

(b) substitutes y = mx + c into $x^2 + y^2 = 20$ to form $(1+m^2)x^2 + 2mcx + (c^2-20) = 0$ **M1**

(b) applies the tangency condition $b^2 - 4ac = 0$ to this quadratic **M1**

(b) obtains the correct equation in m: $2m^2 - 3m - 2 = 0$ oe **A1**

(b) m = 2 **A1**

(b) m = -1/2 **A1**

(b) tangents are y = 2x - 10 and y = -(1/2)x + 5, both correct, ft from their m values **A1**

(b) Answer: y = 2x - 10 and y = -(1/2)x + 5

Question 14

(a) uses ((x1+x2)/2, (y1+y2)/2) **M1**

(a) M = (1, -1) cao **A1**

(a) Answer: M = (1, -1)

(b) uses (-5 - 3)/(4 - (-2)) **M1**

(b) gradient = -4/3 cao **A1**

(b) Answer: Gradient = -4/3

(c) uses perpendicular gradient = 3/4 (negative reciprocal of -4/3) **M1**

(c) forms y - (-1) = (3/4)(x - 1) using M = (1, -1) **M1**

(c) 3x - 4y - 7 = 0 oe with integer coefficients, cao **A1**

(c) Answer: 3x - 4y - 7 = 0

Question 15

(a) uses the midpoint formula on A and B **M1**

(a) centre = (3, 2) cao **A1**

(a) Answer: Centre = (3, 2)

(b) finds length AB = $\sqrt{(7-(-1))^2 + (-2-6)^2}$ **M1**

(b) halves AB to obtain the radius **M1**

(b) radius = $4\sqrt{2}$ exact, oe $\sqrt{32}$ **A1**

(b) Answer: radius = $4\sqrt{2}$

(c) uses centre (3,2) and $r^2 = (4\sqrt{2})^2 = 32$ **M1**

(c) $(x-3)^2 + (y-2)^2 = 32$ cao **A1**

(c) Answer: $(x-3)^2 + (y-2)^2 = 32$

Question 16

uses y - (-3) = 4(x - 2) or substitutes (2,-3) into y = 4x + c **M1**

y = 4x - 11 cao **A1**

Answer: y = 4x - 11

Question 17

finds gradient CD = (-2-4)/(5-(-3)) = -3/4 **M1**

forms y - 4 = -3/4(x + 3) using point C (or point D) **M1**

3x + 4y - 7 = 0 oe with integer coefficients, cao **A1**

Answer: 3x + 4y - 7 = 0

Question 18

uses $r^2 = (4-(-1))^2 + (15-3)^2$ **M1**

$r^2 = 169$ (r = 13) **A1**

$(x+1)^2 + (y-3)^2 = 169$ cao **A1**

Answer: $(x + 1)^2 + (y - 3)^2 = 169$

Question 19

substitutes (8,-5) into $(x-2)^2 + (y+1)^2$, i.e. $(8-2)^2 + (-5+1)^2$ **M1**

compares the result to 50 **M1**

52 > 50 so the point lies outside the circle, cao **A1**

Answer: Outside the circle (52 > 50)

Question 20

(a) midpoint of AB **M1**

(a) centre (0, 0) **A1**

(a) radius squared = $(4)^2 + (-2)^2$ using the distance from the centre to A **M1**

(a) $x^2 + y^2 = 20$ **A1**

(a) Answer: $x^2 + y^2 = 20$

(b) substitutes C into the equation: $(2)^2 + (4)^2$ **M1**

(b) 4 + 16 = 20 (cso) **A1**

(b) Answer: C lies on the circle

(c) gradient AC = -3 and gradient BC = 1/3 **M1**

(c) product of gradients = -3 x 1/3 = -1, so AC is perpendicular to BC **A1**

(c) the angle in a semicircle is a right angle **B1**

(c) Answer: angle ACB = 90 degrees

Question 21

(a) eliminates y between the two equations (substitutes y = (75 - 3x)/4 into the circle) **M1**

(a) substitutes into $x^2 + y^2 = 225$ and simplifies to $25x^2 - 450x + 2025 = 0$ (or equivalent) **M1**

(a) evaluates the discriminant **M1**

(a) discriminant = 0, so there is one repeated root and l is a tangent (cso) **A1**

(a) Answer: discriminant = 0, so l is a tangent

(b) repeated root x = -b/(2a) of the quadratic, or the foot of the perpendicular from the origin **M1**

(b) x = 9 **A1**

(b) y = 12 **A1**

(b) Answer: (9, 12)

Question 22

(a) makes t the subject: t = (x + 4)/-1 **M1**

(a) substitutes into the y equation **M1**

(a) $y = 2(x + 4)^2 - 32$ **A1**

(a) Answer: $y = 2(x + 4)^2 - 32$

(b) sets y = 0, so $t^2 = 16$ **M1**

(b) t = 4 or t = -4 **A1**

(b) (-8, 0) and (0, 0) **A1**

(b) Answer: (-8, 0) and (0, 0)

(c) dx/dt = -1 and dy/dt = 4t **M1**

(c) dy/dx = (dy/dt)/(dx/dt) **M1**

(c) gradient = -8 **A1**

(c) Answer: -8

Question 23

(a) makes t the subject: $t = (x + 3)/4$ **M1**

(a) substitutes into the y equation **M1**

(a) $y = -1/16(x + 3)^2 + 9$ **A1**

(a) Answer: $y = -1/16(x + 3)^2 + 9$

(b) sets $y = 0$, so $t^2 = 9$ **M1**

(b) $t = 3$ or $t = -3$ **A1**

(b) (9, 0) and (-15, 0) **A1**

(b) Answer: (9, 0) and (-15, 0)

(c) $dx/dt = 4$ and $dy/dt = -2t$ **M1**

(c) $dy/dx = (dy/dt)/(dx/dt)$ **M1**

(c) gradient = -3/2 **A1**

(c) Answer: -3/2

Question 24

(a) completes the square in x and y **M1**

(a) $(x + 4)^2 + (y + 5)^2 = 169$ **A1**

(a) centre (-4, -5) **A1**

(a) radius = 13 **A1**

(a) Answer: centre (-4, -5), radius 13

(b) substitutes x = 1, y = -17 **M1**

(b) 0 = 0, so P lies on C (cso) **A1**

(b) Answer: P lies on C

(c) gradient of the radius CP = -12/5 **M1**

(c) uses the perpendicular gradient 5/12 and the point P **M1**

(c) 5x - 12y - 209 = 0 **A1**

(c) Answer: 5x - 12y - 209 = 0

(d) sets y = 0 in the tangent equation **M1**

(d) Q(209/5, 0) **A1**

(d) Answer: Q(209/5, 0)

Question 25

(a) midpoint of AB **M1**

(a) centre (2, 1) **A1**

(a) radius squared = $(3)^2 + (-4)^2$ using the distance from the centre to A **M1**

(a) $(x - 2)^2 + (y - 1)^2 = 25$ **A1**

(a) Answer: $(x - 2)^2 + (y - 1)^2 = 25$

(b) substitutes C into the equation: $(4)^2 + (3)^2$ **M1**

(b) 16 + 9 = 25 (cso) **A1**

(b) Answer: C lies on the circle

(c) gradient AC = 7 and gradient BC = -1/7 **M1**

(c) product of gradients = 7 x -1/7 = -1, so AC is perpendicular to BC **A1**

(c) the angle in a semicircle is a right angle **B1**

(c) Answer: angle ACB = 90 degrees

Question 26

(a) completes the square in x and y **M1**

(a) $(x + 4)^2 + (y + 1)^2 = 100$ **A1**

(a) centre (-4, -1) **A1**

(a) radius = 10 **A1**

(a) Answer: centre (-4, -1), radius 10

(b) substitutes x = -10, y = 7 **M1**

(b) 0 = 0, so P lies on C (cso) **A1**

(b) Answer: P lies on C

(c) gradient of the radius CP = -4/3 **M1**

(c) uses the perpendicular gradient 3/4 and the point P **M1**

(c) 3x - 4y + 58 = 0 **A1**

(c) Answer: 3x - 4y + 58 = 0

(d) sets y = 0 in the tangent equation **M1**

(d) Q(-58/3, 0) **A1**

(d) Answer: Q(-58/3, 0)

Question 27

(a) completes the square in x and y **M1**

(a) $(x + 3)^2 + (y - 1)^2 = 25$ **A1**

(a) centre (-3, 1) **A1**

(a) radius = 5 **A1**

(a) Answer: centre (-3, 1), radius 5

(b) substitutes x = 1, y = -2 **M1**

(b) 0 = 0, so P lies on C (cso) **A1**

(b) Answer: P lies on C

(c) gradient of the radius CP = -3/4 **M1**

(c) uses the perpendicular gradient 4/3 and the point P **M1**

(c) 4x - 3y - 10 = 0 **A1**

(c) Answer: 4x - 3y - 10 = 0

(d) sets y = 0 in the tangent equation **M1**

(d) Q(5/2, 0) **A1**

(d) Answer: Q(5/2, 0)

Question 28

(a) midpoint of AB **M1**

(a) centre (-3, 2) **A1**

(a) radius squared = $(-5)^2 + (6)^2$ using the distance from the centre to A **M1**

(a) $(x + 3)^2 + (y - 2)^2 = 61$ **A1**

(a) Answer: $(x + 3)^2 + (y - 2)^2 = 61$

(b) substitutes C into the equation: $(-6)^2 + (-5)^2$ **M1**

(b) 36 + 25 = 61 (cso) **A1**

(b) Answer: C lies on the circle

(c) gradient AC = 11 and gradient BC = -1/11 **M1**

(c) product of gradients = 11 x -1/11 = -1, so AC is perpendicular to BC **A1**

(c) the angle in a semicircle is a right angle **B1**

(c) Answer: angle ACB = 90 degrees

Question 29

(a) finds midpoint of AB = (5, 5) **M1**

(a) finds gradient of AB = (3-7)/(9-1) = -1/2 **M1**

(a) uses perpendicular gradient 2 with y - 5 = 2(x - 5) **M1**

(a) y = 2x - 5 oe, cao **A1**

(a) Answer: y = 2x - 5

(b) solves y = 2x - 5 and y = x - 1 simultaneously **M1**

(b) centre = (4, 3) **A1**

(b) finds radius using distance from centre to A or B, e.g. $\sqrt{(4-1)^2+(3-7)^2}$ **M1**

(b) $(x-4)^2 + (y-3)^2 = 25$ cao **A1**

(b) Answer: $(x-4)^2 + (y-3)^2 = 25$

Question 30

(a) centre = (1, 1) **B1**

(a) radius = $\sqrt{50} = 5\sqrt{2}$, exact **B1**

(a) Answer: Centre (1, 1), radius $5\sqrt{2}$

(b) rearranges l into the form x - y + 6 = 0, or finds the perpendicular to l through the centre, y = -x + 2 **M1**

(b) applies distance = $|1 - 1 + 6| / \sqrt{1^2 + (-1)^2}$, or solves y = -x + 2 with y = x + 6 for the foot (-2, 4) and finds its distance from (1, 1) **M1**

(b) distance = $3\sqrt{2}$ exact, oe awrt 4.24 **A1**

(b) Answer: Distance = $3\sqrt{2}$ (awrt 4.24)

(c) uses $r^2 - d^2 = 50 - 18 = 32$ **M1**

(c) takes the square root and doubles: $2\sqrt{32}$ **M1**

(c) chord length = $8\sqrt{2}$ cao **A1**

(c) Answer: Chord length = $8\sqrt{2}$

Question 31

completes the square on the x terms: $x^2 - 8x = (x-4)^2 - 16$ **M1**

completes the square on the y terms: $y^2 + 6y = (y+3)^2 - 9$ **M1**

centre = (4, -3) **A1**

radius = $\sqrt{40} = 2\sqrt{10}$, exact **A1**

Answer: Centre (4, -3), radius $2\sqrt{10}$

Question 32

radius of C: $r^2 = 45$ **B1**

finds $OA^2 = 7^2 + 4^2 = 65$, where O is the centre of C **M1**

uses tangent length = $\sqrt{OA^2 - r^2}$ **M1**

tangent length = $\sqrt{20} = 2\sqrt{5}$ exact **A1**

Answer: Tangent length = $2\sqrt{5}$