1.4 Pure: Sequences and Series (incl. Binomial Expansion): 1-hour Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Consider the expansion of $(3 + 3x)^4$ in ascending powers of x.

(a) Find the first four terms in the expansion of $(3 + 3x)^4$, giving each coefficient as an integer. **(4)**

(b) Hence find the coefficient of x^3 in the expansion of $(2 - x)(3 + 3x)^4$. **(3)**

(Total for Question 1 is 7 marks)

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2 This question uses the binomial expansion of $(2+x)^7$.

(a) Find, in ascending powers of x, the first four terms of the binomial expansion of $(2+x)^7$, giving each coefficient as an integer. **(4)**

(b) Hence find an estimate for 2.01^7, giving your answer to 3 significant figures. **(3)**

(Total for Question 2 is 7 marks)

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3 An arithmetic series has first term 8 and 20th term 84. Find the common difference, and find the sum of the first 20 terms.

(Total for Question 3 is 3 marks)

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4 A geometric series has first term 20 and common ratio 0.5. State the sum to infinity.

(Total for Question 4 is 1 mark)

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5 The Kingswood Theatre has rows of seats arranged in an arithmetic sequence. Row 1 (nearest the stage) has 18 seats, the common difference between consecutive rows is d, there are 30 rows, and the total seating capacity is 1410.

(a) Show that d = 2. **(3)**

(b) Find the number of seats in row 30 (the back row). **(2)**

(c) The theatre wants to add extra rows at the back, continuing the same arithmetic sequence (same first term and common difference), so that the total capacity is at least 2000 seats. Find the minimum number of extra rows needed. **(5)**

(Total for Question 5 is 10 marks)

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6 A company's sales form an arithmetic sequence. In month 1, sales are 150 units, and sales increase by 12 units each month. Find the total sales over the first 24 months.

(Total for Question 6 is 4 marks)

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7 $f(x) = (1-3x)^{1/3}$.

(a) Find the binomial expansion of f(x), in ascending powers of x, up to and including the term in x^3, simplifying each coefficient. **(5)**

(b) State the range of values of x for which this expansion is valid. **(1)**

(c) By choosing a suitable value of x, use your expansion to estimate $0.94^{1/3}$ to 5 decimal places, and state, with a reason, whether your estimate is an over-estimate or an under-estimate. **(4)**

(Total for Question 7 is 10 marks)

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8 An arithmetic sequence has first term a and common difference d, where a>0 and d>0. The 1st, 2nd and 4th terms of this sequence, taken in that order, form the first three terms of a geometric sequence.

(a) Show that d=a. **(3)**

(b) Given that a=5, write down the first four terms of the arithmetic sequence and verify that its 1st, 2nd and 4th terms form a geometric sequence, stating the common ratio. **(3)**

(c) A geometric sequence has first term 5 and common ratio 2 (as found in part (b)). Find the smallest value of n for which the sum of the first n terms of this geometric sequence exceeds 100000. **(5)**

(Total for Question 8 is 11 marks)

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Mark scheme · 1.4 Pure: Sequences and Series (incl. Binomial Expansion): 1-hour Pack

Question 1

(a) binomial expansion with the correct form for the x term: 4C1 $(3)^3(3)x$	M1
(a) first two terms 81 + 324x	A1
(a) attempts the x^2 and x^3 terms using 4C2 and 4C3	M1
(a) 81 + 324x + 486x^2 + 324x^3 (all four terms correct)	A1
(a) Answer: 81 + 324x + 486x^2 + 324x^3	
(b) identifies the two contributions to x^3: 2 times the x^3 coefficient and -1 times the x^2 coefficient	M1
(b) 648 - 486	A1
(b) 162	A1
(b) Answer: 162	

Question 2

(a) correct structure using C(7,k) 2^{7-k} x^k	M1
(a) 128 + 448x	A1
(a) 672x^2 term correct	A1
(a) 560x^3 term correct	A1
(a) Answer: 128 + 448x + 672x^2 + 560x^3 (+...)	
(b) substitutes x=0.01 into the expansion from part (a)	M1
(b) correct evaluation of the sum (awrt 132.5)	M1
(b) awrt 133	A1
(b) Answer: 133 (3sf)	

Question 3

uses u20 = a + 19d = 84 to find d	M1
d = 4	A1
S20 = 920 (using the correct sum formula), cao	A1
Answer: d = 4; S20 = 920	

Question 4

40 cao	B1
Answer: 40	

Question 5

(a) correct substitution into S_n = n/2(2a+(n-1)d) with a=18, n=30, S=1410	M1
(a) correct rearrangement to 870 = 435d	M1
(a) d=2 cso	A1
(a) Answer: d = 2 (shown)	
(b) u30 = 18 + 29(2)	M1
(b) 76 cao	A1
(b) Answer: 76 seats	
(c) forms S_n = n/2(36+2(n-1)) = n(n+17) for the extended arrangement	M1
(c) sets up the inequality n(n+17) ≥ 2000	M1
(c) solves the associated quadratic n^2+17n-2000=0 (formula or trial), obtaining n=awrt 37.0	dM1
(c) smallest integer total rows n=38 (since n=37 gives 1998<2000)	A1
(c) minimum extra rows = 8 (38-30) cao	A1
(c) Answer: 8 extra rows	

Question 6

identifies a = 150, d = 12, n = 24	M1
correct substitution into S_n = n/2(2a+(n-1)d)	M1
correct unsimplified value 12(300+276)	A1
6912 cao	A1
Answer: 6912 units	

Question 7

(a) correct structure $(1+u)^n$ = 1+nu+n(n-1)/2 u^2+n(n-1)(n-2)/6 u^3 with n=1/3, u=-3x	M1
(a) correct first two terms 1 - x	A1
(a) correct method for the x^2 coefficient	M1
(a) -x^2 term correct	A1
(a) -(5/3)x^3 term correct	A1
(a) Answer: 1 - x - x^2 - (5/3)x^3 (+...)	
(b) |x| < 1/3 oe (e.g. -1/3 < x < 1/3)	B1
(b) Answer: |x| < 1/3	
(c) identifies x=0.02 (from 1-3x=0.94)	M1
(c) correctly substitutes x=0.02 into the expansion from part (a)	M1
(c) awrt 0.97959	A1
(c) states over-estimate, with valid reason (e.g. the next term in the series is negative, so the truncated sum is larger than the true value)	B1
(c) Answer: 0.97959 (over-estimate)	

Question 8

(a) forms $(a+d)^2$ = a(a+3d) using the given geometric sequence condition	M1
(a) correctly expands to a^2+2ad+d^2=a^2+3ad and rearranges to d^2=ad	M1
(a) d=a cso (dividing by d, valid since d>0)	A1
(a) Answer: d = a (shown)	
(b) arithmetic terms 5, 10, 15, 20 (using d=a=5)	B1
(b) computes both ratios 10/5 and 20/10	M1
(b) both ratios equal 2, confirming a geometric sequence with common ratio r=2	A1
(b) Answer: 5, 10, 15, 20; r = 2	
(c) correct use of S_n = a(r^n-1)/(r-1) = 5(2^n-1)	M1
(c) forms the inequality 5(2^n-1) > 100000, i.e. 2^n > 20001	M1
(c) takes logs of both sides: n > log(20001)/log(2)	M1
(c) n > awrt 14.3	A1
(c) smallest integer n=15 cao	A1
(c) Answer: n = 15	

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