1.4 Pure: Sequences and Series (incl. Binomial Expansion): 2-hour Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The terms of a geometric sequence satisfy $u_4 = 81$ and $u_6 = 729/4$. The common ratio is positive.

(a) Find the common ratio, showing your method clearly. **(2)**

(b) Find the first term and the sum of the first 6 terms. **(4)**

(Total for Question 1 is 6 marks)

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2 The terms of a geometric sequence satisfy $u_3 = 21$ and $u_5 = 7/3$. The common ratio is positive.

(a) Find the common ratio, showing your method clearly. **(2)**

(b) Find the first term and the sum of the first 6 terms. **(4)**

(c) Find the sum to infinity of the series. **(3)**

(Total for Question 2 is 9 marks)

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3 The sequence u_r is defined by $u_r = 7r - 4$, $r = 1, 2, 3, ...$

(a) Write down u_1 and the common difference, and state the type of sequence. **(2)**

(b) Evaluate the sum from $r = 1$ to $r = 17$ of $(7r - 4)$. **(3)**

(c) Find the least value of n for which the sum from $r = 1$ to $r = n$ of $(7r - 4)$ exceeds 350. **(3)**

(Total for Question 3 is 8 marks)

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4 The terms of a geometric sequence satisfy $u_2 = 36$ and $u_6 = 64/9$. The common ratio is positive.

(a) Find the common ratio, showing your method clearly. **(2)**

(b) Find the first term and the sum of the first 6 terms. **(4)**

(c) Find the sum to infinity of the series. **(3)**

(Total for Question 4 is 9 marks)

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5 The sequence u_r is defined by $u_r = 6r - 2$, $r = 1, 2, 3, ...$

(a) Write down u_1 and the common difference, and state the type of sequence. **(2)**

(b) Evaluate the sum from $r = 1$ to $r = 15$ of $(6r - 2)$. **(3)**

(c) Find the least value of n for which the sum from $r = 1$ to $r = n$ of $(6r - 2)$ exceeds 680. **(3)**

(Total for Question 5 is 8 marks)

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6 A charity runner trains for a marathon. In week 1 she runs 3 km, and each week she runs 10 km further than the week before. The pattern continues for 12 weeks.

(a) Find the value in week 12. **(2)**

(b) Find the total over the first 12 weeks. **(3)**

(c) Find the smallest number of weeks for which the total exceeds 4000. **(3)**

(Total for Question 6 is 8 marks)

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7 A stack of tiles has 13 tiles in the first row, and each further row has 2 more tiles than the row before it. The pattern continues for 13 rows.

(a) Find the value in row 13. **(2)**

(b) Find the total over the first 13 rows. **(3)**

(c) Find the smallest number of rows for which the total exceeds 1100. **(3)**

(Total for Question 7 is 8 marks)

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8 Consider the expansion of $(1 + x)^5$ in ascending powers of x.

(a) Find the first four terms in the expansion of $(1 + x)^5$, giving each coefficient as an integer. **(4)**

(b) Hence find the coefficient of x^2 in the expansion of $(4 + x)(1 + x)^5$. **(3)**

(Total for Question 8 is 7 marks)

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9 A stack of tiles has 29 tiles in the first row, and each further row has 8 more tiles than the row before it. The pattern continues for 29 rows.

(a) Find the value in row 29. **(2)**

(b) Find the total over the first 29 rows. **(3)**

(c) Find the smallest number of rows for which the total exceeds 1600. **(3)**

(Total for Question 9 is 8 marks)

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10 An arithmetic sequence has 3rd term 17 and common difference 4. Find the first term.

(Total for Question 10 is 2 marks)

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11 Find the sum of the first 20 positive integers $(1 + 2 + 3 + ... + 20)$.

(Total for Question 11 is 2 marks)

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12 A geometric sequence has first term 4 and common ratio 3. Find the 4th term.

(Total for Question 12 is 1 mark)

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13 A geometric sequence has first term 100 and common ratio 0.5. Find the 3rd term.

(Total for Question 13 is 1 mark)

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14 An arithmetic series has first term 2 and common difference 5. Find the sum of the first 4 terms.

(Total for Question 14 is 1 mark)

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15 A geometric series has first term 3 and common ratio 2. Find the sum of the first 4 terms.

(Total for Question 15 is 1 mark)

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16 Find the coefficient of x^2 in the expansion of $(1+3x)^5$.

(Total for Question 16 is 2 marks)

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17 A geometric sequence has first term 6 and 4th term 162. Find the common ratio, and find the 6th term.

(Total for Question 17 is 3 marks)

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18 Find the sum of the first 15 terms of the arithmetic series 7 + 11 + 15 + ...

(Total for Question 18 is 3 marks)

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19 A geometric series has first term a = 54 and sum to infinity 81. Find the common ratio r, and find the sum of the first 5 terms, giving your answer to 3 significant figures.

(Total for Question 19 is 4 marks)

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20 The first three terms of an arithmetic sequence, in order, are k - 2, 2k + 1 and 5k - 4, where k is a constant. Find the value of k, and hence find the common difference of the sequence.

(Total for Question 20 is 4 marks)

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Mark scheme · 1.4 Pure: Sequences and Series (incl. Binomial Expansion): 2-hour Pack

Question 1

(a) forms $r^2 = u_6/u_4 = 9/4$ **M1**

(a) r = 3/2 (r = -3/2 is rejected because the common ratio is positive) **A1**

(a) Answer: r = 3/2

(b) $a = u_4/r^3$ **M1**

(b) a = 24 **A1**

(b) uses $S_n = a(1 - r^n)/(1 - r)$ **M1**

(b) $S_6 = 1995/4$ **A1**

(b) Answer: a = 24, $S_6 = 1995/4$

Question 2

(a) forms $r^2 = u_5/u_3 = 1/9$ **M1**

(a) r = 1/3 (r = -1/3 is rejected because the common ratio is positive) **A1**

(a) Answer: r = 1/3

(b) $a = u_3/r^2$ **M1**

(b) a = 189 **A1**

(b) uses $S_n = a(1 - r^n)/(1 - r)$ **M1**

(b) $S_6 = 2548/9$ **A1**

(b) Answer: a = 189, $S_6 = 2548/9$

(c) states |r| < 1 so the series converges **B1**

(c) uses $S_\infty = a/(1 - r)$ **M1**

(c) 567/2 **A1**

(c) Answer: 567/2

Question 3

(a) $u_1 = 3$ **B1**

(a) d = 7, arithmetic **B1**

(a) Answer: $u_1 = 3$, d = 7

(b) uses the arithmetic series formula with a = 3, d = 7, n = 17 **M1**

(b) (17/2)(2(3) + 16(7)) **A1**

(b) 1003 **A1**

(b) Answer: 1003

(c) forms (n/2)(2(3) + (n - 1)(7)) > 350 **M1**

(c) solves the quadratic inequality or tests consecutive values **A1**

(c) $S_{10} = 345$ and $S_{11} = 418$, so n = 11 **A1**

(c) Answer: n = 11

Question 4

(a) forms $r^4 = u_6/u_2 = 16/81$ **M1**

(a) r = 2/3 (r = -2/3 is rejected because the common ratio is positive) **A1**

(a) Answer: r = 2/3

(b) $a = u_2/r$ **M1**

(b) a = 54 **A1**

(b) uses $S_n = a(1 - r^n)/(1 - r)$ **M1**

(b) $S_6 = 1330/9$ **A1**

(b) Answer: a = 54, $S_6 = 1330/9$

(c) states |r| < 1 so the series converges **B1**

(c) uses $S_\infty = a/(1 - r)$ **M1**

(c) 162 **A1**

(c) Answer: 162

Question 5

(a) $u_1 = 4$ **B1**

(a) d = 6, arithmetic **B1**

(a) Answer: $u_1 = 4$, d = 6

(b) uses the arithmetic series formula with a = 4, d = 6, n = 15 **M1**

(b) (15/2)(2(4) + 14(6)) **A1**

(b) 690 **A1**

(b) Answer: 690

(c) forms (n/2)(2(4) + (n - 1)(6)) > 680 **M1**

(c) solves the quadratic inequality or tests consecutive values **A1**

(c) $S_{14} = 602$ and $S_{15} = 690$, so n = 15 **A1**

(c) Answer: n = 15

Question 6

(a) uses $u_n = a + (n - 1)d$ with a = 3, d = 10, n = 12 **M1**

(a) 113 **A1**

(a) Answer: 113

(b) uses $S_n = (n/2)(2a + (n - 1)d)$ or $(n/2)(a + l)$ **M1**

(b) 12/2 x (3 + 113) **A1**

(b) 696 **A1**

(b) Answer: 696

(c) forms and solves (m/2)(2(3) + (m - 1)(10)) > 4000 **M1**

(c) critical value m = 28.48 (or tests m = 28 and m = 29) **A1**

(c) $S_{28} = 3864 < 4000$ and $S_{29} = 4147 > 4000$, so m = 29 **A1**

(c) Answer: 29

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Question 7

(a) uses $u_n = a + (n - 1)d$ with a = 13, d = 2, n = 13 **M1**

(a) 37 **A1**

(a) Answer: 37

(b) uses $S_n = (n/2)(2a + (n - 1)d)$ or $(n/2)(a + l)$ **M1**

(b) 13/2 x (13 + 37) **A1**

(b) 325 **A1**

(b) Answer: 325

(c) forms and solves $(m/2)(2(13) + (m - 1)(2)) > 1100$ **M1**

(c) critical value m = 27.70 (or tests m = 27 and m = 28) **A1**

(c) $S_{27} = 1053 < 1100$ and $S_{28} = 1120 > 1100$, so m = 28 **A1**

(c) Answer: 28

Question 8

(a) binomial expansion with the correct form for the x term: $5C1\ (1)^4(1)x$ **M1**

(a) first two terms 1 + 5x **A1**

(a) attempts the x^2 and x^3 terms using 5C2 and 5C3 **M1**

(a) $1 + 5x + 10x^2 + 10x^3$ (all four terms correct) **A1**

(a) Answer: $1 + 5x + 10x^2 + 10x^3$

(b) identifies the two contributions to x^2: 4 times the x^2 coefficient and 1 times the x coefficient **M1**

(b) 40 + 5 **A1**

(b) 45 **A1**

(b) Answer: 45

Question 9

(a) uses $u_n = a + (n - 1)d$ with a = 29, d = 8, n = 29 **M1**

(a) 253 **A1**

(a) Answer: 253

(b) uses $S_n = (n/2)(2a + (n - 1)d)$ or $(n/2)(a + l)$ **M1**

(b) 29/2 x (29 + 253) **A1**

(b) 4089 **A1**

(b) Answer: 4089

(c) forms and solves $(m/2)(2(29) + (m - 1)(8)) > 1600$ **M1**

(c) critical value m = 17.12 (or tests m = 17 and m = 18) **A1**

(c) $S_{17} = 1581 < 1600$ and $S_{18} = 1746 > 1600$, so m = 18 **A1**

(c) Answer: 18

Question 10

uses u3 = a + 2d = 17 **M1**

a = 9 cao **A1**

Answer: a = 9

Question 11

uses $S_n = n(n+1)/2$ (or the arithmetic series formula with a=1, d=1, n=20) **M1**

210 cao **A1**

Answer: 210

Question 12

108 cao **B1**

Answer: 108

Question 13

25 cao **B1**

Answer: 25

Question 14

38 cao **B1**

Answer: 38

Question 15

45 cao **B1**

Answer: 45

Question 16

uses $5C2 \times 3^2$ **M1**

90 cao **A1**

Answer: 90

Question 17

uses $u4 = a\,r^3 = 162$ **M1**

r = 3 **A1**

u6 = 1458 cao **A1**

Answer: r = 3; u6 = 1458

Question 18

identifies a = 7, d = 4 **M1**

correct use of $S_n = n/2(2a+(n-1)d)$ with n = 15 **M1**

525 cao **A1**

Answer: 525

Question 19

uses $S_\infty = a/(1-r) = 81$ **M1**

r = 1/3 **A1**

correct use of $S_n = a(1-r^n)/(1-r)$ with n = 5 **M1**

awrt 80.7 **A1**

Answer: r = 1/3; S5 = 80.7 (3 s.f.)

Question 20

sets consecutive differences equal: (2k+1) - (k-2) = (5k-4) - (2k+1) **M1**

simplifies to a linear equation, k + 3 = 3k - 5 **M1**

k = 4 cao **A1**

common difference = 7 (from terms 2, 9, 16), cao **A1**

Answer: k = 4; common difference = 7

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