1.4 Pure: Sequences and Series (incl. Binomial Expansion): Test Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Priya's training plan for a marathon increases the distance she runs each week by a fixed amount. In week 1 she runs 8 km, and each following week she runs 1.5 km further than the previous week, for a total of 16 weeks.

(a) Find the distance Priya runs in week 16. **(2)**

(b) Find the total distance Priya runs over the 16 weeks. **(3)**

(c) After week 16, Priya changes to running a constant 30.5 km every week (the same distance as week 16) for a further m weeks. Given that her total distance run overall (the original 16 weeks plus the constant-distance weeks) must be at least 800 km, find the least value of m. **(4)**

(Total for Question 1 is 9 marks)

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2 Write down the first two terms, in ascending powers of x, of the binomial expansion of $(1+x)^9$.

(Total for Question 2 is 1 mark)

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3 The binomial expansion of $(1+x)^6$ is to be used to answer this question.

(a) Write out the full expansion of $(1+x)^6$ in ascending powers of x. **(3)**

(b) Hence find the coefficient of x^3 in the expansion of $(1+2x)^6$. **(2)**

(c) Find the term independent of x in the expansion of $(x^2 + 3/x)^6$. **(4)**

(Total for Question 3 is 9 marks)

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4 Find the coefficient of x^3 in the expansion of $(2+x)^6$, giving your answer as an integer.

(Total for Question 4 is 3 marks)

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5 The first three terms of a geometric sequence, in order, are k, k+6 and 4k+9, where k is a positive constant.

(a) Show that $k^2 - k - 12 = 0$. **(3)**

(b) Hence show that k=4. **(2)**

(c) Using k=4, find the common ratio of the sequence and the sum of the first 12 terms, giving your answer to the nearest integer. **(4)**

(Total for Question 5 is 9 marks)

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6 In the binomial expansion, in ascending powers of x, of $(1+cx)^{10}$, where c is a non-zero constant, the coefficient of x^3 is 4 times the coefficient of x^2.

(a) Find the value of c. **(4)**

(b) Using this value of c, find the coefficient of x^4 in the expansion, giving your answer as an exact fraction. **(3)**

(Total for Question 6 is 7 marks)

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7 A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1=3$ and $u_{n+1} = 1/(1-u_n)$ for $n \geq 1$.

(a) Find the values of u_2, u_3 and u_4, and hence show that the sequence is periodic, stating the order (period) of the sequence. **(4)**

(b) Hence find $\sum_{r=1}^{100} u_r$. **(4)**

(c) Find the value of u_{2026}. **(2)**

(Total for Question 7 is 10 marks)

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Mark scheme · 1.4 Pure: Sequences and Series (incl. Binomial Expansion): Test Pack

Question 1

(a) uses $u_n = a + (n-1)d$ with a=8, d=1.5, n=16 **M1**

(a) 30.5 km cao **A1**

(a) Answer: 30.5 km

(b) correct use of $S_n = n/2(2a+(n-1)d)$ with n=16, a=8, d=1.5 **M1**

(b) correct unsimplified value 8(16+22.5) **A1**

(b) 308 km cao **A1**

(b) Answer: 308 km

(c) forms 308 + 30.5m ≥ 800 **M1**

(c) correct rearrangement m ≥ 492/30.5 **M1**

(c) m ≥ awrt 16.13 **A1**

(c) least integer m=17 cao, with reference to context (whole number of weeks) **A1**

(c) Answer: m = 17

Question 2

1 + 9x cao **B1**

Answer: 1 + 9x

Question 3

(a) correct binomial coefficients 1, 6, 15, 20, 15, 6, 1 (Pascal's triangle row 6, or nCr values) **M1**

(a) correct powers of x attached to each coefficient **A1**

(a) fully correct expansion cao **A1**

(a) Answer: $1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$

(b) uses $C(6,3) \times 2^3$ **M1**

(b) 160 cao **A1**

(b) Answer: 160

(c) correct general term $C(6,k)(x^2)^{6-k}(3/x)^k$ **M1**

(c) correct simplified power of x: 12-3k **M1**

(c) sets 12-3k=0 and solves k=4 **M1**

(c) 1215 cao **A1**

(c) Answer: 1215

Question 4

uses the correct general term $6C3\ (2)^3\ x^3$ **M1**

correct evaluation $6C3=20$, $2^3=8$ **A1**

160 cao **A1**

Answer: 160

Question 5

(a) forms $(k+6)^2 = k(4k+9)$ using the geometric sequence property **M1**

(a) correctly expands both sides **M1**

(a) $k^2-k-12=0$ cso, answer given **A1**

(a) Answer: $k^2 - k - 12 = 0$ (shown)

(b) factorises (k-4)(k+3)=0 or uses the quadratic formula **M1**

(b) k=4 cso, rejecting k=-3 since k is positive **A1**

(b) Answer: k = 4

(c) r=2.5 (=10/4, confirmed also by 25/10) **B1**

(c) correct use of $S_n = a(r^n-1)/(r-1)$ with a=4, n=12 **M1**

(c) correct value of r^{12} (awrt 59604.6) **M1**

(c) awrt 158943 **A1**

(c) Answer: r = 2.5; S12 = 158943 (nearest integer)

Question 6

(a) writes the coefficient of x^2 as $C(10,2)c^2$ and the coefficient of x^3 as $C(10,3)c^3$ **M1**

(a) forms the equation $120c^3 = 4(45c^2)$, i.e. $120c^3 = 180c^2$ **M1**

(a) solves $60c^2(2c-3)=0$ **M1**

(a) c=3/2 cao, rejecting c=0 **A1**

(a) Answer: c = 3/2

(b) uses $C(10,4)c^4$ **M1**

(b) substitutes c=3/2 correctly, $(3/2)^4 = 81/16$ **M1**

(b) 8505/8 oe (awrt 1063.13) **A1**

(b) Answer: 8505/8 (=1063.125)

Question 7

(a) correct $u_2=-1/2$ **M1**

(a) correct $u_3=2/3$ **M1**

(a) correct $u_4=3$ $(=u_1)$, demonstrating repetition **A1**

(a) states the sequence is periodic with period (order) 3 **A1**

(a) Answer: $u_2 = -1/2$, $u_3 = 2/3$, $u_4 = 3$; period 3

(b) finds the sum of one period = 3 + (-1/2) + 2/3 = 19/6 **M1**

(b) recognises 100=3(33)+1, giving 33 complete periods plus one extra term **M1**

(b) computes $33(19/6) + u_1$ **dM1**

(b) 215/2 oe (=107.5) cao **A1**

(b) Answer: 215/2 (=107.5)

(c) considers the remainder when 2026 is divided by 3 (2026=3(675)+1) **M1**

(c) $u_{2026}=3$ cao **A1**

(c) Answer: $u_{2026} = 3$

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