1.5 Pure: Trigonometry: 1-hour Pack

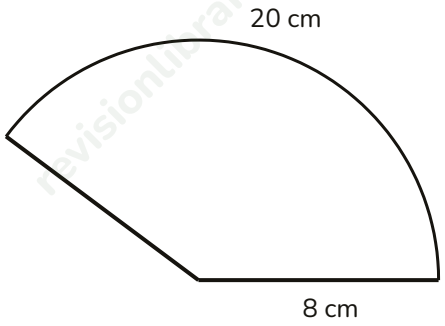
EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A sector has radius 8 cm and arc length 20 cm. Find the angle at the centre, in radians.



(Total for Question 1 is 2 marks)

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2 Solve $2\sin(x) - 1 = 0$ for $0 \text{ degrees} \leq x \leq 360 \text{ degrees}$, giving all solutions.

(Total for Question 2 is 3 marks)

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3 $f(x) = 2 \sin x + 2 \cos x$, where x is in degrees.

(a) Express f(x) in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < 90$ degrees. Give R as an exact value and α to 1 decimal place. **(4)**

(b) Write down the maximum value of f(x), and the smallest positive value of x at which it occurs, to 1 decimal place. **(2)**

(c) Solve $f(x) = 1$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 3 is 10 marks)

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4 Given that θ is small and measured in radians, use small angle approximations to answer part (a). Part (b) asks for an exact value, found on a calculator.

(a) Show that $(1 - \cos 5\theta)/(\theta \tan 3\theta)$ is approximately 25/6. **(3)**

(b) Evaluate $(1 - \cos 5\theta)/(\theta \tan 3\theta)$ at $\theta = 0.05$ radians to 4 significant figures, and find the percentage error in the approximation 25/6, to 2 significant figures. **(4)**

(Total for Question 4 is 7 marks)

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5 This question has two parts.

(a) Show that $\text{cosec}(\theta) - \sin(\theta) = \cos(\theta)\cot(\theta)$, for $0 \text{ degrees} < \theta < 360 \text{ degrees}$, θ not equal to 180 degrees. **(4)**

(b) Hence solve $\text{cosec}(\theta) - \sin(\theta) = \cos(\theta)$ for $0 \text{ degrees} \leq \theta \leq 360 \text{ degrees}$, θ not equal to 0, 180, 360 degrees. **(4)**

(Total for Question 5 is 8 marks)

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6 This question has two parts.

(a) Prove that $\sin(3\text{theta}) = 3\sin(\theta) - 4\sin^3(\theta)$. **(4)**

(b) Hence, or otherwise, solve $4\sin^3(\theta) - 3\sin(\theta) + 0.5 = 0$ for $0 \text{ degrees} \leq \theta \leq 360$ degrees. **(5)**

(Total for Question 6 is 9 marks)

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7 This question concerns the inverse trigonometric functions arcsin and arccos.

(a) State the domain and range of the function $f(x) = \arcsin(x)$. **(2)**

(b) Sketch the graph of $y = \arccos(x)$ for $-1 \leq x \leq 1$, indicating the coordinates of the endpoints of the curve and the point where the curve crosses the y-axis. **(3)**

(c) Solve $\arccos(2x - 1) = \pi/3$, giving x as an exact fraction. **(3)**

(d) Given that $\arcsin(x) = \arccos(x) + \pi/6$, and using the identity $\arcsin(x) + \arccos(x) = \pi/2$, find the exact value of x. **(4)**

(Total for Question 7 is 12 marks)

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Mark scheme · 1.5 Pure: Trigonometry: 1-hour Pack

Question 1

uses θ = s/r	**M1**
2.5 radians cao	**A1**

Answer: 2.5 radians

Question 2

rearranges to sin(x) = 0.5	**M1**
x = 30 degrees	**A1**
x = 150 degrees, no extra values in range	**A1**

Answer: x = 30 degrees, 150 degrees

Question 3

(a) R cos α = 2 and R sin α = 2	**M1**
(a) R = $2\sqrt{2}$	**A1**
(a) tan α = 2/2	**M1**
(a) α = 45.0 degrees	**A1**

(a) Answer: f(x) = $2\sqrt{2}$ sin(x + 45.0)

(b) maximum $2\sqrt{2}$	**B1**
(b) x + 45.0 = 90, so x = 45.0	**B1**

(b) Answer: maximum $2\sqrt{2}$ at x = 45.0

(c) sin(x + 45.0) = 1/($2\sqrt{2}$)	**M1**
(c) x + 45.0 = 20.7 or 159.3 (and adds 360 if needed)	**M1**
(c) x = 114.3	**A1**
(c) x = 335.7	**A1**

(c) Answer: x = 114.3, 335.7

Question 4

(a) uses cos 5theta is approximately 1 - $(5\text{theta})^2/2$	**M1**
(a) numerator is approximately $(25/2)\theta^2$ and tan 3theta is approximately 3theta	**M1**
(a) denominator is approximately 3theta^2, so the ratio is (25/2)/3 = 25/6 (cso)	**A1**

(a) Answer: approximately 25/6

(b) calculates the value with θ = 0.05: 4.11388	**M1**
(b) 4.114	**A1**
(b) percentage error = |4.1667 - 4.1139|/4.1139 x 100	**M1**
(b) 1.3%	**A1**

(b) Answer: 4.114, error 1.3%

Question 5

(a) writes cosec(θ) as 1/sin(θ)	**M1**
(a) combines into a single fraction: $(1 - \sin^2(\theta))/\sin(\theta)$	**M1**
(a) uses $1 - \sin^2(\theta) = \cos^2(\theta)$	**M1**
(a) cso completes to cos(θ)*(cos(θ)/sin(θ)) = cos(θ) cot(θ)	**A1**

(a) Answer: Identity shown (proof)

(b) uses part (a) to form cos(θ)*cot(θ) = cos(θ), i.e. cos(θ)*(cot(θ) - 1) = 0	**M1**
(b) cos(θ) = 0 leading to θ = 90, 270	**M1**
(b) cot(θ) = 1, i.e. tan(θ) = 1, leading to θ = 45, 225	**M1**
(b) all four values correct: θ = 45, 90, 225, 270 degrees cao, no extras	**A1**

(b) Answer: θ = 45, 90, 225, 270 degrees

Question 6

(a) writes sin(3theta) = sin(2theta + θ) = sin(2theta)cos(θ) + cos(2theta)sin(θ)	**M1**
(a) substitutes sin(2theta) = 2sin(θ)cos(θ) and cos(2theta) = $1 - 2\sin^2(\theta)$	**M1**
(a) uses $\cos^2(\theta) = 1 - \sin^2(\theta)$ to write the expression entirely in sin(θ)	**M1**
(a) cso completes correctly to $3\sin(\theta) - 4\sin^3(\theta)$	**A1**

(a) Answer: Identity proved

(b) recognises from (a) that $4\sin^3(\theta) - 3\sin(\theta) = -\sin(3\text{theta})$	**M1**
(b) forms sin(3theta) = 0.5	**M1**
(b) correct method for all solutions of 3theta in [0, 1080] using 30+360k and 150+360k	**M1**
(b) at least 3 of the 6 correct values of θ	**A1**
(b) all six correct: θ = 10, 50, 130, 170, 250, 290 degrees, cao	**A1**

(b) Answer: θ = 10, 50, 130, 170, 250, 290 degrees

Question 7

(a) domain: -1 ≤ x ≤ 1	**B1**
(a) range: -90 degrees ≤ f(x) ≤ 90 degrees (or -π/2 ≤ f(x) ≤ π/2), oe	**B1**

(a) Answer: Domain: -1 ≤ x ≤ 1; Range: -90 ≤ f(x) ≤ 90 degrees (-π/2 to π/2)

(b) smooth decreasing curve shape, no turning points, correct general shape from top-left to bottom-right	**B1**
(b) endpoints correctly labelled: (-1, 180 degrees) [or (-1, π)] and (1, 0)	**B1**
(b) y-intercept correctly labelled: (0, 90 degrees) [or (0, π/2)]	**B1**

(b) Answer: Decreasing curve from (-1, 180 deg) through (0, 90 deg) to (1, 0 deg)

(c) takes cosine of both sides: 2x - 1 = cos(π/3)	**M1**
(c) cos(π/3) = 1/2 (oe) used	**A1**
(c) x = 3/4 cao (exact)	**A1**

(c) Answer: x = 3/4

(d) uses the identity to write arccos(x) = π/2 - arcsin(x)	**M1**
(d) substitutes into given equation and rearranges to 2 arcsin(x) = 2pi/3, i.e. arcsin(x) = π/3	**M1**
(d) x = sin(π/3)	**A1**
(d) x = $\sqrt{3}/2$ cao (exact form)	**A1**

(d) Answer: x = $\sqrt{3}/2$

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