1.5 Pure: Trigonometry

EDEXCEL 9MA0 · Calculator allowed · about 225 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Convert 60 degrees to radians, giving your answer in terms of π.

(Total for Question 1 is 1 mark)

2 Write down the exact value of sin(30 degrees).

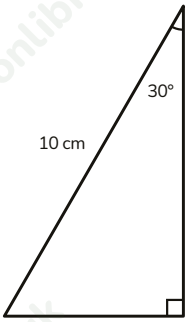
(Total for Question 2 is 1 mark)

3 Write down the exact value of cos(60 degrees).

(Total for Question 3 is 1 mark)

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4 A right-angled triangle has hypotenuse 10 cm and one angle of 30 degrees. Find the length of the side opposite this angle.



(Total for Question 4 is 1 mark)

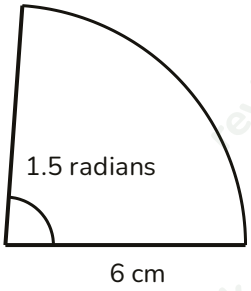
5 Solve the following trigonometric equations, giving all solutions in the given ranges.

(a) Solve $2\sin(3\theta - 20 \text{ degrees}) = 1.2$ for $0 \text{ degrees} \leq \theta \leq 360 \text{ degrees}$, giving your answers to 1 decimal place. **(5)**

(b) Solve $\cos(x/2) = -0.85$ for $0 \text{ degrees} \leq x \leq 720 \text{ degrees}$, giving your answers to 1 decimal place. **(4)**

(Total for Question 5 is 9 marks)

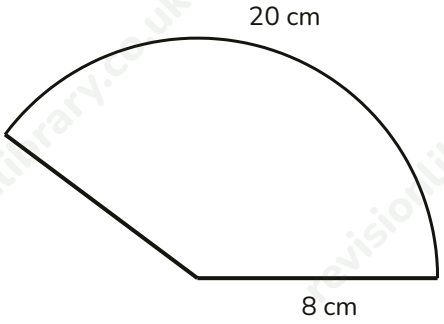
6 A sector has radius 6 cm and angle 1.5 radians at the centre. Find the area of the sector.



(Total for Question 6 is 1 mark)

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7 A sector has radius 8 cm and arc length 20 cm. Find the angle at the centre, in radians.



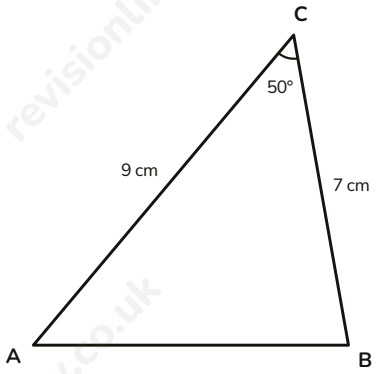
(Total for Question 7 is 2 marks)

8 Solve cos(x) = 0.5 for 0 degrees ≤ x ≤ 360 degrees, giving all solutions.

(Total for Question 8 is 2 marks)

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9 A triangle has sides a = 7 cm and b = 9 cm with included angle C = 50 degrees. Find the area of the triangle, giving your answer to 3 significant figures.



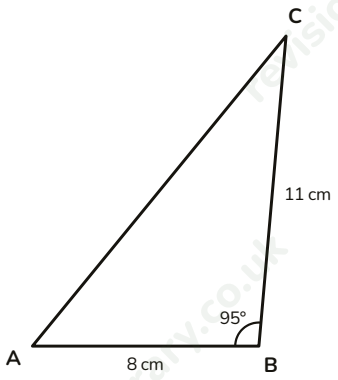
(Total for Question 9 is 2 marks)

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10 Solve 2sin(x) - 1 = 0 for 0 degrees ≤ x ≤ 360 degrees, giving all solutions.

(Total for Question 10 is 3 marks)

11 In triangle ABC, AB = 8 cm, BC = 11 cm and angle ABC = 95 degrees. Calculate the length of AC, giving your answer to 3 significant figures.



(Total for Question 11 is 3 marks)

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12 Solve tan(x) = -1 for 0 degrees ≤ x ≤ 360 degrees, giving all solutions.

(Total for Question 12 is 3 marks)

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13 Solve sin(x) = 0.5 for 0 degrees ≤ x ≤ 90 degrees.

(Total for Question 13 is 1 mark)

14 The depth of water, D metres, in a harbour, t hours after midnight on a particular day, is modelled by $D(t) = 6 + 2.5 \sin(30t)$, for $0 \leq t \leq 24$, where the angle is measured in degrees.

(a) State the maximum depth of water given by the model, and find the times during the 24 hour period at which it occurs. **(3)**

(b) Find the depth of water at 08:30, giving your answer to 3 significant figures. **(2)**

(c) Find the times during the 24 hour period at which the depth of water is exactly 7 m, giving your answers to the nearest minute. **(6)**

(Total for Question 14 is 11 marks)

15 This question is about trigonometric identities and equations.

(a) Show that $(1 + \sin x)/\cos x + \cos x/(1 + \sin x) = 2 \sec x$. **(3)**

(b) Hence solve $(1 + \sin x)/\cos x + \cos x/(1 + \sin x) = 3$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(3)**

(Total for Question 15 is 6 marks)

16 $f(x) = 2 \sin x + 2 \cos x$, where x is in degrees.

(a) Express $f(x)$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < 90$ degrees. Give R as an exact value and α to 1 decimal place. **(4)**

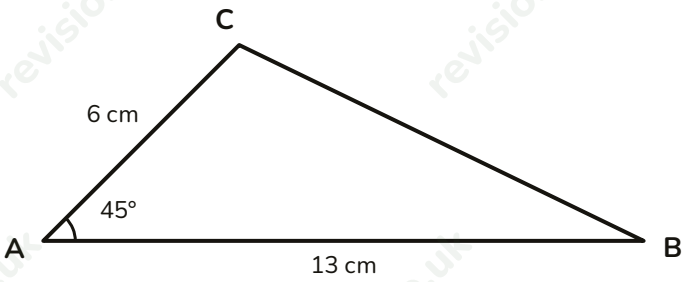
(b) Write down the maximum value of $f(x)$, and the smallest positive value of x at which it occurs, to 1 decimal place. **(2)**

(c) Solve $f(x) = 1$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 16 is 10 marks)

17 In triangle ABC, AB = 13 cm, AC = 6 cm and angle BAC = 45 degrees.

Diagram NOT accurately drawn



(a) Calculate the length of BC, giving your answer to 3 significant figures. **(3)**

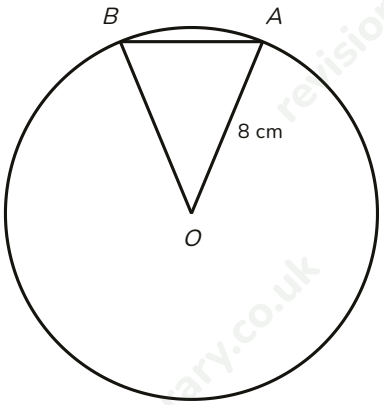
(b) Calculate the size of angle ABC, giving your answer to 1 decimal place. **(2)**

(c) Calculate the area of triangle ABC, giving your answer to 3 significant figures. **(2)**

(Total for Question 17 is 7 marks)

18 The sector OAB of a circle with centre O and radius 8 cm has angle AOB = $\pi/4$ radians.

Diagram NOT accurately drawn



(a) Find the length of the arc AB, giving your answer as an exact multiple of π. **(2)**

(b) Find the area of the sector, giving your answer as an exact multiple of π. **(2)**

(c) Find the exact area of the segment bounded by the chord AB and the arc AB. **(3)**

(d) Find the perimeter of the sector, giving your answer to 3 significant figures. **(2)**

(Total for Question 18 is 9 marks)

19 Given that θ is small and measured in radians, use small angle approximations to answer part (a). Part (b) asks for an exact value, found on a calculator.

(a) Show that (1 - cos 5theta)/(θ tan 3theta) is approximately 25/6. **(3)**

(b) Evaluate (1 - cos 5theta)/(θ tan 3theta) at θ = 0.05 radians to 4 significant figures, and find the percentage error in the approximation 25/6, to 2 significant figures. **(4)**

(Total for Question 19 is 7 marks)

20 Solve $2\cos^2 x + \sin x - 2 = 0$ for $-180 < x \leq 180$, giving all solutions in degrees.

(a) Use $\cos^2 x = 1 - \sin^2 x$ to show that the equation can be written as a quadratic in $\sin x$, and state that quadratic. **(3)**

(b) Hence solve the equation. **(4)**

(Total for Question 20 is 7 marks)

21 $f(x) = 6 \sin x + 7 \cos x$, where x is in degrees.

(a) Express $f(x)$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < 90$ degrees. Give R as an exact value and α to 1 decimal place. **(4)**

(b) Write down the maximum value of $f(x)$, and the smallest positive value of x at which it occurs, to 1 decimal place. **(2)**

(c) Solve $f(x) = 3$ for $0 \leq x < 360$ degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 21 is 10 marks)

22 This question is about trigonometric identities and equations.

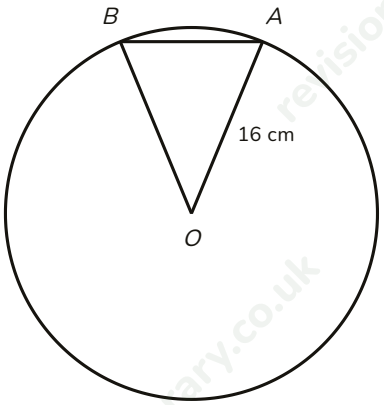
(a) Show that cosec x - sin x = cos x cot x, for sin x ≠ 0. **(3)**

(b) Hence solve cosec x - sin x = 3 cos x for 0 < x < 360 degrees, giving your answers to 1 decimal place. **(4)**

(Total for Question 22 is 7 marks)

23 The sector OAB of a circle with centre O and radius 16 cm has angle AOB = $\pi/4$ radians.

Diagram NOT accurately drawn



(a) Find the length of the arc AB, giving your answer as an exact multiple of π. **(2)**

(b) Find the area of the sector, giving your answer as an exact multiple of π. **(2)**

(c) Find the exact area of the segment bounded by the chord AB and the arc AB. **(3)**

(d) Find the perimeter of the sector, giving your answer to 3 significant figures. **(2)**

(Total for Question 23 is 9 marks)

24 This question has two parts.

(a) Show that $\text{cosec}(\theta) - \sin(\theta) = \cos(\theta)\cot(\theta)$, for $0 \text{ degrees} < \theta < 360 \text{ degrees}$, θ not equal to 180 degrees. **(4)**

(b) Hence solve $\text{cosec}(\theta) - \sin(\theta) = \cos(\theta)$ for $0 \text{ degrees} \leq \theta \leq 360 \text{ degrees}$, θ not equal to 0, 180, 360 degrees. **(4)**

(Total for Question 24 is 8 marks)

25 This question has two parts.

(a) Show that the equation $2\cot^2(\theta) + \operatorname{cosec}(\theta) = 1$ can be written in the form $2\operatorname{cosec}^2(\theta) + \operatorname{cosec}(\theta) - 3 = 0$. **(3)**

(b) Hence solve $2\cot^2(\theta) + \operatorname{cosec}(\theta) = 1$ for 0 degrees $\leq \theta \leq 360$ degrees, giving non-exact answers to 1 decimal place. **(6)**

(Total for Question 25 is 9 marks)

26 This question has two parts.

(a) Prove that $\sin(3\text{theta}) = 3\sin(\theta) - 4\sin^3(\theta)$. **(4)**

(b) Hence, or otherwise, solve $4\sin^3(\theta) - 3\sin(\theta) + 0.5 = 0$ for $0 \text{ degrees} \leq \theta \leq 360$ degrees. **(5)**

(Total for Question 26 is 9 marks)

27 This question concerns the inverse trigonometric functions arcsin and arccos.

(a) State the domain and range of the function $f(x) = \arcsin(x)$. **(2)**

(b) Sketch the graph of $y = \arccos(x)$ for $-1 \leq x \leq 1$, indicating the coordinates of the endpoints of the curve and the point where the curve crosses the y-axis. **(3)**

(c) Solve $\arccos(2x - 1) = \pi/3$, giving x as an exact fraction. **(3)**

(d) Given that $\arcsin(x) = \arccos(x) + \pi/6$, and using the identity $\arcsin(x) + \arccos(x) = \pi/2$, find the exact value of x. **(4)**

(Total for Question 27 is 12 marks)

28 Prove the identity $\sin(\theta)/(1+\cos(\theta)) + (1+\cos(\theta))/\sin(\theta) = 2/\sin(\theta)$, for $\sin(\theta)$ not equal to 0.

(Total for Question 28 is 4 marks)

29 The angle θ is such that $\sin(\theta) = 3/5$ and θ is obtuse. Find the exact value of $\cos(\theta)$ and the exact value of $\tan(\theta)$.

(Total for Question 29 is 4 marks)

30 Consider the equation $2\cos^2(x) + 7\sin(x) - 5 = 0$, for -180 degrees $\leq x \leq 180$ degrees.

(a) Show that the equation $2\cos^2(x) + 7\sin(x) - 5 = 0$ can be written in the form $2\sin^2(x) - 7\sin(x) + 3 = 0$. **(2)**

(b) Hence solve the equation $2\cos^2(x) + 7\sin(x) - 5 = 0$ for -180 degrees $\leq x \leq 180$ degrees, giving a reason why any additional roots of the quadratic in sin(x) must be rejected. **(5)**

(Total for Question 30 is 7 marks)

31 This question has three parts.

(a) Prove that $\cos(A + B)\cos(A - B) = \cos^2(A) - \sin^2(B)$. **(4)**

(b) Using the result in part (a), or otherwise, show that cos(75 degrees) cos(15 degrees) = 1/4. **(3)**

(c) Hence find the exact value of $\cos^2(105 \text{ degrees}) - \sin^2(15 \text{ degrees})$. **(3)**

(Total for Question 31 is 10 marks)

Mark scheme · 1.5 Pure: Trigonometry

Question 1

π/3 cao **B1**

Answer: π/3

Question 2

1/2 cao **B1**

Answer: 1/2

Question 3

1/2 cao **B1**

Answer: 1/2

Question 4

5 cm cao **B1**

Answer: 5 cm

Question 5

(a) rearranges to sin(3theta - 20) = 0.6 **M1**

(a) finds a reference/principal angle, awrt 36.9 degrees **M1**

(a) correct method to generate further values of (3theta - 20) using 180 - x and/or +360k, and converts back by adding 20 and dividing by 3 **M1**

(a) at least 3 of the 6 correct values, awrt 1 d.p. **A1**

(a) all six correct: θ = 19.0, 54.4, 139.0, 174.4, 259.0, 294.4 (awrt), cao, no extra values in range **A1**

(a) Answer: θ = 19.0, 54.4, 139.0, 174.4, 259.0, 294.4 degrees

(b) sets u = x/2 with correct range $0 \leq u \leq 360$ **M1**

(b) finds principal value arccos(-0.85), awrt 148.2 degrees **M1**

(b) correct second value 360 - 148.2 = 211.8 degrees, and doubles both to find x **M1**

(b) x = 296.4 and x = 423.6 (awrt 1 d.p.), cao, no extra values **A1**

(b) Answer: x = 296.4 degrees, 423.6 degrees

Question 6

27 cm^2 cao **B1**

Answer: 27 cm^2

Question 7

uses θ = s/r **M1**

2.5 radians cao **A1**

Answer: 2.5 radians

Question 8

finds the principal value, 60 degrees **M1**

x = 60 and x = 300 degrees, both correct, no extras **A1**

Answer: x = 60 degrees, 300 degrees

Question 9

uses Area = 0.5 * a * b * sin(C) **M1**

awrt 24.1 cm^2 cao **A1**

Answer: 24.1 cm^2 (3 s.f.)

Question 10

rearranges to sin(x) = 0.5 **M1**

x = 30 degrees **A1**

x = 150 degrees, no extra values in range **A1**

Answer: x = 30 degrees, 150 degrees

Question 11

correct statement of the cosine rule, $AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos(ABC)$ **M1**

correct substitution: $AC^2 = 8^2 + 11^2 - 2(8)(11)\cos(95)$ **M1**

AC = awrt 14.2 cm cao **A1**

Answer: AC = 14.2 cm (3 s.f.)

Question 12

finds the reference angle 45 degrees **M1**

x = 135 degrees **A1**

x = 315 degrees, no extra values in range **A1**

Answer: x = 135 degrees, 315 degrees

Question 13

x = 30 degrees cao **B1**

Answer: x = 30 degrees

Question 14

(a) maximum depth = 8.5 m **B1**

(a) sets 30t = 90 (+360k) from sin(30t) = 1 **M1**

(a) t = 3 and t = 15, i.e. 03:00 and 15:00, both correct **A1**

(a) Answer: Maximum depth 8.5 m, at 03:00 and 15:00

(b) substitutes t = 8.5 to give D = 6 + 2.5sin(255) **M1**

(b) D = awrt 3.59 m cao **A1**

(b) Answer: D = 3.59 m (3 s.f.)

(c) forms sin(30t) = 0.4 **M1**

(c) finds principal value arcsin(0.4), awrt 23.6 degrees **M1**

(c) correct method to find all values of 30t within [0, 720] using 23.6+360k and (180-23.6)+360k **M1**

(c) two correct times in the first 12 hours (00:47 and 05:13, ft) **A1**

(c) two correct times in the second 12 hours (12:47 and 17:13, ft) **A1**

(c) all four times correct to the nearest minute cao: 00:47, 05:13, 12:47, 17:13 **A1**

(c) Answer: 00:47, 05:13, 12:47, 17:13

Question 15

(a) writes over a common denominator cos x (1 + sin x) **M1**

(a) numerator $(1 + \sin x)^2 + \cos^2 x = 2 + 2 \sin x$ **M1**

(a) factorises 2(1 + sin x) and cancels, giving 2/cos x = 2 sec x (cso) **A1**

(a) Answer: expression = 2/cos x = 2 sec x

(b) uses the identity: 2 sec x = 3, so cos x = 2/3 **M1**

(b) x = 48.2 **A1**

(b) x = 311.8 **A1**

(b) Answer: x = 48.2, 311.8

Question 16

(a) R cos α = 2 and R sin α = 2 **M1**

(a) R = $2\sqrt{2}$ **A1**

(a) tan α = 2/2 **M1**

(a) α = 45.0 degrees **A1**

(a) Answer: f(x) = $2\sqrt{2}$ sin(x + 45.0)

(b) maximum $2\sqrt{2}$ **B1**

(b) x + 45.0 = 90, so x = 45.0 **B1**

(b) Answer: maximum $2\sqrt{2}$ at x = 45.0

(c) sin(x + 45.0) = 1/($2\sqrt{2}$) **M1**

(c) x + 45.0 = 20.7 or 159.3 (and adds 360 if needed) **M1**

(c) x = 114.3 **A1**

(c) x = 335.7 **A1**

(c) Answer: x = 114.3, 335.7

Question 17

(a) uses the cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ **M1**

(a) $BC^2 = 36 + 169 - 2(6)(13) \cos 45 = 94.691$ **A1**

(a) BC = 9.73 cm **A1**

(a) Answer: BC = 9.73 cm

(b) uses the sine rule or the cosine rule with the calculated side **M1**

(b) angle ABC = 25.8 degrees **A1**

(b) Answer: 25.8 degrees

(c) uses area = (1/2) bc sin A **M1**

(c) 27.6 cm^2 **A1**

(c) Answer: 27.6 cm^2

Question 18

(a) uses s = r θ **M1**

(a) 2pi cm **A1**

(a) Answer: 2pi cm

(b) uses A = (1/2) r^2 θ **M1**

(b) 8pi cm^2 **A1**

(b) Answer: 8pi cm^2

(c) area of segment = area of sector - area of triangle OAB **M1**

(c) area of triangle = (1/2)$(8)^2$ sin(π/4) **M1**

(c) 8pi - $16\sqrt{2}$ cm^2 **A1**

(c) Answer: 8pi - $16\sqrt{2}$ cm^2

(d) perimeter = 2r + arc length **M1**

(d) 22.3 cm **A1**

(d) Answer: 22.3 cm

Question 19

(a) uses cos 5theta is approximately 1 - $(5theta)^2/2$ **M1**

(a) numerator is approximately $(25/2)\theta^2$ and tan 3theta is approximately 3theta **M1**

(a) denominator is approximately $3theta^2$, so the ratio is (25/2)/3 = 25/6 (cso) **A1**

(a) Answer: approximately 25/6

(b) calculates the value with θ = 0.05: 4.11388 **M1**

(b) 4.114 **A1**

(b) percentage error = |4.1667 - 4.1139|/4.1139 x 100 **M1**

(b) 1.3% **A1**

(b) Answer: 4.114, error 1.3%

Question 20

(a) replaces $\cos^2 x$ by 1 - $\sin^2 x$ **M1**

(a) collects terms **M1**

(a) $2s^2 - s = 0$ where s = sin x **A1**

(a) Answer: $2s^2 - s = 0$

(b) solves the quadratic **M1**

(b) sin x = 1/2 or sin x = 0 **A1**

(b) uses inverse sine and symmetry for each value **M1**

(b) 0, 30, 150, 180 degrees **A1**

(b) Answer: 0, 30, 150, 180 degrees

Question 21

(a) R cos α = 6 and R sin α = 7 **M1**

(a) R = $\sqrt{85}$ **A1**

(a) tan α = 7/6 **M1**

(a) α = 49.4 degrees **A1**

(a) Answer: f(x) = $\sqrt{85}$ sin(x + 49.4)

(b) maximum $\sqrt{85}$ **B1**

(b) x + 49.4 = 90, so x = 40.6 **B1**

(b) Answer: maximum $\sqrt{85}$ at x = 40.6

(c) sin(x + 49.4) = 3/$\sqrt{85}$ **M1**

(c) x + 49.4 = 19.0 or 161.0 (and adds 360 if needed) **M1**

(c) x = 111.6 **A1**

(c) x = 329.6 **A1**

(c) Answer: x = 111.6, 329.6

Question 22

(a) writes cosec x - sin x = (1 - $\sin^2 x$)/sin x **M1**

(a) uses 1 - $\sin^2 x = \cos^2 x$ **A1**

(a) $\cos^2 x$/sin x = cos x (cos x/sin x) = cos x cot x (cso) **A1**

(a) Answer: cosec x - sin x = cos x cot x

(b) uses the identity: cos x cot x = 3 cos x, so cos x (cot x - 3) = 0 **M1**

(b) cos x = 0 gives x = 90.0 and 270.0 **A1**

(b) cot x = 3, so tan x = 1/3 **M1**

(b) x = 18.4 and 198.4 **A1**

(b) Answer: x = 18.4, 90.0, 198.4, 270.0

Question 23

(a) uses s = r θ **M1**

(a) 4pi cm **A1**

(a) Answer: 4pi cm

(b) uses A = (1/2) r^2 θ **M1**

(b) 32pi cm^2 **A1**

(b) Answer: 32pi cm^2

(c) area of segment = area of sector - area of triangle OAB **M1**

(c) area of triangle = (1/2)$(16)^2$ sin(π/4) **M1**

(c) 32pi - $64\sqrt{2}$ cm^2 **A1**

(c) Answer: 32pi - $64\sqrt{2}$ cm^2

(d) perimeter = 2r + arc length **M1**

(d) 44.6 cm **A1**

(d) Answer: 44.6 cm

Question 24

(a) writes cosec(θ) as 1/sin(θ) **M1**

(a) combines into a single fraction: $(1 - \sin^2(\theta))/\sin(\theta)$ **M1**

(a) uses $1 - \sin^2(\theta) = \cos^2(\theta)$ **M1**

(a) cso completes to $\cos(\theta)*(\cos(\theta)/\sin(\theta)) = \cos(\theta)\cot(\theta)$ **A1**

(a) Answer: Identity shown (proof)

(b) uses part (a) to form $\cos(\theta)*\cot(\theta) = \cos(\theta)$, i.e. $\cos(\theta)*(\cot(\theta) - 1) = 0$ **M1**

(b) $\cos(\theta) = 0$ leading to $\theta = 90, 270$ **M1**

(b) $\cot(\theta) = 1$, i.e. $\tan(\theta) = 1$, leading to $\theta = 45, 225$ **M1**

(b) all four values correct: $\theta = 45, 90, 225, 270$ degrees cao, no extras **A1**

(b) Answer: θ = 45, 90, 225, 270 degrees

Question 25

(a) uses the identity $\cot^2(\theta) = \text{cosec}^2(\theta) - 1$ **M1**

(a) substitutes to give $2(\text{cosec}^2(\theta) - 1) + \text{cosec}(\theta) = 1$ **M1**

(a) cso correct rearrangement to $2\text{cosec}^2(\theta) + \text{cosec}(\theta) - 3 = 0$ **A1**

(a) Answer: Shown (identity rearrangement)

(b) lets c = cosec(θ) and factorises $2c^2 + c - 3 = 0$ as $(2c+3)(c-1) = 0$ **M1**

(b) cosec(θ) = 1 or cosec(θ) = -3/2 (i.e. sin(θ) = 1 or sin(θ) = -2/3), both stated **A1**

(b) sin(θ) = 1 leads to θ = 90 degrees **M1**

(b) finds reference angle for sin(θ) = -2/3, awrt 41.8 degrees, and correct method for third/fourth quadrant solutions **M1**

(b) θ = 221.8 and 318.2 (awrt), both correct **A1**

(b) all three final solutions correct: θ = 90, 221.8, 318.2 degrees, cao, no extras **A1**

(b) Answer: θ = 90, 221.8, 318.2 degrees

Question 26

(a) writes sin(3theta) = sin(2theta + θ) = sin(2theta)cos(θ) + cos(2theta)sin(θ) **M1**

(a) substitutes $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ and $\cos(2\theta) = 1 - 2\sin^2(\theta)$ **M1**

(a) uses $\cos^2(\theta) = 1 - \sin^2(\theta)$ to write the expression entirely in sin(θ) **M1**

(a) cso completes correctly to $3\sin(\theta) - 4\sin^3(\theta)$ **A1**

(a) Answer: Identity proved

(b) recognises from (a) that $4\sin^3(\theta) - 3\sin(\theta) = -\sin(3\theta)$ **M1**

(b) forms sin(3theta) = 0.5 **M1**

(b) correct method for all solutions of 3theta in [0, 1080] using 30+360k and 150+360k **M1**

(b) at least 3 of the 6 correct values of θ **A1**

(b) all six correct: θ = 10, 50, 130, 170, 250, 290 degrees, cao **A1**

(b) Answer: θ = 10, 50, 130, 170, 250, 290 degrees

Question 27

(a) domain: $-1 \le x \le 1$ **B1**

(a) range: -90 degrees $\le f(x) \le 90$ degrees (or $-\pi/2 \le f(x) \le \pi/2$), oe **B1**

(a) Answer: Domain: -1 ≤ x ≤ 1; Range: -90 ≤ f(x) ≤ 90 degrees (-π/2 to π/2)

(b) smooth decreasing curve shape, no turning points, correct general shape from top-left to bottom-right **B1**

(b) endpoints correctly labelled: (-1, 180 degrees) [or (-1, π)] and (1, 0) **B1**

(b) y-intercept correctly labelled: (0, 90 degrees) [or (0, π/2)] **B1**

(b) Answer: Decreasing curve from (-1, 180 deg) through (0, 90 deg) to (1, 0 deg)

(c) takes cosine of both sides: $2x - 1 = \cos(\pi/3)$ **M1**

(c) $\cos(\pi/3) = 1/2$ (oe) used **A1**

(c) x = 3/4 cao (exact) **A1**

(c) Answer: x = 3/4

(d) uses the identity to write $\arccos(x) = \pi/2 - \arcsin(x)$ **M1**

(d) substitutes into given equation and rearranges to $2\arcsin(x) = 2\text{pi}/3$, i.e. $\arcsin(x) = \pi/3$ **M1**

(d) $x = \sin(\pi/3)$ **A1**

(d) $x = \sqrt{3}/2$ cao (exact form) **A1**

(d) Answer: $x = \sqrt{3}/2$

Question 28

combines the two fractions over a common denominator $\sin(\theta)(1+\cos(\theta))$ **M1**

expands the numerator $\sin^2(\theta) + (1+\cos(\theta))^2$ **M1**

uses $\sin^2(\theta)+\cos^2(\theta)=1$ to simplify the numerator to $2+2\cos(\theta)$ **A1**

cancels the common factor $(1+\cos(\theta))$ to complete the proof, cso **A1**

Answer: Identity proved

Question 29

uses $\sin^2(\theta) + \cos^2(\theta) = 1$ to find $\cos^2(\theta) = 16/25$ **M1**

$\cos(\theta) = -4/5$ (negative, since θ is obtuse) **A1**

uses $\tan(\theta) = \sin(\theta)/\cos(\theta)$ **M1**

$\tan(\theta) = -3/4$ **A1**

Answer: cos(θ) = -4/5; tan(θ) = -3/4

Question 30

(a) substitutes $\cos^2(x) = 1 - \sin^2(x)$ **M1**

(a) cso correct rearrangement (with sign change) to $2\sin^2(x) - 7\sin(x) + 3 = 0$ **A1**

(a) Answer: Shown (rearrangement)

(b) factorises $2\sin^2(x) - 7\sin(x) + 3 = 0$ as $(2\sin(x) - 1)(\sin(x) - 3) = 0$ **M1**

(b) $\sin(x) = 1/2$, with $\sin(x) = 3$ rejected because sin(x) is never greater than 1 **A1**

(b) finds principal value x = 30 degrees **M1**

(b) finds second value x = 150 degrees (using 180 - 30) **A1**

(b) confirms these are the only two solutions in $-180 \le x \le 180$ (no solutions from the negative part of the range, since $\sin(x) = 1/2 > 0$) **B1**

(b) Answer: x = 30 degrees, 150 degrees

Question 31

(a) expands cos(A+B) and cos(A-B) using the compound angle formulae **M1**

(a) multiplies to form the difference of two squares: $\cos^2(A)\cos^2(B) - \sin^2(A)\sin^2(B)$ **M1**

(a) uses $\cos^2(B) = 1 - \sin^2(B)$ and $\sin^2(A) = 1 - \cos^2(A)$ appropriately **M1**

(a) cso simplifies fully to $\cos^2(A) - \sin^2(B)$ **A1**

(a) Answer: Identity proved

(b) identifies A + B = 75 and A - B = 15, giving A = 45, B = 30 **M1**

(b) substitutes into result of (a): $\cos^2(45) - \sin^2(30) = 1/2 - 1/4$ **M1**

(b) cso completes to 1/4 exactly **A1**

(b) Answer: cos75 cos15 = 1/4 (shown)

(c) identifies A = 105, B = 15, giving A + B = 120, A - B = 90 **M1**

(c) evaluates cos(120) = -1/2 and cos(90) = 0 **M1**

(c) $\cos^2(105) - \sin^2(15) = 0$ cao (exact) **A1**

(c) Answer: 0