1.6 Pure: Exponentials and Logarithms: 1-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Solve $2^{3x-1} = 32$.

(Total for Question 1 is 2 marks)

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2 This question tests basic index and logarithm equations.

(a) Solve $2^x = 32$, without using a calculator. **(2)**

(b) Solve $3^{x-1} = 9$, without using a calculator. **(2)**

(c) Express $3\ln(2) - \ln(4)$ as a single logarithm in its simplest exact form. **(2)**

(d) Solve $\log_5(2x + 3) = 2$. **(3)**

(Total for Question 2 is 9 marks)

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3 Solve the following equations, using logarithms where necessary and giving each answer to 3 significant figures.

(a) Solve $5^x = 30$. **(3)**

(b) Solve $3^{2x+1} = 50$. **(4)**

(c) Solve $7^x = 4^{x+1}$. **(4)**

(Total for Question 3 is 11 marks)

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4 Solve $e^{2x} = 15$, giving your answer to 3 significant figures.

(Total for Question 4 is 2 marks)

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5 Using the substitution $y = e^x$, solve the equation $e^{2x} - 4e^x - 5 = 0$, giving your answer as an exact value of x.

(Total for Question 5 is 3 marks)

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6 A scientist believes that two variables x and y are related by the equation $y = A*x^n$, where A and n are constants. Taking logarithms gives $\log_{10}(y) = n*\log_{10}(x) + \log_{10}(A)$. A graph of $\log_{10}(y)$ against $\log_{10}(x)$ is a straight line passing through the points (0, 1.2) and (2, 3.6). Find the value of n and the value of A, giving A to 3 significant figures.

(Total for Question 6 is 4 marks)

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7 Solve the equation $3^{2x} - 10(3^x) + 24 = 0$, giving your answers in terms of logarithms to base 3, and as decimals to 3 significant figures.

(a) Show that the equation can be written as a quadratic in 3^x. **(2)**

(b) Hence solve the equation. **(4)**

(Total for Question 7 is 6 marks)

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8 The value, V pounds, of a machine t years after purchase is modelled by $V = A\,e^{kt}$. Initially the value is 76, and after 4 years it is 25.3333.

(a) Write down the value of A, and show that $k = (1/4)\ln(1/3)$. **(3)**

(b) Find the value of k to 3 significant figures, and interpret its sign. **(3)**

(c) Find the time at which the value is 1/10 times its initial value, giving your answer to 3 significant figures. **(3)**

(Total for Question 8 is 9 marks)

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9 A curve C has equation $y = 2\ln(x - 1) + 3$, for $x > 1$. State the equation of the asymptote of C, and find the exact x-coordinate of the point where C crosses the x-axis.

(Total for Question 9 is 4 marks)

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Mark scheme · 1.6 Pure: Exponentials and Logarithms: 1-hour Pack

Question 1

writes 32 as 2^5 and equates powers, 3x - 1 = 5	**M1**
x = 2 cao	**A1**

Answer: x = 2

Question 2

(a) writes 32 as 2^5	**M1**
(a) x = 5 cao	**A1**

(a) Answer: x = 5

(b) writes 9 as 3^2 and equates exponents x-1 = 2	**M1**
(b) x = 3 cao	**A1**

(b) Answer: x = 3

(c) writes 3ln(2) as ln(8) and combines to ln(8/4)	**M1**
(c) ln(2) cao	**A1**

(c) Answer: ln(2)

(d) writes 2x + 3 = 5^2	**M1**
(d) rearranges to isolate x	**M1**
(d) x = 11 cao	**A1**

(d) Answer: x = 11

Question 3

(a) takes logs (or ln) of both sides: x ln(5) = ln(30)	**M1**
(a) rearranges to x = ln(30)/ln(5)	**M1**
(a) x = awrt 2.11	**A1**

(a) Answer: x = awrt 2.11

(b) takes logs of both sides: (2x+1)ln(3) = ln(50)	**M1**
(b) rearranges to 2x + 1 = ln(50)/ln(3)	**M1**
(b) correctly isolates x	**M1**
(b) x = awrt 1.28	**A1**

(b) Answer: x = awrt 1.28

(c) takes logs of both sides: x ln(7) = (x+1) ln(4)	**M1**
(c) expands the right-hand side: x ln(7) = x ln(4) + ln(4)	**M1**
(c) collects x terms and factorises: x(ln(7) - ln(4)) = ln(4)	**M1**
(c) x = awrt 2.48	**A1**

(c) Answer: x = awrt 2.48

Question 4

takes natural logs of both sides, 2x = ln15	**M1**
awrt 1.35	**A1**

Answer: x = 1.35 (3 s.f.)

Question 5

substitutes to form the quadratic y^2 - 4y - 5 = 0	**M1**
factorises (y - 5)(y + 1) = 0 and rejects y = -1 since $e^x > 0$	**M1**
x = ln5 cao (exact)	**A1**

Answer: x = ln5

Question 6

finds the gradient n = (3.6 - 1.2)/(2 - 0)	**M1**
n = 1.2 cao	**A1**
identifies the y-intercept $\log_{10}(A) = 1.2$ and forms $A = 10^{1.2}$	**M1**
awrt 15.8	**A1**

Answer: n = 1.2, A = 15.8 (3 s.f.)

Question 7

(a) uses $3^{2x} = (3^x)^2$	**M1**
(a) y^2 - 10y + 24 = 0 where $y = 3^x$ (cso)	**A1**

(a) Answer: y^2 - 10y + 24 = 0, $y = 3^x$

(b) solves the quadratic in y by factorising	**M1**
(b) y = 4 or y = 6	**A1**
(b) takes logarithms of each root	**M1**
(b) x = $\log_3(6)$ or x = $\log_3(4)$, i.e. x = 1.26 or 1.63	**A1**

(b) Answer: x = $\log_3(6)$ or x = $\log_3(4)$, i.e. x = 1.26 or 1.63

Question 8

(a) A = 76	**B1**
(a) substitutes t = 4: 76 e^{4k} = 25.3333	**M1**
(a) e^{4k} = 1/3, so k = (1/4) ln(1/3) (cso)	**A1**

(a) Answer: A = 76, k = (1/4) ln(1/3)

(b) k = ln(1/3)/4	**M1**
(b) k = -0.275	**A1**
(b) k < 0 means decay	**B1**

(b) Answer: k = -0.275

(c) forms e^{kt} = 1/10	**M1**
(c) takes natural logs: t = ln(1/10)/k	**M1**
(c) t = 8.38 years	**A1**

(c) Answer: 8.38 years

Question 9

asymptote x = 1	**B1**
sets y = 0 and rearranges to ln(x - 1) = -1.5	**M1**
takes exponentials of both sides, x - 1 = $e^{-1.5}$	**M1**
x = 1 + $e^{-1.5}$ cao (exact)	**A1**

Answer: Asymptote x = 1; curve crosses the x-axis at x = 1 + $e^{-1.5}$

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