1.6 Pure: Exponentials and Logarithms: 2-hour Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 An investment grows according to the model $V = 4000 * (1.06)^t$, where V pounds is the value of the investment after t complete years. Find the value of the investment after 10 years, giving your answer to the nearest pound, and find the number of complete years after which the value of the investment first exceeds £9000.

(Total for Question 1 is 4 marks)

---

2 Solve the following equations involving natural logarithms.

(a) Solve $e^{3x} = 20$, giving your answer to 3 significant figures. **(3)**

(b) Solve $\ln(2x - 1) = 4$, giving your answer to 3 significant figures. **(3)**

(c) Solve $\ln(x) + \ln(x + 3) = \ln(10)$, stating why one of the solutions of the resulting quadratic must be rejected. **(4)**

(Total for Question 2 is 10 marks)

---

3 A radioactive isotope decays according to the model $m = m0 * e^{-\lambda * t}$, where m0 grams is the initial mass, t is the time in years and λ is a positive constant. A sample initially has mass 80 g. After 50 years the mass has reduced to 65 g.

(a) Show that $\lambda = (1/50) * \ln(80/65)$, giving the value of λ to 3 significant figures. **(3)**

(b) Find the half-life of the isotope, giving your answer to 3 significant figures. **(3)**

(c) Find the time taken for the mass to reduce to 10% of its initial value, giving your answer to the nearest year. **(3)**

(Total for Question 3 is 9 marks)

© Revision Library | revisionlibrary.co.uk

4 A microbiologist called Priya is growing a culture of bacteria. The number of bacteria, N, present t hours after the culture is first observed is modelled by $N = 200 * e^{0.15*t}$.

(a) State the number of bacteria present when the culture is first observed. **(1)**

(b) Find the number of bacteria present after 8 hours, giving your answer to the nearest whole number. **(2)**

(c) Find the time taken for the number of bacteria to reach 1000, giving your answer to 3 significant figures. **(3)**

(d) Show that $dN/dt = 30*e^{0.15*t}$, and hence find the rate of increase of the number of bacteria when t = 8, giving your answer to 3 significant figures. **(4)**

(e) Comment on a limitation of this model for large values of t. **(1)**

(Total for Question 4 is 11 marks)

---

5 Solve the simultaneous equations $\log_2(x) + \log_2(y) = 5$ and $\log_2(x) - \log_2(y) = 1$, giving x and y as exact values.

(a) Solve the simultaneous equations, giving x and y as exact values. **(6)**

(Total for Question 5 is 6 marks)

---

6 Rebecca collects data on two variables x and y, which she believes are related by the equation $y = A*x^n$, where A and n are constants. She plots $\log_{10}(y)$ against $\log_{10}(x)$ and obtains a straight line passing through the points (0, 1.20) and (2, 3.60).

(a) Show that taking logarithms of $y = A*x^n$ gives $\log_{10}(y) = n*\log_{10}(x) + \log_{10}(A)$, and explain why this represents a linear relationship between $\log_{10}(y)$ and $\log_{10}(x)$. **(2)**

(b) Find the value of n. **(2)**

(c) Find the value of A, giving your answer to 3 significant figures. **(3)**

(d) Hence write down the equation connecting y and x. **(1)**

(Total for Question 6 is 8 marks)

---

7 Solve the equation $(\log_2(x))^2 - 5(\log_2(x)) + 6 = 0$, giving your answers as exact values of x.

(a) Solve the equation, giving your answers as exact values of x. **(6)**

(Total for Question 7 is 6 marks)

© Revision Library | revisionlibrary.co.uk

8 Jamal owns a rare coin. The value, V pounds, of the coin t years after purchase is modelled by $V = 250*e^{k*t}$, where k is a positive constant. The value of the coin doubles every 12 years.

(a) Show that $k = \ln(2)/12$, giving the value of k to 3 significant figures. **(4)**

(b) Find the value of the coin after 20 years, giving your answer to the nearest pound. **(3)**

(c) Find the number of complete years after purchase before the coin's value first exceeds £1000, justifying your answer carefully. **(4)**

(Total for Question 8 is 11 marks)

---

9 Solve $\log_2(x - 10) + \log_2(x - 11) = 1$.

(a) Show that the equation can be written as a quadratic and find the quadratic. **(3)**

(b) Hence solve the equation, rejecting any invalid root, and give a reason. **(4)**

(Total for Question 9 is 7 marks)

---

10 Solve $\log_3(x + 15) + \log_3(x - 11) = 3$.

(a) Show that the equation can be written as a quadratic and find the quadratic. **(3)**

(b) Hence solve the equation, rejecting any invalid root, and give a reason. **(4)**

(Total for Question 10 is 7 marks)

---

11 The variables x and y are related by $y = ax^n$, where a and n are constants. A graph of log10 y against log10 x is a straight line with gradient 0.6 and vertical intercept 1.2.

(a) Show that $\log10\ y = n \log10\ x + \log10\ a$. **(2)**

(b) State the values of n and log10 a, and hence find a to 3 significant figures. **(3)**

(c) Use the model, with your unrounded value of a, to find y when $x = 6$, to 3 significant figures. **(2)**

(Total for Question 11 is 7 marks)

© Revision Library | revisionlibrary.co.uk

12 Solve the equation $2^{2x} - 6(2^x) + 8 = 0$, giving your answers in terms of logarithms to base 2, and as decimals to 3 significant figures.

(a) Show that the equation can be written as a quadratic in 2^x. **(2)**

(b) Hence solve the equation. **(4)**

(Total for Question 12 is 6 marks)

---

13 This question tests manipulation of logarithms.

(a) Given that $\log_2(x) = p$, express $\log_2(4x^3)$ in terms of p. **(3)**

(b) Solve the equation $\log_3(x + 5) - \log_3(x - 1) = 2$, stating clearly any restriction on the value of x. **(5)**

(Total for Question 13 is 8 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 1.6 Pure: Exponentials and Logarithms: 2-hour Pack

Question 1

substitutes t = 10 into the model **M1**

£7163 cao (nearest pound) **A1**

sets up the inequality $1.06^t > 2.25$ and takes logs to find t > 13.9 (3 s.f.) **M1**

t = 14 cao **A1**

Answer: Value after 10 years is £7163; the value first exceeds £9000 after 14 complete years

Question 2

(a) takes ln of both sides: 3x = ln(20) **M1**

(a) rearranges x = ln(20)/3 **M1**

(a) x = awrt 0.999 **A1**

(a) Answer: x = awrt 0.999

(b) writes $2x - 1 = e^4$ **M1**

(b) rearranges to $x = (e^4 + 1)/2$ **M1**

(b) x = awrt 27.8 **A1**

(b) Answer: x = awrt 27.8

(c) combines logs: ln(x(x+3)) = ln(10) **M1**

(c) forms the quadratic $x^2 + 3x - 10 = 0$ **M1**

(c) factorises to give x = 2 or x = -5 **A1**

(c) rejects x = -5 since ln(x) is undefined for $x \leq 0$; x = 2 cao **B1**

(c) Answer: x = 2

Question 3

(a) forms the equation $65 = 80 * e^{-50*\lambda}$ **M1**

(a) takes ln of both sides and rearranges to $\lambda = (1/50)\ln(80/65)$, cso **M1**

(a) λ = awrt 0.00415 **A1**

(a) Answer: $\lambda = (1/50)\ln(80/65)$ = awrt 0.00415

(b) sets $0.5 = e^{-\lambda*T}$ using the half-life condition **M1**

(b) rearranges to $T = \ln(2)/\lambda$ **M1**

(b) T = awrt 167 (years), using the unrounded value of λ **A1**

(b) Answer: T = awrt 167 years

(c) sets $0.1 = e^{-\lambda*t}$ **M1**

(c) rearranges to $t = \ln(10)/\lambda$ **M1**

(c) t = awrt 554 (years), using the unrounded value of λ **A1**

(c) Answer: t = awrt 554 years

Question 4

(a) N = 200 **B1**

(a) Answer: 200

(b) substitutes t = 8 into the model **M1**

(b) N = awrt 664 **A1**

(b) Answer: N = awrt 664

(c) sets $1000 = 200*e^{0.15*t}$ and simplifies to $5 = e^{0.15*t}$ **M1**

(c) rearranges to $t = \ln(5)/0.15$ **M1**

(c) t = awrt 10.7 (hours) **A1**

(c) Answer: t = awrt 10.7 hours

(d) differentiates N with respect to t using the chain rule **M1**

(d) $dN/dt = 200*0.15*e^{0.15*t} = 30*e^{0.15*t}$, cso **A1**

(d) substitutes t = 8 into dN/dt **M1**

(d) rate = awrt 99.6 bacteria per hour **A1**

(d) Answer: $dN/dt = 30e^{0.15t}$; rate at t=8 is awrt 99.6 per hour

(e) any valid limitation, e.g. the model predicts unlimited growth, but in reality growth would be restricted by limited nutrients, space or competition **B1**

(e) Answer: the model assumes unrestricted exponential growth, which is unrealistic over long periods as resources are finite

Question 5

(a) combines first equation: $\log_2(xy) = 5$, so xy = 32 **M1**

(a) combines second equation: $\log_2(x/y) = 1$, so x/y = 2, i.e. x = 2y **M1**

(a) substitutes x = 2y into xy = 32 to form $2y^2 = 32$ **M1**

(a) y = 4 cao **A1**

(a) x = 8 cao **A1**

(a) rejects y = -4 since $\log_2(y)$ requires y > 0 **B1**

(a) Answer: x = 8, y = 4

Question 6

(a) takes $\log_{10}$ of both sides correctly: $\log_{10}(y) = \log_{10}(A) + n*\log_{10}(x)$ **B1**

(a) identifies this as linear in the form Y = mX + c, with $Y = \log_{10}(y)$, $X = \log_{10}(x)$, gradient n and intercept $\log_{10}(A)$ **B1**

(a) Answer: $\log_{10}(y) = n*\log_{10}(x) + \log_{10}(A)$, which is linear with gradient n and intercept $\log_{10}(A)$

(b) calculates the gradient of the line using the two given points **M1**

(b) n = 1.20 cao **A1**

(b) Answer: n = 1.20

(c) identifies the y-intercept of the line as $\log_{10}(A) = 1.20$ **M1**

(c) rearranges to $A = 10^{1.20}$ **M1**

(c) A = awrt 15.8 **A1**

(c) Answer: A = awrt 15.8

(d) $y = 15.8*x^{1.2}$, ft from parts (b) and (c) **B1ft**

(d) Answer: $y = 15.8x^{1.2}$

© Revision Library | revisionlibrary.co.uk

Question 7

(a) uses the substitution u = $\log_2(x)$ to form u^2 - 5u + 6 = 0 **M1**

(a) factorises as (u-2)(u-3) = 0 **M1**

(a) u = 2 and u = 3 **A1**

(a) converts back to x using $\log_2(x)$ = u **M1**

(a) x = 4 (exact) **A1**

(a) x = 8 (exact) cao **A1**

(a) Answer: x = 4 or x = 8

Question 8

(a) forms the equation 500 = 250*e^{12k} using the doubling condition **M1**

(a) simplifies to 2 = e^{12k} and takes ln of both sides **M1**

(a) k = ln(2)/12 cso, answer printed **A1**

(a) k = awrt 0.0578 **B1**

(a) Answer: k = ln(2)/12 = awrt 0.0578

(b) substitutes t = 20 into V = 250*e^{k*t} using the unrounded value of k **M1**

(b) evaluates e^{20k} correctly **M1**

(b) V = awrt 794 (pounds) **A1**

(b) Answer: V = awrt £794

(c) sets up 1000 = 250*e^{k*t}, simplifying to 4 = e^{k*t} **M1**

(c) solves using k = ln(2)/12 to obtain t*ln(2)/12 = ln(4) = 2ln(2) **M1**

(c) t = 24 (exact), the time at which V equals exactly 1000 **A1**

(c) concludes the value first exceeds £1000 at t = 25 complete years, since at t = 24 the value equals exactly 1000, not more **B1**

(c) Answer: 25 complete years

Question 9

(a) uses log a + log b = log(ab) **M1**

(a) converts to index form: (x - 10)(x - 11) = 2 **M1**

(a) x^2 - 21x + 108 = 0 **A1**

(a) Answer: x^2 - 21x + 108 = 0

(b) solves the quadratic **M1**

(b) x = 12 or x = 9 **A1**

(b) x = 9 makes the argument of a logarithm negative or zero, so it is rejected **B1**

(b) x = 12 **A1**

(b) Answer: x = 12

Question 10

(a) uses log a + log b = log(ab) **M1**

(a) converts to index form: (x + 15)(x - 11) = 3^3 = 27 **M1**

(a) x^2 + 4x - 192 = 0 **A1**

(a) Answer: x^2 + 4x - 192 = 0

(b) solves the quadratic **M1**

(b) x = 12 or x = -16 **A1**

(b) x = -16 makes the argument of a logarithm negative or zero, so it is rejected **B1**

(b) x = 12 **A1**

(b) Answer: x = 12

Question 11

(a) takes logs of both sides of y = ax^n **M1**

(a) log10 y = log10 a + n log10 x (cso) **A1**

(a) Answer: log10 y = n log10 x + log10 a

(b) n = 0.6 **B1**

(b) log10 a = 1.2, so a = $10^{1.2}$ **M1**

(b) a = 15.8 **A1**

(b) Answer: n = 0.6, a = 15.8

(c) y = a x $6^{0.6}$ with the unrounded a = $10^{1.2}$ **M1**

(c) y = 46.4 **A1**

(c) Answer: y = 46.4

Question 12

(a) uses $2^{2x} = (2^x)^2$ **M1**

(a) y^2 - 6y + 8 = 0 where y = 2^x (cso) **A1**

(a) Answer: y^2 - 6y + 8 = 0, y = 2^x

(b) solves the quadratic in y by factorising **M1**

(b) y = 2 or y = 4 **A1**

(b) takes logarithms of each root **M1**

(b) x = $\log_2(4)$ or x = $\log_2(2)$, i.e. x = 1.00 or 2.00 **A1**

(b) Answer: x = $\log_2(4)$ or x = $\log_2(2)$, i.e. x = 1.00 or 2.00

Question 13

(a) splits $\log_2(4x^3)$ as $\log_2(4)$ + $\log_2(x^3)$ **M1**

(a) uses $\log_2(4)$ = 2 and $\log_2(x^3)$ = 3p **M1**

(a) 2 + 3p oe cao **A1**

(a) Answer: 2 + 3p

(b) combines logs: $\log_3((x+5)/(x-1))$ = 2 **M1**

(b) removes the log: (x+5)/(x-1) = 9 **M1**

(b) solves the resulting linear equation **M1**

(b) x = 7/4 oe cao **A1**

(b) states restriction x > 1 (required for both logs to be defined), consistent with x = 7/4 **B1**

(b) Answer: x = 7/4, with restriction x > 1

© Revision Library | revisionlibrary.co.uk