# 1.6 Pure: Exponentials and Logarithms: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

**Answer ALL questions. Show all your working.**

**1** Write $\log_2(64) = 6$ in index form.

**(Total for Question 1 is 1 mark)**

**2** Write down the value of $\log_{10}(1000)$.

**(Total for Question 2 is 1 mark)**

**3** Simplify $\log_2(40) - \log_2(5)$, giving your answer as an integer.

**(Total for Question 3 is 1 mark)**

**4** Solve $2^{x+1} = 16$.

**(Total for Question 4 is 1 mark)**

**5** Find the value of $3\log_2(4) - \log_2(8)$, giving your answer as an integer.

**(Total for Question 5 is 2 marks)**

**6** Solve $\log_3(4x - 1) = 3$.

**(Total for Question 6 is 2 marks)**

**7** A car's value depreciates according to the model $V = 16000 * (0.88)^t$, where V pounds is the value of the car t years after purchase. Find the time taken for the car's value to fall to half of its original value, giving your answer to 3 significant figures.

**(Total for Question 7 is 3 marks)**

**8** Consider the equation $2^{2x} - 9(2^x) + 8 = 0$.

**(a)** Using the substitution $y = 2^x$, show that the equation can be written as $y^2 - 9y + 8 = 0$. **(2)**

**(b)** Hence solve $2^{2x} - 9(2^x) + 8 = 0$, giving your answers as exact values of x. **(5)**

**(Total for Question 8 is 7 marks)**


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**9** Given that $x = \ln(3 + 2*\sqrt{2})$, show that $e^x + e^{-x} = 6$.

**(Total for Question 9 is 5 marks)**

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**10** The population, P thousand, of a colony of birds t years after it is first counted is modelled by $P = A e^{kt}$. Initially the population is 382, and after 6 years it is 254.667.

**(a)** Write down the value of A, and show that $k = (1/6) \ln(2/3)$. **(3)**

**(b)** Find the value of k to 3 significant figures, and interpret its sign. **(3)**

**(c)** Find the time at which the population is 1/5 times its initial value, giving your answer to 3 significant figures. **(3)**

**(Total for Question 10 is 9 marks)**

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**11** Solve the equation $e^{2x} - 19e^x + 90 = 0$, giving your answers as exact values.

**(a)** Show that the equation can be written as a quadratic in $e^x$. **(2)**

**(b)** Hence solve the equation. **(4)**

**(Total for Question 11 is 6 marks)**

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**12** A curve C has equation $y = \ln(x + 2) - 3$, for $x > -2$.

**(a)** State the equation of the asymptote of C. **(1)**

**(b)** Find the coordinates of the point where C crosses the y-axis, giving the y-coordinate to 3 significant figures. **(2)**

**(c)** Find the exact coordinates of the point where C crosses the x-axis. **(3)**

**(d)** Sketch the graph of C, showing clearly the asymptote from part (a) and the axis-crossing points found in parts (b) and (c). **(3)**

**(Total for Question 12 is 9 marks)**


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# Mark scheme · 1.6 Pure: Exponentials and Logarithms: Test Pack

## Question 1

$2^6 = 64$ stated correctly **B1**

**Answer: $2^6 = 64$**

## Question 2

3 cao **B1**

**Answer: 3**

## Question 3

3 cao **B1**

**Answer: 3**

## Question 4

3 cao **B1**

**Answer: x = 3**

## Question 5

uses the power law to write $3\log_2(4) = \log_2(4^3) = \log_2(64)$ **M1**

3 cao **A1**

**Answer: 3**

## Question 6

rewrites in index form, $4x - 1 = 3^3$ **M1**

x = 7 cao **A1**

**Answer: x = 7**

## Question 7

sets up the equation $0.88^t = 0.5$ **M1**

takes logs of both sides, $t \ln(0.88) = \ln(0.5)$ **M1**

awrt 5.42 **A1**

**Answer: t = 5.42 years (3 s.f.)**

## Question 8

(a) recognises $2^{2x} = (2^x)^2 = y^2$ **B1**

(a) substitutes correctly to obtain $y^2 - 9y + 8 = 0$, cso, answer printed **B1**

**(a) Answer: $y^2 - 9y + 8 = 0$ (shown)**

(b) factorises $y^2 - 9y + 8$ as $(y-1)(y-8)$ **M1**

(b) y = 1 and y = 8 **A1**

(b) converts back to x using $2^x = 1$ and $2^x = 8$ **M1**

(b) x = 0 (exact) **A1**

(b) x = 3 (exact) cao **A1**

**(b) Answer: x = 0 or x = 3**

## Question 9

states $e^x = 3 + 2\sqrt{2}$ directly from the definition of x **M1**

writes $e^{-x} = 1/(3 + 2\sqrt{2})$ **M1**

rationalises the denominator by multiplying by $(3 - 2\sqrt{2})/(3 - 2\sqrt{2})$ **M1**

$e^{-x} = 3 - 2\sqrt{2}$ cao, since $(3+2\sqrt{2})(3-2\sqrt{2}) = 9 - 8 = 1$ **A1**

$e^x + e^{-x} = (3+2\sqrt{2}) + (3-2\sqrt{2}) = 6$, cso, answer printed **A1**

**Answer: 6 (shown)**

## Question 10

(a) A = 382 **B1**

(a) substitutes t = 6: $382\,e^{6k} = 254.667$ **M1**

(a) $e^{6k} = 2/3$, so $k = (1/6)\ln(2/3)$ (cso) **A1**

**(a) Answer: A = 382, k = (1/6) ln(2/3)**

(b) $k = \ln(2/3)/6$ **M1**

(b) k = -0.0676 **A1**

(b) k < 0 means decay **B1**

**(b) Answer: k = -0.0676**

(c) forms $e^{kt} = 1/5$ **M1**

(c) takes natural logs: $t = \ln(1/5)/k$ **M1**

(c) t = 23.8 years **A1**

**(c) Answer: 23.8 years**

## Question 11

(a) uses $e^{2x} = (e^x)^2$ **M1**

(a) $y^2 - 19y + 90 = 0$ where $y = e^x$ (cso) **A1**

**(a) Answer: $y^2 - 19y + 90 = 0$, $y = e^x$**

(b) solves the quadratic in y by factorising **M1**

(b) y = 9 or y = 10 **A1**

(b) takes logarithms of each root **M1**

(b) x = ln 9 or x = ln 10 **A1**

**(b) Answer: x = ln 9 or x = ln 10**

## Question 12

(a) x = -2 **B1**

**(a) Answer: x = -2**

(b) substitutes x = 0 into the equation of C **M1**

(b) (0, awrt -2.31) **A1**

**(b) Answer: (0, -2.31)**

(c) sets y = 0: $\ln(x+2) = 3$ **M1**

(c) rearranges to $x + 2 = e^3$ **M1**

(c) $(e^3 - 2, 0)$ exact cao **A1**

**(c) Answer: $(e^3 - 2, 0)$**

(d) correct increasing, concave-down logarithmic shape **B1**

(d) vertical asymptote drawn at x = -2, with curve approaching but not crossing it **B1**

(d) both axis-crossing points correctly plotted, ft from parts (b) and (c) **B1ft**

**(d) Answer: sketch showing asymptote x=-2, y-intercept (0,-2.31), x-intercept ($e^3$-2, 0)**


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