1.7 Pure: Differentiation: 1-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The curve C has equation $y = (x^2 + 1)(x - 3)^3$.

(a) Using the product rule, show that $dy/dx = (x - 3)^2 (5x^2 - 6x + 3)$. **(5)**

(b) Find the gradient of C at the point where $x = 4$. **(2)**

(Total for Question 1 is 7 marks)

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2 $y = e^{3x} \sin(4x)$.

(a) Find dy/dx. **(3)**

(b) Find the exact gradient of the curve at the point where $x = \pi/8$. **(2)**

(Total for Question 2 is 5 marks)

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3 The curve C has equation $y = 2x^3 + 15x^2 + 36x + 3$.

(a) Find dy/dx and d^2y/dx^2. **(3)**

(b) Find the coordinates of the stationary points of C. **(4)**

(c) Determine the nature of each stationary point. **(2)**

(d) Find the range of values of k for which the equation $2x^3 + 15x^2 + 36x + 3 = k$ has three distinct real roots. **(2)**

(Total for Question 3 is 11 marks)

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4 The curve C has equation $y = 4x^2 + 4/x + 1$, $x \neq 0$.

(a) Find dy/dx. **(2)**

(b) Find an equation of the tangent to C at the point where $x = 1$, giving your answer in the form $ax + by + c = 0$ with integer coefficients. **(4)**

(c) Find an equation of the normal to C at this point, and the coordinates of the point where the normal meets the x-axis. **(4)**

(Total for Question 4 is 10 marks)

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5 $f(x) = 4x^2 - 2x$.

(a) Use differentiation from first principles to show that $f'(x) = 8x - 2$. **(4)**

(b) Hence find the gradient of the curve $y = f(x)$ at the point where $x = 4$. **(2)**

(Total for Question 5 is 6 marks)

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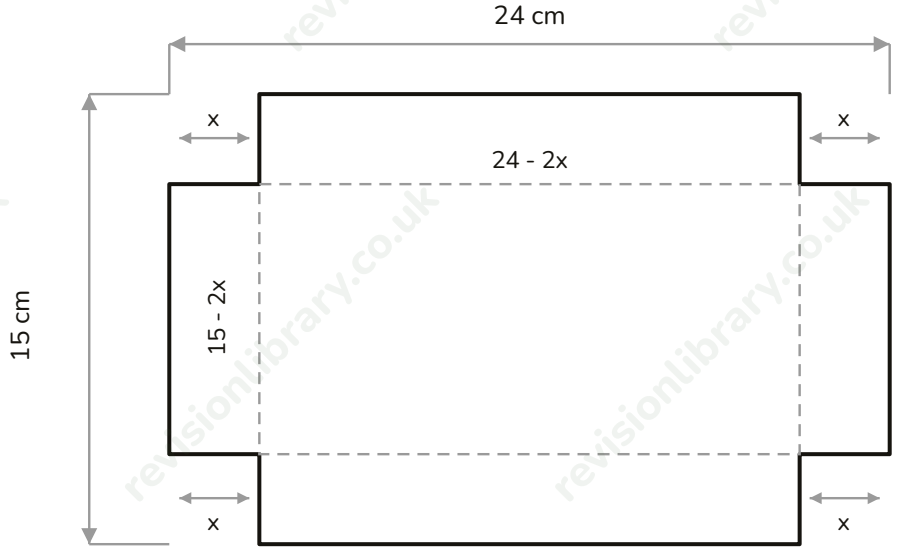
6 Differentiate $y = \ln(5x)$ with respect to x, for $x > 0$.

(Total for Question 6 is 2 marks)

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7 An open-topped box is made from a rectangular sheet of card measuring 24 cm by 15 cm, by cutting a square of side x cm from each corner and folding up the sides, where $0 < x < 7.5$.

Diagram NOT accurately drawn



Dashed rectangle: base of box, folds up along dashed lines

(a) Show that the volume of the box, V cm^3, is given by $V = 4x^3 - 78x^2 + 360x$. **(3)**

(b) Find dV/dx. **(2)**

(c) Using dV/dx, find the value of x that maximises the volume of the box, justifying that it gives a maximum. **(5)**

(d) Hence find the maximum volume of the box. **(2)**

(Total for Question 7 is 12 marks)

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8 A curve has parametric equations $x = t^2 - 3$, $y = t^3 + 2t$. Show that the point (1, 12) lies on the curve, stating the value of t at this point. Find dy/dx in terms of t, and hence find the gradient of the curve at the point (1, 12).

(Total for Question 8 is 4 marks)

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Mark scheme · 1.7 Pure: Differentiation: 1-hour Pack

Question 1

(a) applies product rule u'v + uv' with u = x^2+1, v = $(x-3)^3$ correctly identified **M1**

(a) u' = 2x correct **A1**

(a) v' = $3(x-3)^2$ correct **A1**

(a) factorises out the common factor $(x-3)^2$ from $2x(x-3)^3 + 3(x^2+1)(x-3)^2$ (dependent on M1) **dM1**

(a) cso: simplifies fully to $(x-3)^2(5x^2 - 6x + 3)$ **A1**

(a) Answer: dy/dx = $(x - 3)^2 (5x^2 - 6x + 3)$

(b) substitutes x = 4 into their dy/dx **M1**

(b) cao: gradient = 59 **A1**

(b) Answer: 59

Question 2

(a) uses the product rule **M1**

(a) $3e^{3x}$ sin(4x) **A1**

(a) + $4e^{3x}$ cos(4x) **A1**

(a) Answer: dy/dx = e^{3x} (3 sin(4x) + 4 cos(4x))

(b) substitutes x = π/8, where sin(4x) = 1 and cos(4x) = 0 **M1**

(b) $3e^{3/8pi}$ **A1**

(b) Answer: $3e^{3/8pi}$

Question 3

(a) differentiates, reducing the power of each term by 1 **M1**

(a) dy/dx = $6x^2$ + 30x + 36 **A1**

(a) d^2y/dx^2 = 12x + 30 **A1**

(a) Answer: dy/dx = $6x^2$ + 30x + 36, d^2y/dx^2 = 12x + 30

(b) sets dy/dx = 0 **M1**

(b) x = -3 and x = -2 **A1**

(b) substitutes into the equation of the curve **M1**

(b) (-3, -24) and (-2, -25) **A1**

(b) Answer: (-3, -24) and (-2, -25)

(c) evaluates d^2y/dx^2 at each stationary point **M1**

(c) (-3, -24) is a maximum (d^2y/dx^2 = -6 < 0) and (-2, -25) is a minimum (d^2y/dx^2 = 6 > 0) **A1**

(c) Answer: maximum at x = -3, minimum at x = -2

(d) the horizontal line y = k must cut the curve three times, so lies between the minimum and maximum values **M1**

(d) -25 < k < -24 **A1**

(d) Answer: -25 < k < -24

Question 4

(a) differentiates, writing B/x as Bx^{-1} **M1**

(a) dy/dx = 8x - $4/x^2$ **A1**

(a) Answer: dy/dx = 8x - $4/x^2$

(b) y = 9 when x = 1 **B1**

(b) gradient = 4 **M1**

(b) uses y - y1 = m(x - x1) **M1**

(b) 4x - y + 5 = 0 **A1**

(b) Answer: 4x - y + 5 = 0

(c) normal gradient = -1/(4) **M1**

(c) x + 4y - 37 = 0 **A1**

(c) sets y = 0 **M1**

(c) (37, 0) **A1**

(c) Answer: x + 4y - 37 = 0; (37, 0)

Question 5

(a) forms [f(x + h) - f(x)]/h **M1**

(a) expands f(x + h) = $4(x^2 + 2xh + h^2)$ - 2(x + h) **M1**

(a) simplifies the quotient to 8x + 4h - 2 **dM1**

(a) lets h tend to 0: f'(x) = 8x - 2 (cso) **A1**

(a) Answer: f'(x) = 8x - 2

(b) substitutes x = 4 into f'(x) **M1**

(b) 30 **A1**

(b) Answer: 30

Question 6

uses the chain rule, dy/dx = 5/(5x) **M1**

1/x cao **A1**

Answer: dy/dx = 1/x

Question 7

(a) expands (24-2x)(15-2x) = 360 - 78x + $4x^2$ (oe) **M1**

(a) multiplies by x: V = x(24-2x)(15-2x) **M1**

(a) cso: V = $4x^3$ - $78x^2$ + 360x (answer printed, full working required) **A1**

(a) Answer: V = $4x^3$ - $78x^2$ + 360x (shown)

(b) attempts to differentiate term by term **M1**

(b) cao: dV/dx = $12x^2$ - 156x + 360 **A1**

(b) Answer: dV/dx = $12x^2$ - 156x + 360

(c) sets dV/dx = 0 **M1**

(c) simplifies to x^2 - 13x + 30 = 0 (divides by 12, or equivalent) **M1**

(c) solves to x = 3 or x = 10 (dependent on previous M1) **dM1**

(c) rejects x = 10 since 0 < x < 7.5, selects x = 3 **A1**

(c) justifies maximum: d^2V/dx^2 = 24x - 156, which at x=3 is -84 < 0, hence a maximum **A1**

(c) Answer: x = 3

(d) substitutes x=3 into V(x) or into x(24-2x)(15-2x) **M1**

(d) cao: V = 486 cm^3 **A1**

(d) Answer: 486 cm^3

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Question 8

substitutes t=2 into both parametric equations to verify x=1 and y=12	**M1**
t = 2 (rejecting t = -2, which gives y = -12, not 12)	**B1**
finds dx/dt = 2t and dy/dt = $3t^2 + 2$ and forms dy/dx = $(3t^2+2)/(2t)$	**M1**
3.5 cao, oe 7/2	**A1**

Answer: dy/dx = $(3t^2+2)/(2t)$; gradient at (1,12) is 3.5

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