1.7 Pure: Differentiation: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question concerns differentiation from first principles.

(a) Using differentiation from first principles, show that if $f(x) = x^2$ then $f'(x) = 2x$. **(4)**

(b) Hence, or otherwise, find the gradient of the curve $y = x^2 - 5x + 3$ at the point where $x = 4$. **(2)**

(Total for Question 1 is 6 marks)

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2 The curve C has equation $y = (2x - 1)/(x^2 + 3)$.

(a) Using the quotient rule, find dy/dx, giving your answer as a single fraction in simplest form. **(5)**

(b) Hence find the x-coordinates of the stationary points of C, giving your answers to 3 significant figures. **(3)**

(Total for Question 2 is 8 marks)

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3 $y = x^4 \ln(4x)$, $x > 0$.

(a) Find dy/dx. **(3)**

(b) Show that the curve has a single stationary point and find its exact coordinates. **(4)**

(Total for Question 3 is 7 marks)

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4 The curve C has equation $y = 2x^3 - 9x^2 - 8$.

(a) Find dy/dx and d^2y/dx^2. **(3)**

(b) Find the coordinates of the stationary points of C. **(4)**

(c) Determine the nature of each stationary point. **(2)**

(d) Find the range of values of k for which the equation $2x^3 - 9x^2 - 8 = k$ has three distinct real roots. **(2)**

(Total for Question 4 is 11 marks)

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5 Find the gradient of the curve $y = x^3 - 4x + 1$ at the point where $x = 2$.

(Total for Question 5 is 1 mark)

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6 Differentiate $y = e^{3x}$ with respect to x.

(Total for Question 6 is 1 mark)

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7 Find d^2y/dx^2 for $y = x^4 - 6x^2 + 3x$.

(Total for Question 7 is 2 marks)

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8 Using differentiation from first principles, show that if $f(x) = 3x^2$, then $f'(x) = 6x$. Hence find the gradient of the curve $y = 3x^2 - 5x + 2$ at the point where $x = -1$.

(Total for Question 8 is 3 marks)

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9 The curve C has equation $y = \sqrt{3x + 4}$, for $x > -4/3$. Find the equation of the normal to C at the point where $x = 4$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(Total for Question 9 is 3 marks)

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10 Find the gradient of the curve $y = (3x - 1)^3$ at the point where $x = 1$.

(Total for Question 10 is 2 marks)

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11 A closed cylindrical can has radius r cm and height h cm, and a fixed volume of 250 cm^3. Show that the surface area S cm^2 of the can is given by $S = 2*\pi*r^2 + 500/r$. Find the value of r that minimises S, giving your answer to 3 significant figures, and find the minimum surface area, giving your answer to the nearest whole number.

(Total for Question 11 is 4 marks)

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Mark scheme · 1.7 Pure: Differentiation: Test Pack

Question 1

(a) attempts $f(x+h) = (x+h)^2$ (oe, may be unsimplified) **M1**

(a) forms $[f(x+h) - f(x)]/h$ and expands numerator to $2xh + h^2$ (oe) **M1**

(a) simplifies the quotient to $2x + h$ (dependent on previous M1) **dM1**

(a) cso: states as $h \to 0$, $f'(x) = 2x$ (answer printed; limit statement required) **A1**

(a) Answer: f'(x) = 2x

(b) $dy/dx = 2x - 5$ **M1**

(b) gradient = 3 (cao) **A1**

(b) Answer: 3

Question 2

(a) correct quotient rule statement with $u = 2x-1$, $u' = 2$, $v = x^2+3$, $v' = 2x$ identified **M1**

(a) numerator (unsimplified): $2(x^2+3) - (2x-1)(2x)$ **A1**

(a) expands and simplifies the numerator (dependent on M1) **dM1**

(a) numerator simplifies to $-2x^2 + 2x + 6$ (oe $-2(x^2-x-3)$) **A1**

(a) cao: $dy/dx = (-2x^2+2x+6)/(x^2+3)^2$ (oe $-2(x^2-x-3)/(x^2+3)^2$) **A1**

(a) Answer: $dy/dx = -2(x^2 - x - 3)/(x^2 + 3)^2$

(b) sets numerator of dy/dx equal to zero and simplifies to a 3-term quadratic, e.g. $x^2 - x - 3 = 0$ **M1**

(b) exact solutions $x = (1+\sqrt{13})/2$ and $x = (1-\sqrt{13})/2$ (oe, via quadratic formula) **A1**

(b) awrt $x = 2.30$ and $x = -1.30$ (both required, ft) **A1**

(b) Answer: $x = 2.30$ or $x = -1.30$ (awrt, exact: $x = (1 \pm \sqrt{13})/2$)

Question 3

(a) uses the product rule with $u = x^4$ and $v = \ln(4x)$ **M1**

(a) $du/dx = 4x^3$ and $dv/dx = 1/x$ **A1**

(a) $dy/dx = x^3(4 \ln(4x) + 1)$ **A1**

(a) Answer: $dy/dx = x^3(4 \ln(4x) + 1)$

(b) sets $dy/dx = 0$, with $x^3 > 0$ so $4 \ln(4x) + 1 = 0$ **M1**

(b) $\ln(4x) = -1/4$ **A1**

(b) $x = e^{-1/4}/4$ **M1**

(b) $y = x^4 (-1/4) = -1/(1024e)$ **A1**

(b) Answer: $x = e^{-1/4}/4$, $y = -1/(1024e)$

Question 4

(a) differentiates, reducing the power of each term by 1 **M1**

(a) $dy/dx = 6x^2 - 18x$ **A1**

(a) $d^2y/dx^2 = 12x - 18$ **A1**

(a) Answer: $dy/dx = 6x^2 - 18x$, $d^2y/dx^2 = 12x - 18$

(b) sets $dy/dx = 0$ **M1**

(b) $x = 0$ and $x = 3$ **A1**

(b) substitutes into the equation of the curve **M1**

(b) (0, -8) and (3, -35) **A1**

(b) Answer: (0, -8) and (3, -35)

(c) evaluates d^2y/dx^2 at each stationary point **M1**

(c) (0, -8) is a maximum ($d^2y/dx^2 = -18 < 0$) and (3, -35) is a minimum ($d^2y/dx^2 = 18 > 0$) **A1**

(c) Answer: maximum at x = 0, minimum at x = 3

(d) the horizontal line $y = k$ must cut the curve three times, so lies between the minimum and maximum values **M1**

(d) $-35 < k < -8$ **A1**

(d) Answer: $-35 < k < -8$

Question 5

8 cao **B1**

Answer: 8

Question 6

$3e^{3x}$ cao **B1**

Answer: $dy/dx = 3e^{3x}$

Question 7

finds $dy/dx = 4x^3 - 12x + 3$ **M1**

$12x^2 - 12$ cao **A1**

Answer: $d^2y/dx^2 = 12x^2 - 12$

Question 8

forms $f(x+h) = 3(x+h)^2$ and expands, then finds $f(x+h) - f(x) = 6xh + 3h^2$ **M1**

divides by h to get $6x + 3h$ and lets $h \to 0$ to show $f'(x) = 6x$ **M1**

-11 cao **A1**

Answer: f'(x) = 6x (shown); gradient at x = -1 is -11

Question 9

differentiates using the chain rule, $dy/dx = 3/(2*\sqrt{3x+4})$ **M1**

evaluates the gradient at x=4 as 3/8 and takes the negative reciprocal, -8/3, for the normal **M1**

$8x + 3y - 44 = 0$ cao, oe **A1**

Answer: 8x + 3y - 44 = 0

Question 10

uses the chain rule to find $dy/dx = 9(3x-1)^2$ **M1**

36 cao **A1**

Answer: 36

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Question 11

differentiates $S = 2*\pi*r^2 + 500/r$ to get $dS/dr = 4*\pi*r - 500/r^2$	**M1**
sets $dS/dr = 0$ and rearranges to $r^3 = 125/\pi$	**M1**
awrt 3.41	**A1**
awrt 220	**A1**

Answer: r = 3.41 cm (3 s.f.); minimum S = 220 cm^2 (nearest whole number)

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