1.7 Pure: Differentiation

EDEXCEL 9MA0 · Calculator allowed · about 235 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $y = 3x^4 - 2/x^3 + 4\sqrt{x} - 7$, for $x > 0$.

(a) Find dy/dx, writing each term with a positive or negative index or a root as appropriate. **(4)**

(b) Hence find the exact value of dy/dx when $x = 4$, giving your answer as a single fraction in its simplest form. **(3)**

(Total for Question 1 is 7 marks)

2 The curve C has equation $y = (3x^2 - 4x + 1)^5$.

(a) Using the chain rule, find dy/dx. **(3)**

(b) Find the exact value of the gradient of C at the point where x = 2. **(2)**

(c) Find the equation of the tangent to C at the point where x = 2, giving your answer in the form y = mx + c. **(3)**

(Total for Question 2 is 8 marks)

3 The curve C has equation $y = (x^2 + 1)(x - 3)^3$.

(a) Using the product rule, show that $dy/dx = (x - 3)^2 (5x^2 - 6x + 3)$. **(5)**

(b) Find the gradient of C at the point where $x = 4$. **(2)**

(Total for Question 3 is 7 marks)

4 Differentiate $y = x^5$ with respect to x.

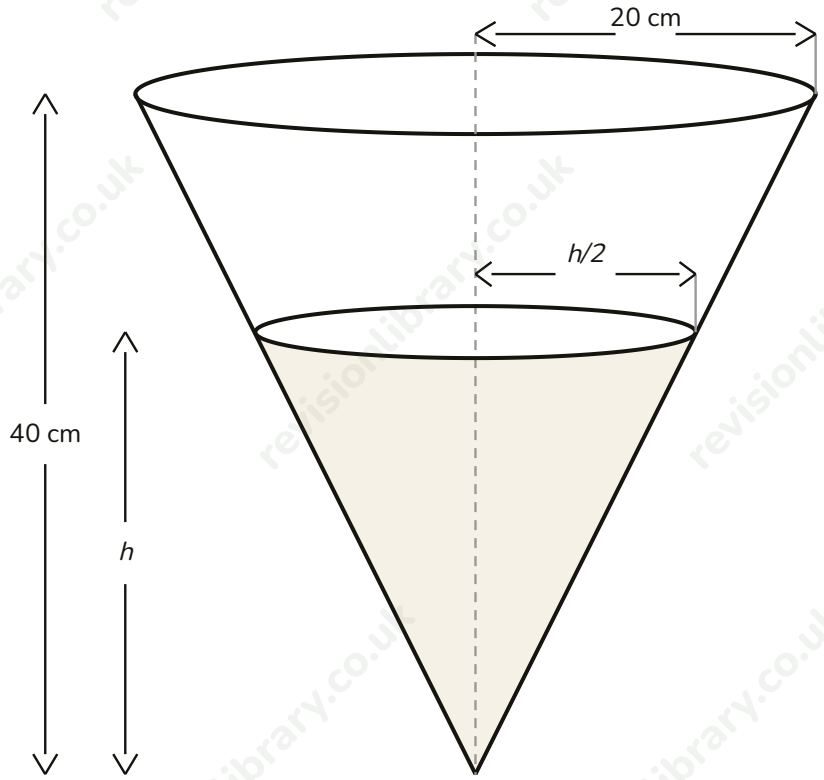
(Total for Question 4 is 1 mark)

---

5 Differentiate $y = 7x^3 - 2x$ with respect to x.

(Total for Question 5 is 1 mark)

6 Water is poured into an inverted conical container at a constant rate of 30 cm^3/s. The container has height 40 cm and radius 20 cm at the top. By similar triangles, when the depth of water is h cm the radius of the water surface is h/2 cm, so the volume of water is $V = (\pi h^3)/12\ cm^3$.



(a) Show that $dV/dh = (\pi h^2)/4$. **(2)**

(b) Given that $dV/dt = 30$, find an expression for dh/dt in terms of h. **(3)**

(c) Find the rate at which the depth of the water is increasing at the instant when h = 10 cm, giving your answer to 3 significant figures. **(2)**

(d) State one assumption made in modelling this situation that could affect the accuracy of your answer to part (c). **(1)**

(Total for Question 6 is 8 marks)

---

7 $y = e^{3x} \sin(4x)$.

(a) Find dy/dx. **(3)**

(b) Find the exact gradient of the curve at the point where $x = \pi/8$. **(2)**

(Total for Question 7 is 5 marks)

8 The curve C has equation $y = 2x^3 + 15x^2 + 36x + 3$.

(a) Find dy/dx and d^2y/dx^2. **(3)**

(b) Find the coordinates of the stationary points of C. **(4)**

(c) Determine the nature of each stationary point. **(2)**

(d) Find the range of values of k for which the equation $2x^3 + 15x^2 + 36x + 3 = k$ has three distinct real roots. **(2)**

(Total for Question 8 is 11 marks)

9 $f(x) = x^2 - 2x$.

(a) Use differentiation from first principles to show that $f'(x) = 2x - 2$. **(4)**

(b) Hence find the gradient of the curve $y = f(x)$ at the point where $x = 2$. **(2)**

(Total for Question 9 is 6 marks)

10 The curve C has equation $y = 4x^2 + 4/x + 1$, $x \neq 0$.

(a) Find dy/dx. **(2)**

(b) Find an equation of the tangent to C at the point where $x = 1$, giving your answer in the form $ax + by + c = 0$ with integer coefficients. **(4)**

(c) Find an equation of the normal to C at this point, and the coordinates of the point where the normal meets the x-axis. **(4)**

(Total for Question 10 is 10 marks)

11 $f(x) = 4x^2 - 2x$.

(a) Use differentiation from first principles to show that $f'(x) = 8x - 2$. **(4)**

(b) Hence find the gradient of the curve $y = f(x)$ at the point where $x = 4$. **(2)**

(Total for Question 11 is 6 marks)

12 The curve C has equation $y = -x^2 + 3/x - 6$, $x \neq 0$.

(a) Find dy/dx. **(2)**

(b) Find an equation of the tangent to C at the point where $x = -2$, giving your answer in the form $ax + by + c = 0$ with integer coefficients. **(4)**

(c) Find an equation of the normal to C at this point, and the coordinates of the point where the normal meets the x-axis. **(4)**

(Total for Question 12 is 10 marks)

13 The curve C has equation $x^2 + 2xy + 2y^2 = 5$. The point P(-1, 2) lies on C.

(a) Show that $dy/dx = -(2x + 2y)/(2x + 4y)$. **(5)**

(b) Find the gradient of C at P. **(2)**

(c) Find an equation of the normal to C at P, in the form $ax + by + c = 0$ with integer coefficients. **(3)**

(Total for Question 13 is 10 marks)

14 $y = 2\sin(3x) - \cos^2(x)$, where x is measured in radians.

(a) Find dy/dx. **(4)**

(b) Find the exact value of dy/dx when $x = \pi/6$. **(3)**

(c) Using the double angle identity $\sin(2x) = 2\sin(x)\cos(x)$, show that $dy/dx = 6\cos(3x) + \sin(2x)$. **(2)**

(Total for Question 14 is 9 marks)

15 $y = \ln(3x^2 + 2) + e^{1 - 2x}$.

(a) Find dy/dx. **(4)**

(b) Find the exact value of dy/dx when $x = 0$. **(3)**

(c) Find the equation of the tangent to the curve at the point where $x = 0$, giving your answer in the form $y = mx + c$, where c is given in terms of e and ln(2). **(3)**

(Total for Question 15 is 10 marks)

16 The curve C has equation $y = 2x^3 - 9x^2 + 12x + 5$.

(a) Find dy/dx and d^2y/dx^2. **(3)**

(b) Find the coordinates of the stationary points of C. **(4)**

(c) Using the second derivative, determine the nature of each stationary point. **(2)**

(Total for Question 16 is 9 marks)

17 The curve C has equation $y = \sqrt{2x + 5}$, for $x > -2.5$.

(a) Find dy/dx. **(3)**

(b) Find the equation of the tangent to C at the point where $x = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

(c) Find the equation of the normal to C at the point where $x = 2$, giving your answer in the form $y = mx + c$. **(4)**

(Total for Question 17 is 11 marks)

18 Differentiate $y = \ln(5x)$ with respect to x, for $x > 0$.

(Total for Question 18 is 2 marks)

19 Find dy/dx for $y = 4x^{1/2}$.

(Total for Question 19 is 1 mark)

20 Find dy/dx for $y = 6x^{-2}$.

(Total for Question 20 is 1 mark)

21 Differentiate $y = \sin(4x)$ with respect to x.

(Total for Question 21 is 1 mark)

22 Using the chain rule, find dy/dx for $y = (2x + 3)^4$.

(Total for Question 22 is 2 marks)

23 A curve has parametric equations $x = t^2 + 1$, $y = t^3 - 3t$.

(a) Find dy/dx in terms of t, giving your answer as a single fraction in simplest factorised form. **(4)**

(b) Show that the point (5, 2) lies on the curve, and find the value of t at this point. **(2)**

(c) Find the gradient of the curve at the point (5, 2). **(2)**

(d) Find the equation of the tangent to the curve at the point (5, 2), giving your answer in the form $y = mx + c$. **(3)**

(Total for Question 23 is 11 marks)

24 The curve C is defined by the equation $x^2 + 3xy - y^2 = 9$. The point P(2, 1) lies on C.

(a) Show that P(2, 1) lies on C. **(1)**

(b) Find dy/dx in terms of x and y. **(5)**

(c) Find the gradient of C at the point P(2, 1). **(2)**

(d) Find the equation of the normal to C at P(2, 1), giving your answer in the form ax + by + c = 0, where a, b and c are integers. **(3)**

(Total for Question 24 is 11 marks)

25 Find dy/dx for $y = x^2 * e^x$, using the product rule, giving your answer in a fully factorised form.

(Total for Question 25 is 2 marks)

26 Find dy/dx for $y = (4x - 1)/(x + 2)$, using the quotient rule, giving your answer as a single fraction in simplest form.

(Total for Question 26 is 2 marks)

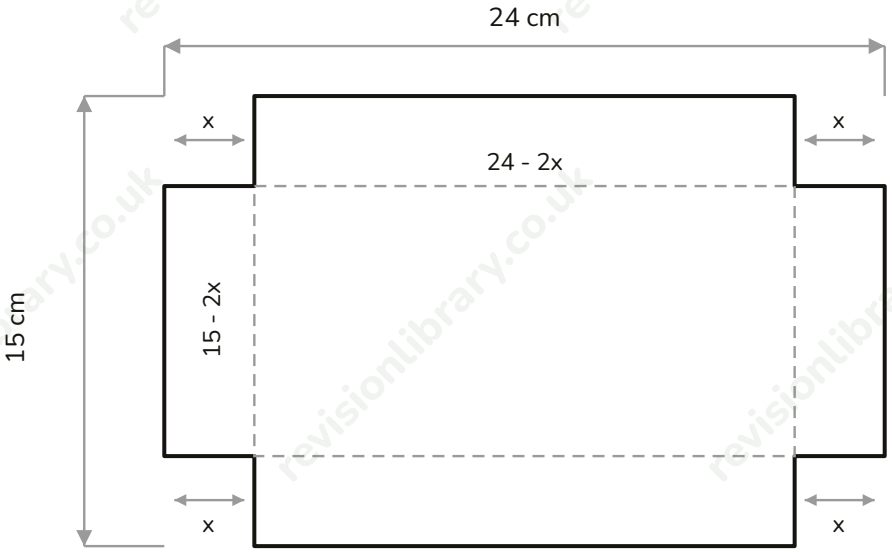
---

27 The curve C has equation $y = x^3 - 3x^2 - 9x + 4$. Find the x-coordinates of the stationary points of C, and use the second derivative to determine the nature of each.

(Total for Question 27 is 3 marks)

28 An open-topped box is made from a rectangular sheet of card measuring 24 cm by 15 cm, by cutting a square of side x cm from each corner and folding up the sides, where $0 < x < 7.5$.

Diagram NOT accurately drawn



Dashed rectangle: base of box, folds up along dashed lines

(a) Show that the volume of the box, V cm^3, is given by $V = 4x^3 - 78x^2 + 360x$. **(3)**

(b) Find dV/dx. **(2)**

(c) Using dV/dx, find the value of x that maximises the volume of the box, justifying that it gives a maximum. **(5)**

(d) Hence find the maximum volume of the box. **(2)**

(Total for Question 28 is 12 marks)

29 A spherical balloon is being inflated so that its volume increases at a constant rate of 200 cm^3/s. Show that $dr/dt = 50/(\pi * r^2)$, where r cm is the radius of the balloon at time t seconds. Find the rate of increase of the radius at the instant when r = 10 cm, giving your answer to 3 significant figures.

(Total for Question 29 is 4 marks)

---

30 A curve has parametric equations $x = t^2 - 3$, $y = t^3 + 2t$. Show that the point (1, 12) lies on the curve, stating the value of t at this point. Find dy/dx in terms of t, and hence find the gradient of the curve at the point (1, 12).

(Total for Question 30 is 4 marks)

Mark scheme · 1.7 Pure: Differentiation

Question 1

(a) attempts to differentiate at least one term of the form ax^n using $ax^n \rightarrow anx^{n-1}$ **M1**

(a) $3x^4 \rightarrow 12x^3$ correct **A1**

(a) $-2x^{-3} \rightarrow 6x^{-4}$ (oe $6/x^4$) correct **A1**

(a) $4x^{1/2} \rightarrow 2x^{-1/2}$ (oe $2/\sqrt{x}$) correct, with $-7 \rightarrow 0$ **A1**

(a) Answer: $dy/dx = 12x^3 + 6x^{-4} + 2x^{-1/2}$ (oe $12x^3 + 6/x^4 + 2/\sqrt{x}$)

(b) substitutes $x = 4$ into their dy/dx **M1**

(b) evaluates all three terms correctly: 768, 3/128, 1 (oe, dependent on M1) **dM1**

(b) cao: $dy/dx = 98435/128$ **A1**

(b) Answer: 98435/128

Question 2

(a) attempts chain rule of the form $n(3x^2 - 4x + 1)^{n-1} \times$ (derivative of bracket) **M1**

(a) $5(3x^2 - 4x + 1)^4$ correct outer part **A1**

(a) $(6x - 4)$ correct derivative of the bracket, combined cao **A1**

(a) Answer: $dy/dx = 5(3x^2 - 4x + 1)^4 (6x - 4)$

(b) substitutes $x = 2$ into their dy/dx: $5(5)^4(8)$ **M1**

(b) cao: gradient = 25000 **A1**

(b) Answer: 25000

(c) finds y-coordinate at $x = 2$: $y = 5^5 = 3125$ **M1**

(c) uses $y - 3125 = 25000(x - 2)$ (ft their gradient) **M1**

(c) cao: $y = 25000x - 46875$ **A1**

(c) Answer: y = 25000x - 46875

Question 3

(a) applies product rule u'v + uv' with $u = x^2+1$, $v = (x-3)^3$ correctly identified **M1**

(a) $u' = 2x$ correct **A1**

(a) $v' = 3(x-3)^2$ correct **A1**

(a) factorises out the common factor $(x-3)^2$ from $2x(x-3)^3 + 3(x^2+1)(x-3)^2$ (dependent on M1) **dM1**

(a) cso: simplifies fully to $(x-3)^2(5x^2 - 6x + 3)$ **A1**

(a) Answer: $dy/dx = (x - 3)^2 (5x^2 - 6x + 3)$

(b) substitutes $x = 4$ into their dy/dx **M1**

(b) cao: gradient = 59 **A1**

(b) Answer: 59

Question 4

$5x^4$ cao **B1**

Answer: $dy/dx = 5x^4$

Question 5

$21x^2 - 2$ cao **B1**

Answer: $dy/dx = 21x^2 - 2$

Question 6

(a) differentiates $V = (\pi h^3)/12$ to get $(\pi/12)(3h^2)$ **M1**

(a) cso: $dV/dh = (\pi h^2)/4$ **A1**

(a) Answer: $dV/dh = (\pi h^2)/4$ (shown)

(b) states the chain rule $dh/dt = (dV/dt) / (dV/dh)$ **M1**

(b) substitutes to get $dh/dt = 30 / ((\pi h^2)/4)$ **M1**

(b) cao: $dh/dt = 120/(\pi h^2)$ **A1**

(b) Answer: $dh/dt = 120/(\pi h^2)$

(c) substitutes $h = 10$ into their dh/dt **M1**

(c) awrt 0.382 cm/s **A1**

(c) Answer: 0.382 cm/s (awrt)

(d) valid modelling assumption, e.g. the container is a perfect cone with no leaks/deformation, or water is poured in at a genuinely constant rate, or no water is lost to splashing/evaporation **B1**

(d) Answer: e.g. the cone is assumed to be a perfect rigid cone with no leaks and the inflow rate is exactly constant

Question 7

(a) uses the product rule **M1**

(a) $3e^{3x} \sin(4x)$ **A1**

(a) $+ 4e^{3x} \cos(4x)$ **A1**

(a) Answer: $dy/dx = e^{3x} (3 \sin(4x) + 4 \cos(4x))$

(b) substitutes $x = \pi/8$, where $\sin(4x) = 1$ and $\cos(4x) = 0$ **M1**

(b) $3e^{3/8pi}$ **A1**

(b) Answer: $3e^{3/8pi}$

Question 8

(a) differentiates, reducing the power of each term by 1 **M1**

(a) $dy/dx = 6x^2 + 30x + 36$ **A1**

(a) $d^2y/dx^2 = 12x + 30$ **A1**

(a) Answer: $dy/dx = 6x^2 + 30x + 36$, $d^2y/dx^2 = 12x + 30$

(b) sets $dy/dx = 0$ **M1**

(b) $x = -3$ and $x = -2$ **A1**

(b) substitutes into the equation of the curve **M1**

(b) (-3, -24) and (-2, -25) **A1**

(b) Answer: (-3, -24) and (-2, -25)

(c) evaluates d^2y/dx^2 at each stationary point **M1**

(c) (-3, -24) is a maximum ($d^2y/dx^2 = -6 < 0$) and (-2, -25) is a minimum ($d^2y/dx^2 = 6 > 0$) **A1**

(c) Answer: maximum at x = -3, minimum at x = -2

(d) the horizontal line $y = k$ must cut the curve three times, so lies between the minimum and maximum values **M1**

(d) $-25 < k < -24$ **A1**

(d) Answer: $-25 < k < -24$

Question 9

(a) forms [f(x + h) - f(x)]/h **M1**

(a) expands $f(x + h) = 1(x^2 + 2xh + h^2) - 2(x + h)$ **M1**

(a) simplifies the quotient to 2x + h - 2 **dM1**

(a) lets h tend to 0: f'(x) = 2x - 2 (cso) **A1**

(a) Answer: f'(x) = 2x - 2

(b) substitutes x = 2 into f'(x) **M1**

(b) 2 **A1**

(b) Answer: 2

Question 10

(a) differentiates, writing B/x as Bx^{-1} **M1**

(a) $dy/dx = 8x - 4/x^2$ **A1**

(a) Answer: $dy/dx = 8x - 4/x^2$

(b) y = 9 when x = 1 **B1**

(b) gradient = 4 **M1**

(b) uses y - y1 = m(x - x1) **M1**

(b) 4x - y + 5 = 0 **A1**

(b) Answer: 4x - y + 5 = 0

(c) normal gradient = -1/(4) **M1**

(c) x + 4y - 37 = 0 **A1**

(c) sets y = 0 **M1**

(c) (37, 0) **A1**

(c) Answer: x + 4y - 37 = 0; (37, 0)

Question 11

(a) forms [f(x + h) - f(x)]/h **M1**

(a) expands $f(x + h) = 4(x^2 + 2xh + h^2) - 2(x + h)$ **M1**

(a) simplifies the quotient to 8x + 4h - 2 **dM1**

(a) lets h tend to 0: f'(x) = 8x - 2 (cso) **A1**

(a) Answer: f'(x) = 8x - 2

(b) substitutes x = 4 into f'(x) **M1**

(b) 30 **A1**

(b) Answer: 30

Question 12

(a) differentiates, writing B/x as Bx^{-1} **M1**

(a) $dy/dx = -2x - 3/x^2$ **A1**

(a) Answer: $dy/dx = -2x - 3/x^2$

(b) y = -23/2 when x = -2 **B1**

(b) gradient = 13/4 **M1**

(b) uses y - y1 = m(x - x1) **M1**

(b) 13x - 4y - 20 = 0 **A1**

(b) Answer: 13x - 4y - 20 = 0

(c) normal gradient = -1/(13/4) **M1**

(c) 8x + 26y + 315 = 0 **A1**

(c) sets y = 0 **M1**

(c) (-315/8, 0) **A1**

(c) Answer: 8x + 26y + 315 = 0; (-315/8, 0)

Question 13

(a) differentiates x^2 to 2x **M1**

(a) differentiates 2xy using the product rule: 2y + 2x dy/dx **M1**

(a) differentiates $2y^2$ to 4y dy/dx **M1**

(a) collects the dy/dx terms and factorises **M1**

(a) dy/dx = -(2x + 2y)/(2x + 4y) (cso) **A1**

(a) Answer: dy/dx = -(2x + 2y)/(2x + 4y)

(b) substitutes x = -1, y = 2 **M1**

(b) -1/3 **A1**

(b) Answer: -1/3

(c) normal gradient = -1/(-1/3) **M1**

(c) uses the point P **M1**

(c) 3x - y + 5 = 0 **A1**

(c) Answer: 3x - y + 5 = 0

Question 14

(a) 2sin(3x) -> 6cos(3x) **B1**

(a) attempts chain rule on $\cos^2(x)$: 2cos(x) x (-sin(x)) **M1**

(a) $-\cos^2(x)$ -> 2sin(x)cos(x) (oe sin(2x)) correct **A1**

(a) cao: dy/dx = 6cos(3x) + 2sin(x)cos(x) fully combined **A1**

(a) Answer: dy/dx = 6cos(3x) + 2sin(x)cos(x)

(b) substitutes x = π/6 into their dy/dx **M1**

(b) uses exact values sin(π/6) = 1/2, cos(π/6) = $\sqrt{3}/2$, cos(π/2) = 0 **M1**

(b) cao: dy/dx = $\sqrt{3}/2$ **A1**

(b) Answer: $\sqrt{3}/2$

(c) substitutes sin(2x) = 2sin(x)cos(x) into their expression from part (a) **M1**

(c) cso: dy/dx = 6cos(3x) + sin(2x) **A1**

(c) Answer: dy/dx = 6cos(3x) + sin(2x) (shown)

Question 15

(a) differentiates $\ln(3x^2+2)$ using the chain rule, form $(6x)/(3x^2+2)$ **M1**

(a) $6x/(3x^2+2)$ correct **A1**

(a) differentiates e^{1-2x} using the chain rule, form e^{1-2x} x (-2) **M1**

(a) $-2e^{1-2x}$ correct, combined cao **A1**

(a) Answer: $dy/dx = 6x/(3x^2+2) - 2e^{1-2x}$

(b) substitutes x = 0 into their dy/dx **M1**

(b) evaluates each term: 6(0)/2 = 0 and $-2e^1 = -2e$ **M1**

(b) cao: dy/dx = -2e **A1**

(b) Answer: -2e

(c) finds y-coordinate at x=0: y = ln(2) + e **M1**

(c) uses y - (ln2+e) = -2e(x - 0) (ft their gradient) **M1**

(c) cao: y = -2ex + (e + ln2) **A1**

(c) Answer: y = -2ex + (e + ln2)

Question 16

(a) attempts to differentiate term by term **M1**

(a) $dy/dx = 6x^2 - 18x + 12$ **A1**

(a) ft: $d^2y/dx^2 = 12x - 18$ **B1**

(a) Answer: $dy/dx = 6x^2 - 18x + 12$, $d^2y/dx^2 = 12x - 18$

(b) sets dy/dx = 0 and simplifies to a 3-term quadratic, e.g. $x^2 - 3x + 2 = 0$ **M1**

(b) solves to x = 1 and x = 2 (factorising or formula) **M1**

(b) y = 10 when x = 1 **A1**

(b) y = 9 when x = 2 **A1**

(b) Answer: (1, 10) and (2, 9)

(c) substitutes x = 1 and x = 2 into their d^2y/dx^2 (ft) **M1**

(c) cao: (1,10) is a maximum since $d^2y/dx^2 = -6 < 0$; (2,9) is a minimum since $d^2y/dx^2 = 6 > 0$ **A1**

(c) Answer: (1,10) maximum; (2,9) minimum

Question 17

(a) rewrites $y = (2x+5)^{1/2}$ **M1**

(a) applies chain rule: $(1/2)(2x+5)^{-1/2} \times 2$ **M1**

(a) cao: $dy/dx = (2x+5)^{-1/2}$ (oe $1/\sqrt{2x+5}$) **A1**

(a) Answer: $dy/dx = 1/\sqrt{2x + 5}$

(b) finds y-coordinate at x=2: $y = \sqrt{9} = 3$ **M1**

(b) finds gradient at x=2: dy/dx = 1/3 **M1**

(b) uses y - 3 = (1/3)(x-2) **M1**

(b) cao: x - 3y + 7 = 0 (oe with integer a,b,c) **A1**

(b) Answer: x - 3y + 7 = 0

(c) normal gradient = -1/(their tangent gradient) = -3 **M1**

(c) uses y - 3 = -3(x - 2) **M1**

(c) expands correctly to y = -3x + 6 + 3 **A1**

(c) cao: y = -3x + 9 **A1**

(c) Answer: y = -3x + 9

Question 18

uses the chain rule, dy/dx = 5/(5x) **M1**

1/x cao **A1**

Answer: dy/dx = 1/x

Question 19

$2x^{-1/2}$ cao, oe $2/\sqrt{x}$ **B1**

Answer: $dy/dx = 2x^{-1/2}$

Question 20

$-12x^{-3}$ cao **B1**

Answer: $dy/dx = -12x^{-3}$

Question 21

4cos(4x) cao **B1**

Answer: dy/dx = 4cos(4x)

Question 22

differentiates using the chain rule, $4(2x+3)^3 * 2$ **M1**

$8(2x+3)^3$ cao **A1**

Answer: $dy/dx = 8(2x+3)^3$

Question 23

(a) dx/dt = 2t **M1**

(a) $dy/dt = 3t^2 - 3$ **A1**

(a) uses dy/dx = (dy/dt)/(dx/dt) **M1**

(a) cao: dy/dx = 3(t-1)(t+1)/(2t) (oe $3(t^2-1)/(2t)$) **A1**

(a) Answer: dy/dx = 3(t - 1)(t + 1)/(2t)

(b) solves $t^2 + 1 = 5$ to get t = 2 or t = -2 **M1**

(b) cso: selects t = 2 using $y = t^3-3t = 2$ (t=-2 gives y=-2, rejected) **A1**

(b) Answer: t = 2

(c) substitutes t = 2 into their dy/dx **M1**

(c) cao: gradient = 9/4 **A1**

(c) Answer: 9/4

(d) uses y - 2 = (9/4)(x - 5) (ft their gradient) **M1**

(d) rearranges to isolate y **M1**

(d) cao: y = (9/4)x - 37/4 **A1**

(d) Answer: y = (9/4)x - 37/4

Question 24

(a) cso: substitutes x=2, y=1 to get 4 + 6 - 1 = 9, confirming P lies on C **B1**

(a) Answer: 4 + 6 - 1 = 9 (confirmed)

(b) differentiates x^2 implicitly to get 2x **M1**

(b) differentiates 3xy using the product rule to get 3y + 3x(dy/dx) **M1**

(b) differentiates $-y^2$ implicitly to get -2y(dy/dx) **M1**

(b) collects dy/dx terms and factorises: dy/dx(3x - 2y) = -2x - 3y **M1**

(b) cao: dy/dx = -(2x+3y)/(3x-2y) (oe (2x+3y)/(2y-3x)) **A1**

(b) Answer: dy/dx = -(2x + 3y)/(3x - 2y)

(c) substitutes x=2, y=1 into their dy/dx **M1**

(c) cao: gradient = -7/4 **A1**

(c) Answer: -7/4

(d) normal gradient = -1/(their tangent gradient) = 4/7 **M1**

(d) uses y - 1 = (4/7)(x - 2) (ft) **M1**

(d) cao: 4x - 7y - 1 = 0 (oe with integer a,b,c) **A1**

(d) Answer: 4x - 7y - 1 = 0

Question 25

applies the product rule, $2x*e^x + x^2*e^x$ **M1**

$x(x+2)e^x$ cao, oe **A1**

Answer: $dy/dx = x(x+2)e^x$

Question 26

applies the quotient rule, $[4(x+2) - (4x-1)(1)] / (x+2)^2$ **M1**

$9/(x+2)^2$ cao **A1**

Answer: $dy/dx = 9/(x+2)^2$

Question 27

sets $dy/dx = 3x^2 - 6x - 9 = 0$ and factorises to 3(x-3)(x+1) = 0 **M1**

x = 3 and x = -1 cao **A1**

$d^2y/dx^2 = 6x-6$ gives minimum at x=3 (d2y/dx2=12>0) and maximum at x=-1 (d2y/dx2=-12<0) **A1**

Answer: Minimum at x = 3, maximum at x = -1

Question 28

(a) expands $(24-2x)(15-2x) = 360 - 78x + 4x^2$ (oe)	**M1**
(a) multiplies by x: $V = x(24-2x)(15-2x)$	**M1**
(a) cso: $V = 4x^3 - 78x^2 + 360x$ (answer printed, full working required)	**A1**

(a) Answer: $V = 4x^3 - 78x^2 + 360x$ (shown)

(b) attempts to differentiate term by term	**M1**
(b) cao: $dV/dx = 12x^2 - 156x + 360$	**A1**

(b) Answer: $dV/dx = 12x^2 - 156x + 360$

(c) sets $dV/dx = 0$	**M1**
(c) simplifies to $x^2 - 13x + 30 = 0$ (divides by 12, or equivalent)	**M1**
(c) solves to x = 3 or x = 10 (dependent on previous M1)	**dM1**
(c) rejects x = 10 since $0 < x < 7.5$, selects x = 3	**A1**
(c) justifies maximum: $d^2V/dx^2 = 24x - 156$, which at x=3 is $-84 < 0$, hence a maximum	**A1**

(c) Answer: x = 3

(d) substitutes x=3 into V(x) or into $x(24-2x)(15-2x)$	**M1**
(d) cao: $V = 486$ cm^3	**A1**

(d) Answer: 486 cm^3

Question 29

differentiates $V = (4/3)\pi*r^3$ to find $dV/dr = 4*\pi*r^2$	**M1**
uses the chain rule $dr/dt = (dV/dt)/(dV/dr) = 200/(4*\pi*r^2)$, simplifying to $50/(\pi*r^2)$	**M1**
substitutes r = 10 into the expression for dr/dt	**M1**
awrt 0.159	**A1**

Answer: $dr/dt = 50/(\pi*r^2)$ (shown); rate = 0.159 cm/s (3 s.f.)

Question 30

substitutes t=2 into both parametric equations to verify x=1 and y=12	**M1**
t = 2 (rejecting t = -2, which gives y = -12, not 12)	**B1**
finds $dx/dt = 2t$ and $dy/dt = 3t^2 + 2$ and forms $dy/dx = (3t^2+2)/(2t)$	**M1**
3.5 cao, oe 7/2	**A1**

Answer: $dy/dx = (3t^2+2)/(2t)$; gradient at (1,12) is 3.5