1.8 Pure: Integration: 1-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The curve C has equation $y = 2x^2 + 2x - 3$ and the line l has equation $y = 18x - 27$. The curve and the line meet at the points A and B.

(a) Find the coordinates of A and B. **(3)**

(b) Find the area of the finite region bounded by the curve and the line. **(4)**

(Total for Question 1 is 7 marks)

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2 Use the substitution $u = x^2$ to answer this question.

(a) Find integral of $x\, e^{x^2}\, dx$. **(3)**

(b) Hence find the exact value of the integral from $x = 0$ to $x = 1$ of $x\, e^{x^2}\, dx$. **(2)**

(Total for Question 2 is 5 marks)

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3 The curve C has equation $y = 2x^2 - 3x + 5$ and the line l has equation $y = 5x - 1$. The curve and the line meet at the points A and B.

(a) Find the coordinates of A and B. **(3)**

(b) Find the area of the finite region bounded by the curve and the line. **(4)**

(Total for Question 3 is 7 marks)

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4 $f(x) = x^2 - 4x^{-1/2} - 1$, $x > 0$.

(a) Find integral of $f(x)\, dx$, simplifying your answer. **(3)**

(b) Hence find the exact value of the integral from $x = 1$ to $x = 4$ of $f(x)\, dx$. **(3)**

(Total for Question 4 is 6 marks)

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5 $f(x) = (x - 1)/((x - 2)(x - 3))$, $x > 3$.

(a) Express f(x) in partial fractions. **(3)**

(b) Hence find the exact value of the integral from $x = 4$ to $x = 7$ of $f(x)\, dx$. **(4)**

(Total for Question 5 is 7 marks)

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6 The curve C has equation $y = x(x - 2)(x - 3)$. The curve crosses the x-axis at the origin and at the points A(2, 0) and B(3, 0). The finite region R_1 lies between the origin and A, and the finite region R_2 lies between A and B.

(a) Show that $y = x^3 - 5x^2 + 6x$. **(2)**

(b) Find integral of y dx. **(2)**

(c) Find the exact area of R_1 and the exact area of R_2. **(4)**

(d) Hence find the total area enclosed between the curve and the x-axis from x = 0 to x = 3. **(2)**

(Total for Question 6 is 10 marks)

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7 Evaluate the definite integral, from x = 1 to x = 3, of 2x dx.

(Total for Question 7 is 1 mark)

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8 Using inspection (the reverse chain rule), find the integral of $(2x + 5)^3$ with respect to x.

(Total for Question 8 is 2 marks)

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9 The curve C has equation $y = 8x - x^2$, and the line l has equation y = 3x. Find the coordinates of the points of intersection of C and l, and find the area of the region enclosed by C and l.

(Total for Question 9 is 3 marks)

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10 Using the substitution u = 2x + 1, find the integral of $x*\sqrt{2x + 1}$ with respect to x. Hence evaluate the definite integral, from x = 0 to x = 4, of $x*\sqrt{2x + 1}$ dx, giving your answer as an exact value.

(Total for Question 10 is 3 marks)

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Mark scheme · 1.8 Pure: Integration: 1-hour Pack

Question 1

(a) equates the expressions for y and rearranges to a quadratic	**M1**
(a) x = 2 and x = 6	**A1**
(a) (2, 9) and (6, 81)	**A1**
(a) Answer: A(2, 9), B(6, 81)	
(b) area = integral of (line - curve) or of (curve - line) between the x values	**M1**
(b) integrand $2x^2 - 16x + 24$	**A1**
(b) integrates and substitutes the limits	**M1**
(b) 64/3	**A1**
(b) Answer: 64/3	

Question 2

(a) du = 2x dx	**M1**
(a) integral of e^u du x 1/2	**M1**
(a) $1/2e^{x^2} + c$	**A1**
(a) Answer: $1/2e^{x^2} + c$	
(b) substitutes the limits	**M1**
(b) 1/2(e - 1)	**A1**
(b) Answer: 1/2(e - 1)	

Question 3

(a) equates the expressions for y and rearranges to a quadratic	**M1**
(a) x = 1 and x = 3	**A1**
(a) (1, 4) and (3, 14)	**A1**
(a) Answer: A(1, 4), B(3, 14)	
(b) area = integral of (line - curve) or of (curve - line) between the x values	**M1**
(b) integrand $2x^2 - 8x + 6$	**A1**
(b) integrates and substitutes the limits	**M1**
(b) 8/3	**A1**
(b) Answer: 8/3	

Question 4

(a) raises each power by 1 and divides by the new power	**M1**
(a) $(1/3)x^3$	**A1**
(a) $- 8x^{1/2} - x + C$	**A1**
(a) Answer: $(1/3)x^3 - 8x^{1/2} - x + C$	
(b) F(4) - F(1)	**M1**
(b) F(4) = 4/3	**A1**
(b) F(1) = -26/3, so the value is 10	**A1**
(b) Answer: 10	

Question 5

(a) A/(x - p) + B/(x - q) and equating numerators	**M1**
(a) A = -1	**A1**
(a) B = 2	**A1**
(a) Answer: -1/(x - 2) + 2/(x - 3)	
(b) integrates each term to a logarithm: A ln|x - p| + B ln|x - q|	**M1**
(b) -1 ln|x - 2| + 2 ln|x - 3| (+ c)	**A1**
(b) substitutes the limits and combines logs	**M1**
(b) -ln(5/2) + 2 ln(4)	**A1**
(b) Answer: -ln(5/2) + 2 ln(4)	

Question 6

(a) multiplies out two brackets	**M1**
(a) $x^3 - 5x^2 + 6x$ (cso)	**A1**
(a) Answer: $y = x^3 - 5x^2 + 6x$	
(b) integrates term by term, raising each power by 1	**M1**
(b) $x^4/4 - 5/3x^3 + 3x^2$ (+ c)	**A1**
(b) Answer: $(1/4)x^4 - (5/3)x^3 + 3x^2 + c$	
(c) evaluates the integral between 0 and 2 for R_1	**M1**
(c) area of R_1 = 8/3 (the integral is 8/3)	**A1**
(c) evaluates the integral between 2 and 3 for R_2	**M1**
(c) area of R_2 = 5/12 (the integral is -5/12, negative as the curve is below the axis)	**A1**
(c) Answer: $R_1 = 8/3$, $R_2 = 5/12$	
(d) adds the magnitudes of the two areas	**M1**
(d) 37/12	**A1**
(d) Answer: 37/12	

Question 7

8 cao	**B1**
Answer: 8	

Question 8

recognises the form $k(2x+5)^4$ and finds k by differentiating to check	**M1**
$(1/8)(2x+5)^4 + c$ cao	**A1**
Answer: $(1/8)(2x+5)^4 + c$	

Question 9

sets $8x - x^2 = 3x$ and solves to find x = 0 and x = 5	**M1**
sets up and integrates the area as the integral of $(5x - x^2)$ dx	**M1**
125/6 cao, oe awrt 20.8	**A1**
Answer: Points (0,0) and (5,15); area = 125/6	

Question 10

substitutes x = (u-1)/2, dx = (1/2)du to form (1/4) times the integral of $(u-1)*u^{1/2}$ du	**M1**
integrates to $(1/10)u^{5/2} - (1/6)u^{3/2}$ + c and substitutes the limits u=1 (x=0) and u=9 (x=4)	**M1**
298/15 cao, oe awrt 19.9	**A1**
Answer: 298/15	

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