1.8 Pure: Integration: 30-minute Pack

EDEXCEL 9MA0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question tests integration by inspection (reverse chain rule).

(a) Find integral of $(2x + 1)^5$ dx. **(3)**

(b) Find integral of $x/(x^2 + 4)$ dx. **(3)**

(c) Find integral of $\cos(3x - 1)$ dx. **(3)**

(Total for Question 1 is 9 marks)

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2 The curve C has equation $y = x(x - 6)(x - 7)$. The curve crosses the x-axis at the origin and at the points A(6, 0) and B(7, 0). The finite region R_1 lies between the origin and A, and the finite region R_2 lies between A and B.

(a) Show that $y = x^3 - 13x^2 + 42x$. **(2)**

(b) Find integral of y dx. **(2)**

(c) Find the exact area of R_1 and the exact area of R_2. **(4)**

(d) Hence find the total area enclosed between the curve and the x-axis from $x = 0$ to $x = 7$. **(2)**

(Total for Question 2 is 10 marks)

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3 The table shows values of $y = \sqrt{1 + x^3}$, for $0 \leq x \leq 2$.

x	0	0.5	1	1.5	2
y	1	1.0607	?	2.0917	3

x	y
0	1
0.5	1.0607
1	?
1.5	2.0917
2	3

(a) Complete the table by finding the value of y at x = 1, giving your answer to 4 decimal places. **(2)**

(b) Using the trapezium rule with all five values in the table (strip width h = 0.5), find an estimate for the definite integral, from x = 0 to x = 2, of $\sqrt{1 + x^3}\ dx$, giving your answer to 3 significant figures. **(3)**

(c) State, with a reason, whether the trapezium rule estimate found in part (b) is an overestimate or an underestimate of the true value of the integral. **(2)**

(Total for Question 3 is 7 marks)

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Mark scheme · 1.8 Pure: Integration: 30-minute Pack

A correct answer earns full marks unless the question asks for working, a method, proof or explanation.

Question 1

(a) attempt at $(2x+1)^6$ form	**M1**
(a) divide by 12 (their coefficient from the chain rule), dep	**M1**
(a) $(2x+1)^6 / 12 + c$, cao	**A1**
(a) Answer: $(2x + 1)^6 / 12 + c$	
(b) recognise numerator is (1/2) x derivative of denominator	**M1**
(b) (1/2) $\ln(x^2 + 4)$	**A1**
(b) + c present	**B1**
(b) Answer: (1/2) $\ln(x^2 + 4) + c$	
(c) sin(3x - 1) form seen	**M1**
(c) coefficient 1/3 correct	**A1**
(c) + c present	**B1**
(c) Answer: (1/3) sin(3x - 1) + c	

Question 2

(a) multiplies out two brackets	**M1**
(a) $x^3 - 13x^2 + 42x$ (cso)	**A1**
(a) Answer: $y = x^3 - 13x^2 + 42x$	
(b) integrates term by term, raising each power by 1	**M1**
(b) $x^4/4 - 13/3x^3 + 21x^2$ (+ c)	**A1**
(b) Answer: $(1/4)x^4 - (13/3)x^3 + 21x^2 + c$	
(c) evaluates the integral between 0 and 6 for R_1	**M1**
(c) area of $R_1 = 144$ (the integral is 144)	**A1**
(c) evaluates the integral between 6 and 7 for R_2	**M1**
(c) area of $R_2 = 13/12$ (the integral is -13/12, negative as the curve is below the axis)	**A1**
(c) Answer: $R_1 = 144$, $R_2 = 13/12$	
(d) adds the magnitudes of the two areas	**M1**
(d) 1741/12	**A1**
(d) Answer: 1741/12	

Question 3

(a) substitute x = 1 into $\sqrt{1 + x^3}$	**M1**
(a) awrt 1.4142	**A1**
(a) Answer: 1.4142 (awrt)	
(b) correct trapezium rule structure: (h/2)[(y0 + y4) + 2(y1 + y2 + y3)]	**M1**
(b) correct substitution of h = 0.5 and all five table values	**M1**
(b) awrt 3.28	**A1**
(b) Answer: 3.28 (awrt)	
(c) overestimate	**B1**
(c) correct reason: the curve is convex (concave up, gradient increasing) on this interval, so the trapezia lie above the curve	**B1**
(c) Answer: Overestimate, since the curve is convex (concave up) over [0, 2], so each trapezium lies above the curve.	

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