1.8 Pure: Integration: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 70 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question tests integration of expressions with fractional and negative indices.

(a) Find integral of $(3x^2 - 4x^{-1/2} + 5)\,dx$, giving your answer in simplified form. **(3)**

(b) Show that integral of $(2x - 3)/\sqrt{x}\,dx = (4/3)x^{3/2} - 6x^{1/2} + c$ **(3)**

(c) Hence evaluate the definite integral, from $x = 1$ to $x = 4$, of $(2x - 3)/\sqrt{x}\,dx$, giving your answer as an exact value. **(2)**

(Total for Question 1 is 8 marks)

2 Use the substitution $u = 2x^2 + 5$ to answer this question.

(a) Show that integral of $x(2x^2 + 5)^2\,dx = (2x^2 + 5)^3/12 + c$. **(3)**

(b) Hence find the exact value of the integral from $x = 0$ to $x = 2$ of $x(2x^2 + 5)^2\,dx$. **(3)**

(Total for Question 2 is 6 marks)

3 Evaluate the definite integral, from $x = 0$ to $x = 2$, of $(3x^2 + 1)\,dx$.

(Total for Question 3 is 2 marks)

4 $f(x) = (5x - 14) / ((x - 1)(x - 4))$, for $x > 4$.

(a) Express $f(x)$ in the form $A/(x - 1) + B/(x - 4)$, where A and B are integers to be found. **(4)**

(b) Hence find integral of $f(x)\,dx$. **(4)**

(c) Hence show that the definite integral, from $x = 5$ to $x = 6$, of $f(x)\,dx$ equals $\ln(k)$, and find the exact value of k. **(4)**

(Total for Question 4 is 12 marks)

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5 The curve C has equation $y = x/(1 + x^2)$, for $x \geq 0$.

(a) Show that integral of $x/(1 + x^2)\, dx = (1/2)*\ln(1 + x^2) + c$. **(2)**

(b) Hence find the exact area of the region bounded by C, the x-axis and the lines $x = 0$ and $x = \sqrt{3}$, giving your answer in the form $k*\ln(2)$, where k is a constant to be found. **(3)**

(c) Find the exact area of the region bounded by C, the x-axis and the lines $x = \sqrt{3}$ and $x = 3$, giving your answer in the form $(1/2)*\ln(p/q)$, where p and q are integers in lowest terms. **(3)**

(Total for Question 5 is 8 marks)

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6 The temperature, θ degrees Celsius, of a cup of tea t minutes after it is made satisfies the differential equation $d\theta/dt = -k(\theta - 18)$, where k is a positive constant and 18 is the room temperature. When $t = 0$, $\theta = 90$.

(a) Using separation of variables, show that $\theta = 18 + 72*e^{-kt}$. **(5)**

(b) Given that $\theta = 60$ when $t = 5$, show that $k = (1/5)\ln(12/7)$. **(3)**

(c) Find the time taken, in minutes, for the temperature of the tea to fall to 25 degrees Celsius, giving your answer to 3 significant figures. **(3)**

(Total for Question 6 is 11 marks)

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7 Using inspection (the reverse chain rule), find the integral of $x/(x^2 + 9)$ with respect to x, giving your answer in terms of a natural logarithm.

(Total for Question 7 is 2 marks)

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8 The number of bacteria, N, in a culture is modelled by the differential equation $dN/dt = N/50$, where t is measured in hours. Initially there are 200 bacteria.

(a) By separating the variables, show that $N = 200\ e^{t/50}$. **(2)**

(b) Find the time taken, in hours to 3 significant figures, for the population to reach 500. **(2)**

(Total for Question 8 is 4 marks)

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9 Find the integral of x*cos(3x) with respect to x, using integration by parts. Hence evaluate the definite integral, from x = 0 to x = π/6, of x*cos(3x) dx, giving your answer as an exact value in terms of π.

(Total for Question 9 is 4 marks)

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Mark scheme · 1.8 Pure: Integration: Test Pack

Question 1

(a) attempt to integrate at least two terms, index raised by 1 throughout **M1**

(a) $x^3 - 8x^{1/2}$ with both coefficients correct oe **A1**

(a) +5x + c present and correct **B1**

(a) Answer: $x^3 - 8\sqrt{x} + 5x + c$

(b) rewrite integrand as $2x^{1/2} - 3x^{-1/2}$ **M1**

(b) integrate to $(4/3)x^{3/2}$ and $-6x^{1/2}$, indices raised by 1 (dep) **M1**

(b) cso, fully correct with +c matching the printed result **A1**

(b) Answer: $(4/3)x^{3/2} - 6x^{1/2} + c$ (given, cso)

(c) substitute limits x = 4 and x = 1 into the result from part (b) **M1**

(c) 10/3 oe, cao **A1**

(c) Answer: 10/3

Question 2

(a) du/dx = 4x, so x dx = du/4 **M1**

(a) the integral becomes (1/4) integral of u^2 du **M1**

(a) = $u^3/12$ + c and substitutes back (cso) **A1**

(a) Answer: $(2x^2 + 5)^3/12 + c$

(b) substitutes the limits into the result of part (a) **M1**

(b) $[(13)^3 - 5^3]/12$ **A1**

(b) 518/3 **A1**

(b) Answer: 518/3

Question 3

finds the antiderivative $x^3 + x$ **M1**

10 cao **A1**

Answer: 10

Question 4

(a) set up 5x - 14 = A(x - 4) + B(x - 1) **M1**

(a) substitute x = 4 or x = 1 to find one constant **M1**

(a) one of A = 3, B = 2 correct **A1**

(a) both A = 3 and B = 2 correct **A1**

(a) Answer: A = 3, B = 2

(b) integrate A/(x-1) term to A ln(x-1) **M1**

(b) integrate B/(x-4) term to B ln(x-4) **M1**

(b) 3 ln(x - 1) + 2 ln(x - 4) all correct **A1**

(b) + c present **B1**

(b) Answer: 3 ln(x - 1) + 2 ln(x - 4) + c

(c) substitute x = 6 and x = 5 into their antiderivative from part (b) **M1**

(c) combine logs using the laws of logarithms: 3ln5 + 2ln2 - 3ln4 **M1**

(c) simplify to ln(125/16), oe **A1**

(c) k = 125/16, cao **A1**

(c) Answer: k = 125/16

Question 5

(a) recognise the integrand as (1/2) times the derivative of $\ln(1+x^2)$ (or use the substitution $u=1+x^2$, du=2x dx) **M1**

(a) $(1/2)*\ln(1+x^2)$ + c cso **A1**

(a) Answer: $(1/2)*\ln(1 + x^2) + c$

(b) substitute limits $x=\sqrt{3}$ and x=0 into the result from part (a) (ft) **M1**

(b) (1/2)*ln(4) - (1/2)*ln(1) obtained **A1**

(b) k = 1, i.e. ln(2), cao **A1**

(b) Answer: ln(2), i.e. k = 1

(c) substitute limits x=3 and $x=\sqrt{3}$ into the result from part (a) (ft) **M1**

(c) (1/2)*ln(10) - (1/2)*ln(4) obtained **A1**

(c) (1/2)*ln(5/2), i.e. p=5, q=2, cao **A1**

(c) Answer: (1/2)*ln(5/2), i.e. p = 5, q = 2

Question 6

(a) separate variables correctly: integral of 1/(θ-18) dtheta = integral of -k dt **M1**

(a) ln(θ - 18) = -kt (+ constant) **A1**

(a) rearrange (exponentiate) to $\theta - 18 = A*e^{-kt}$ **M1**

(a) use t = 0, θ = 90 to find A = 72 **M1**

(a) cso, $\theta = 18 + 72*e^{-kt}$ **A1**

(a) Answer: $\theta = 18 + 72*e^{-kt}$ (given, cso)

(b) substitute t = 5, θ = 60 and rearrange to $e^{-5k} = 7/12$ **M1**

(b) take ln of both sides **M1**

(b) cso, k = (1/5) ln(12/7) **A1**

(b) Answer: k = (1/5) ln(12/7) (given, cso)

(c) substitute θ = 25 into $\theta = 18+72e^{-kt}$ and rearrange to $e^{-kt} = 7/72$ **M1**

(c) take logs and use their value of k (ft) to solve for t **M1**

(c) awrt 21.6 (minutes) **A1**

(c) Answer: t = awrt 21.6 minutes

Question 7

recognises that the numerator is (1/2) of the derivative of the denominator **M1**

$(1/2)\ln(x^2+9)$ + c cao **A1**

Answer: $(1/2)\ln(x^2+9) + c$

Question 8

(a) separates variables and integrates: ln N = t/50 + C **M1**

(a) cso: uses N=200 at t=0 to obtain $N = 200e^{t/50}$ **A1**

(a) Answer: $N = 200e^{t/50}$

(b) sets $200e^{t/50} = 500$ and takes logarithms of both sides **M1**

(b) awrt 45.8 hours **A1**

(b) Answer: awrt 45.8 hours

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Question 9

sets u = x, dv/dx = cos(3x), so du/dx = 1, v = (1/3)sin(3x), and applies the by-parts formula	**M1**
(x/3)sin(3x) + (1/9)cos(3x) + c cao	**A1**
substitutes the limits x = π/6 and x = 0	**M1**
(π-2)/18 cao, oe awrt 0.0634	**A1**

Answer: (π-2)/18

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