1.9 Pure: Numerical Methods: 30-minute Pack

EDEXCEL 9MA0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $f(x) = x^3 - 3x - 9$, and the equation $f(x) = 0$ has a single real root, α.

(a) Show that α lies between $x = 2$ and $x = 3$. **(2)**

(b) Show that the equation can be rearranged as $x = (3x + 9)^{1/3}$. **(2)**

(c) Use $x_{n+1} = (3x_n + 9)^{1/3}$ with $x_0 = 2$ to find x_1, x_2 and x_3, giving each to 4 decimal places. **(3)**

(d) By considering the change of sign of f at $x = 2.545$ and $x = 2.555$, show that $\alpha = 2.55$ to 2 decimal places. **(2)**

(Total for Question 1 is 9 marks)

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2 The table shows values of $y = \sqrt{x + 2}$, rounded to 4 decimal places, for equally spaced values of x.

x	0	0.4	0.8	1.2	1.6	2
y	1.4142	1.5492	1.6733	1.7889	1.8974	2.0000

(a) Use the trapezium rule with 5 strips to estimate the integral from $x = 0$ to $x = 2$ of $\sqrt{x + 2}\ dx$, giving your answer to 4 decimal places. **(4)**

(b) State, with a reason, whether the trapezium rule gives an overestimate or an underestimate. **(2)**

(c) Find the percentage error in your estimate, given that the exact value is 3.4477, to 2 significant figures. **(2)**

(Total for Question 2 is 8 marks)

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3 $f(x) = x^3 + x - 7$. The equation $f(x) = 0$ has a root α near $x = 3$.

(a) Find $f'(x)$. **(2)**

(b) Use the Newton-Raphson method with $x_0 = 3$ to find x_1, x_2 and x_3, giving each answer to 4 decimal places. **(4)**

(c) Hence state the value of α correct to 3 decimal places, and justify your answer by evaluating f at two suitable values. **(3)**

(Total for Question 3 is 9 marks)

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Mark scheme · 1.9 Pure: Numerical Methods: 30-minute Pack

Question 1

(a) evaluates f(2) and f(3) **M1**

(a) f(2) = -7 and f(3) = 9, a change of sign, and f is continuous, so a root lies in the interval (cso) **A1**

(a) Answer: f(2) = -7 < 0 < f(3) = 9

(b) adds 3x + 9 to both sides: $x^3 = 3x + 9$ **M1**

(b) takes cube roots: $x = (3x + 9)^{1/3}$ (cso) **A1**

(b) Answer: $x = (3x + 9)^{1/3}$

(c) $x_1 = (3 \times 2 + 9)^{1/3}$ **M1**

(c) $x_1 = 2.4662$ **A1**

(c) $x_2 = 2.5406$ and $x_3 = 2.5521$ **A1**

(c) Answer: $x_1 = 2.4662$, $x_2 = 2.5406$, $x_3 = 2.5521$

(d) evaluates f(2.545) and f(2.555) **M1**

(d) f(2.545) = -0.1510 and f(2.555) = 0.0141, a change of sign, so α = 2.55 to 2 d.p. (cso) **A1**

(d) Answer: α = 2.55

Question 2

(a) h = 2/5 = 0.4 **B1**

(a) trapezium rule: $(h/2)[y_0 + y_n + 2(\text{sum of the middle values})]$ **M1**

(a) sum of middle values = 6.9088 **A1**

(a) 3.4464 **A1**

(a) Answer: 3.4464

(b) underestimate **B1**

(b) the curve is concave (concave down), so the chords lie below the curve **B1**

(b) Answer: underestimate

(c) |estimate - exact|/exact x 100 **M1**

(c) 0.038% (accept 0.039% using the unrounded estimate and exact value) **A1**

(c) Answer: 0.038% (accept 0.039% using the unrounded estimate and exact value)

Question 3

(a) differentiates each term **M1**

(a) $f'(x) = 3x^2 + 1$ **A1**

(a) Answer: $f'(x) = 3x^2 + 1$

(b) $x_{n+1} = x_n - f(x_n)/f'(x_n)$ **M1**

(b) $x_1 = 2.1786$ **A1**

(b) $x_2 = 1.8164$ **A1**

(b) $x_3 = 1.7421$ **A1**

(b) Answer: $x_1 = 2.1786$, $x_2 = 1.8164$, $x_3 = 1.7421$

(c) α = 1.739 **B1**

(c) evaluates f(1.7385) and f(1.7395) **M1**

(c) values -0.0071 and 0.0030 have opposite signs **A1**

(c) Answer: α = 1.739

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