1.9 Pure: Numerical Methods: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The equation $x^3 + 2x - 5 = 0$ has a single real root, α.

(a) Show that α lies between $x = 1$ and $x = 2$. **(2)**

(b) Show that the equation $x^3 + 2x - 5 = 0$ can be rearranged into the iterative form $x_{n+1} = (5 - 2x_n)^{1/3}$. **(2)**

(c) Using $x_0 = 1.5$, apply the iteration formula from part (b) to find x_1, x_2 and x_3, giving each answer to 4 decimal places. **(4)**

(d) Show that $\alpha = 1.328$ correct to 3 decimal places. **(2)**

(Total for Question 1 is 10 marks)

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2 $f(x) = x^3 - 3x - 8$, and the equation $f(x) = 0$ has a single real root, α.

(a) Show that α lies between $x = 2$ and $x = 3$. **(2)**

(b) Show that the equation can be rearranged as $x = (3x + 8)^{1/3}$. **(2)**

(c) Use $x_{n+1} = (3x_n + 8)^{1/3}$ with $x_0 = 1$ to find x_1, x_2 and x_3, giving each to 4 decimal places. **(3)**

(d) By considering the change of sign of f at $x = 2.485$ and $x = 2.495$, show that $\alpha = 2.49$ to 2 decimal places. **(2)**

(Total for Question 2 is 9 marks)

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3 The curve $y = e^{-x}$ is shown for $0 \leq x \leq 2$. The table gives values of y at x = 0, 0.5, 1, 1.5 and 2.

x : 0 0.5 1 1.5 2

y : 1 p 0.3679 q 0.1353

(a) Find the values of p and q, each correct to 4 decimal places. **(2)**

(b) Use the trapezium rule with all 4 strips to find an estimate for the integral of e^{-x} from 0 to 2, giving your answer to 4 decimal places. **(3)**

(c) Given that the exact value of the integral of e^{-x} from 0 to 2 is $1 - e^{-2}$, calculate the percentage error in the trapezium rule estimate found in part (b), giving your answer to 1 decimal place. **(3)**

(d) Explain, with reference to the shape of the curve $y = e^{-x}$, why the trapezium rule gives an overestimate of the true value of the integral on this interval. **(2)**

(Total for Question 3 is 10 marks)

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4 Fahima records the velocity, v m/s, of her car t seconds after moving away from a junction.

t : 0 10 20 30 40

v : 0 4.2 7.8 10.1 11.5

(a) Use the trapezium rule with all 4 strips to estimate the total distance travelled by the car in the first 40 seconds. **(3)**

(b) State, giving a reason, whether your answer to part (a) is likely to be an overestimate or an underestimate of the true distance travelled. **(2)**

(c) Explain why the trapezium rule would give the exact distance travelled if the velocity had increased at a constant rate throughout the 40 seconds. **(2)**

(Total for Question 4 is 7 marks)

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5 State the condition on f(a) and f(b) required for the change-of-sign test to conclude that a root of the continuous function f lies between x = a and x = b.

(Total for Question 5 is 1 mark)

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6 Write down the general iteration formula used in the Newton-Raphson method for solving f(x) = 0.

(Total for Question 6 is 1 mark)

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7 Let $f(x) = x^3 - 6x + 2$. Evaluate $f(2)$.

(Total for Question 7 is 1 mark)

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8 Using $x_0 = 2$ and the Newton-Raphson iteration $x_{n+1} = x_n - (x_n^2 - 7)/(2x_n)$, find x_1 and x_2, each to 4 decimal places.

(Total for Question 8 is 2 marks)

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9 Let $f(x) = x^3 - 4x + 1$. Show that $f(x) = 0$ has a root between $x = 1$ and $x = 2$. Using $x_0 = 2$ and $f'(x) = 3x^2 - 4$, apply the Newton-Raphson method to find x_1 and x_2, giving each answer to 4 decimal places.

(Total for Question 9 is 3 marks)

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10 The curve $y = 1/(x+2)$ is defined for $x > -2$. The table gives values of y at $x = 0, 1, 2, 3, 4$. x: 0, 1, 2, 3, 4 y: 0.5, 0.3333, 0.25, 0.2, 0.1667. Use the trapezium rule with all four strips (h=1) to find an estimate for the integral of $1/(x+2)$ from $x=0$ to $x=4$, giving your answer to 4 decimal places. By considering the sign of the second derivative of $y = 1/(x+2)$, state, with a reason, whether this trapezium rule estimate is an overestimate or an underestimate of the true value of the integral.

(Total for Question 10 is 4 marks)

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Mark scheme · 1.9 Pure: Numerical Methods: Test Pack

Question 1

(a) evaluates f(1) and f(2) **M1**

(a) f(1) = -2, f(2) = 7; opposite signs and f continuous on [1,2], so by the change of sign rule a root lies in (1,2) cso **A1**

(a) Answer: f(1) = -2, f(2) = 7, sign change confirms a root in (1,2)

(b) rearranges to $x^3 = 5 - 2x$ **M1**

(b) takes the cube root of both sides to give $x = (5 - 2x)^{1/3}$ cso **A1**

(b) Answer: $x_{n+1} = (5 - 2x_n)^{1/3}$

(c) correct substitution of $x_0 = 1.5$ into the iteration formula **M1**

(c) $x_1 = 1.2599$ (awrt 4dp) **A1**

(c) $x_2 = 1.3536$ (awrt 4dp) **A1**

(c) $x_3 = 1.3186$ (awrt 4dp) **A1**

(c) Answer: $x_1 = 1.2599$, $x_2 = 1.3536$, $x_3 = 1.3186$

(d) evaluates f(1.3275) and f(1.3285) **M1**

(d) f(1.3275) = -0.0056 (<0) and f(1.3285) = +0.0017 (>0); change of sign in (1.3275, 1.3285) confirms α = 1.328 to 3dp cso **A1**

(d) Answer: α = 1.328 (3dp)

Question 2

(a) evaluates f(2) and f(3) **M1**

(a) f(2) = -6 and f(3) = 10, a change of sign, and f is continuous, so a root lies in the interval (cso) **A1**

(a) Answer: f(2) = -6 < 0 < f(3) = 10

(b) adds 3x + 8 to both sides: $x^3 = 3x + 8$ **M1**

(b) takes cube roots: $x = (3x + 8)^{1/3}$ (cso) **A1**

(b) Answer: $x = (3x + 8)^{1/3}$

(c) $x_1 = (3 \times 1 + 8)^{1/3}$ **M1**

(c) $x_1 = 2.2240$ **A1**

(c) $x_2 = 2.4481$ and $x_3 = 2.4849$ **A1**

(c) Answer: $x_1 = 2.2240$, $x_2 = 2.4481$, $x_3 = 2.4849$

(d) evaluates f(2.485) and f(2.495) **M1**

(d) f(2.485) = -0.1096 and f(2.495) = 0.0464, a change of sign, so α = 2.49 to 2 d.p. (cso) **A1**

(d) Answer: α = 2.49

Question 3

(a) p = 0.6065 (awrt 4dp) **B1**

(a) q = 0.2231 (awrt 4dp) **B1**

(a) Answer: p = 0.6065, q = 0.2231

(b) correct trapezium structure with h = 0.5: $(h/2)[y_0 + y_4 + 2(y_1+y_2+y_3)]$ **M1**

(b) correct value inside the bracket, 6.5904... (or equivalent unsimplified form) **A1**

(b) 0.8826 (awrt 4dp) **A1**

(b) Answer: Estimate = 0.8826

(c) exact value = 0.8647 (awrt 4dp) **B1**

(c) percentage error = |estimate - exact| / exact x 100 **M1**

(c) 2.1% (awrt 1dp) **A1**

(c) Answer: Percentage error = 2.1%

(d) states that $y = e^{-x}$ is convex (curves upwards / concave up) on [0,2] **B1**

(d) hence each chord joining consecutive points lies above the curve, so the trapezium rule overestimates the area/integral **B1**

(d) Answer: The curve is convex, so the chords lie above the curve and the trapezium rule overestimates.

Question 4

(a) correct trapezium structure with h = 10: $(h/2)[v_0 + v_4 + 2(v_1+v_2+v_3)]$ **M1**

(a) correct value inside the bracket, 55.7 **A1**

(a) 278.5 (m) cao **A1**

(a) Answer: Distance approx 278.5 m

(b) states underestimate **B1**

(b) valid reason, e.g. the rate of increase of v is slowing (successive differences 4.2, 3.6, 2.3, 1.4 are decreasing) so the v-t graph is concave, meaning the curve lies above the chords, giving an underestimate **B1**

(b) Answer: Underestimate, since the v-t graph is concave (the increases in v are getting smaller).

(c) a constant rate of increase means the v-t graph is a straight line between recorded points **B1**

(c) the trapezium rule fits a straight line exactly between each pair of points, so there is no approximation error and the area under a straight line segment is exact **B1**

(c) Answer: With constant acceleration the v-t graph is linear, and the trapezium rule is exact for a straight line, so no error arises.

Question 5

f(a) and f(b) have opposite signs (one positive, one negative) **B1**

Answer: f(a) and f(b) must have opposite signs.

Question 6

$x_{n+1} = x_n - f(x_n)/f'(x_n)$ cao **B1**

Answer: $x_{n+1} = x_n - f(x_n)/f'(x_n)$

Question 7

-2 cao **B1**

Answer: -2

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Question 8

finds $x_1 = 2.75$ **M1**

x_2 = awrt 2.6477 **A1**

Answer: $x_1 = 2.7500$, $x_2 = 2.6477$

Question 9

evaluates f(1) = -2 and f(2) = 1, a change of sign, so a root lies between x=1 and x=2 **M1**

substitutes into $x_1 = x_0 - f(x_0)/f'(x_0) = 2 - 1/8$ **M1**

$x_1 = 1.8750$, x_2 = awrt 1.8610 **A1**

Answer: Root between x=1 and x=2 (shown); $x_1 = 1.8750$, $x_2 = 1.8610$

Question 10

substitutes into the trapezium rule, $(1/2)[(y_0+y_4) + 2(y_1+y_2+y_3)]$ **M1**

awrt 1.1167 **A1**

finds the second derivative, $d^2y/dx^2 = 2/(x+2)^3$, and notes it is positive for x > -2 **M1**

states the curve is convex (concave up), so the chords lie above the curve, and the trapezium rule gives an overestimate **A1**

Answer: Trapezium estimate = 1.1167; this is an overestimate since $d^2y/dx^2 > 0$ for x > -2

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