2.3 Statistics: Probability

EDEXCEL 9MA0 · Calculator allowed · about 225 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 At a sixth form college, 120 students were surveyed about the subjects they study. Let M be the event that a randomly chosen student studies Mathematics and P be the event that a randomly chosen student studies Physics. From the survey, P(M) = 0.65, P(P) = 0.40 and P(M and P) = 0.25.

(a) Find P(M or P). **(2)**

(b) Find P(M and not P), the probability that a randomly chosen student studies Mathematics but not Physics. **(2)**

(c) Determine, showing your working, whether the events M and P are independent. **(3)**

(d) Find P(not M and not P). **(2)**

(Total for Question 1 is 9 marks)

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2 A bag contains 6 red counters and 3 blue counters. Two counters are taken at random from the bag, one after the other, without replacement.

(a) Draw a tree diagram, labelling each branch with its probability, and state the probability that both counters are red. **(4)**

(b) Find the probability that the two counters are the same colour. **(2)**

(c) Given that the two counters are different colours, find the probability that the first counter was red. **(2)**

(Total for Question 2 is 8 marks)

3 136 club members were asked which of three sports they prefer, and whether they are left-handed or right-handed. The results are:

Sport: Swimming, Tennis, Golf

Left-handed: 19, 38, 10

Right-handed: 18, 34, 17

(a) A member is chosen at random. Find the probability that the member prefers tennis. **(2)**

(b) Given that the member prefers tennis, find the probability that the member is left-handed. **(2)**

(c) Given that the member is left-handed, find the probability that the member prefers swimming. **(2)**

(Total for Question 3 is 6 marks)

4 In a survey of 100 students, 33 study French (F) and 41 study Spanish (S). 15 students study both languages.

(a) Complete the Venn diagram by finding the number of students in each region (F only, S only, both, neither). **(3)**

(b) A student is chosen at random. Find the probability that the student studies at least one of the languages. **(2)**

(c) Given that a randomly chosen student studies Spanish, find the probability that the student also studies French. **(2)**

(d) Determine whether studying French and studying Spanish are statistically independent events. **(2)**

(Total for Question 4 is 9 marks)

5 A bag contains 6 red counters and 6 blue counters. Two counters are taken at random from the bag, one after the other, without replacement.

(a) Draw a tree diagram, labelling each branch with its probability, and state the probability that both counters are red. **(4)**

(b) Find the probability that the two counters are the same colour. **(2)**

(c) Given that the two counters are different colours, find the probability that the first counter was red. **(2)**

(Total for Question 5 is 8 marks)

6 112 club members were asked which of three sports they prefer, and whether they are left-handed or right-handed. The results are:

Sport: Swimming, Tennis, Golf

Left-handed: 22, 9, 34

Right-handed: 28, 8, 11

(a) A member is chosen at random. Find the probability that the member prefers tennis. **(2)**

(b) Given that the member prefers tennis, find the probability that the member is left-handed. **(2)**

(c) Given that the member is left-handed, find the probability that the member prefers swimming. **(2)**

(Total for Question 6 is 6 marks)

7 In a survey of 40 students, 10 study French (F) and 13 study Spanish (S). 6 students study both languages.

(a) Complete the Venn diagram by finding the number of students in each region (F only, S only, both, neither). **(3)**

(b) A student is chosen at random. Find the probability that the student studies at least one of the languages. **(2)**

(c) Given that a randomly chosen student studies Spanish, find the probability that the student also studies French. **(2)**

(d) Determine whether studying French and studying Spanish are statistically independent events. **(2)**

(Total for Question 7 is 9 marks)

8 A bag contains 6 red counters and 7 blue counters. Two counters are taken at random from the bag, one after the other, without replacement.

(a) Draw a tree diagram, labelling each branch with its probability, and state the probability that both counters are red. **(4)**

(b) Find the probability that the two counters are the same colour. **(2)**

(c) Given that the two counters are different colours, find the probability that the first counter was red. **(2)**

(Total for Question 8 is 8 marks)

9 138 club members were asked which of three sports they prefer, and whether they are left-handed or right-handed. The results are:
Sport: Swimming, Tennis, Golf
Left-handed: 38, 15, 25
Right-handed: 12, 29, 19

(a) A member is chosen at random. Find the probability that the member prefers tennis. **(2)**

(b) Given that the member prefers tennis, find the probability that the member is left-handed. **(2)**

(c) Given that the member is left-handed, find the probability that the member prefers swimming. **(2)**

(Total for Question 9 is 6 marks)

10 The events A and B are independent, with P(A) = 1/5 and P(B) = 2/7.

(a) Find P(A and B) and P(A or B). **(4)**

(b) The independent events C and D satisfy P(D) = 2/7 and P(C or D) = 3/7. Find P(C). **(3)**

(c) Find P(A | B') and state what this shows about A and B. **(3)**

(Total for Question 10 is 10 marks)

11 150 club members were asked which of three sports they prefer, and whether they are left-handed or right-handed. The results are:
Sport: Swimming, Tennis, Golf
Left-handed: 20, 15, 36
Right-handed: 26, 13, 40

(a) A member is chosen at random. Find the probability that the member prefers tennis. **(2)**

(b) Given that the member prefers tennis, find the probability that the member is left-handed. **(2)**

(c) Given that the member is left-handed, find the probability that the member prefers swimming. **(2)**

(Total for Question 11 is 6 marks)

12 The events A and B are independent, with P(A) = 3/10 and P(B) = 3/4.

(a) Find P(A and B) and P(A or B). **(4)**

(b) The independent events C and D satisfy P(D) = 3/4 and P(C or D) = 33/40. Find P(C). **(3)**

(c) Find P(A | B') and state what this shows about A and B. **(3)**

(Total for Question 12 is 10 marks)

13 In a survey of 20 students, 7 study French (F) and 6 study Spanish (S). 3 students study both languages.

(a) Complete the Venn diagram by finding the number of students in each region (F only, S only, both, neither). **(3)**

(b) A student is chosen at random. Find the probability that the student studies at least one of the languages. **(2)**

(c) Given that a randomly chosen student studies Spanish, find the probability that the student also studies French. **(2)**

(d) Determine whether studying French and studying Spanish are statistically independent events. **(2)**

(Total for Question 13 is 9 marks)

14 In a survey of 40 students, 15 study French (F) and 10 study Spanish (S). 5 students study both languages.

(a) Complete the Venn diagram by finding the number of students in each region (F only, S only, both, neither). **(3)**

(b) A student is chosen at random. Find the probability that the student studies at least one of the languages. **(2)**

(c) Given that a randomly chosen student studies Spanish, find the probability that the student also studies French. **(2)**

(d) Determine whether studying French and studying Spanish are statistically independent events. **(2)**

(Total for Question 14 is 9 marks)

15 A bag contains 5 red counters and 7 blue counters. Two counters are drawn at random, one after another, without replacement.

(a) Find the probability that both counters drawn are the same colour. **(3)**

(b) Find the probability that at least one of the two counters drawn is red. **(3)**

(c) Given that both counters drawn are the same colour, find the probability that both are red. **(3)**

(Total for Question 15 is 9 marks)

16 The random variable $X \sim B(10, 0.3)$.

(a) Calculate $P(X = 3)$, giving your answer to 3 significant figures. **(2)**

(b) Calculate $P(X \leq 2)$. **(3)**

(c) Calculate $P(X \geq 4)$. **(3)**

(Total for Question 16 is 8 marks)

17 The discrete random variable X has probability distribution given by $P(X = x) = kx$ for $x = 1, 2, 3, 4$, and $P(X = x) = 0$ otherwise, where k is a constant.

(a) Show that $k = 0.1$. **(2)**

(b) Find $P(X \geq 3)$. **(2)**

(c) Find E(X). **(3)**

(d) Find Var(X). **(4)**

(Total for Question 17 is 11 marks)

18 The random variable X ~ B(n, p). Given that E(X) = 6 and Var(X) = 4.2.

(a) Show that p = 0.3. **(3)**

(b) Find the value of n. **(2)**

(c) Find P(X = 8). **(3)**

(Total for Question 18 is 8 marks)

19 In a survey of 200 people, C is the event that a randomly chosen person owns a car and Bi is the event that a randomly chosen person owns a bicycle. It is known that P(C) = 0.55, P(Bi) = 0.30 and P(neither a car nor a bicycle) = 0.20.

(a) Show that P(C and Bi) = 0.05. **(3)**

(b) Find the number of the 200 people who own a bicycle but not a car. **(2)**

(c) Two people are selected at random, without replacement, from those in the survey who own a car. Find the probability that both of the people selected also own a bicycle. **(4)**

(Total for Question 19 is 9 marks)

20 Events A and B are independent, with P(A) = p and P(B) = p + 0.2, where $0 < p < 0.8$. Given that P(A or B) = 0.75.

(a) Show that p satisfies the equation $p^2 - 1.8p + 0.55 = 0$. **(4)**

(b) Hence find the value of p, giving a reason why the other root of the equation should be rejected. **(3)**

(c) State, with a reason, the value of P(A | B'). **(2)**

(Total for Question 20 is 9 marks)

21 A box contains n balls, of which 3 are red and the rest are blue, where $n > 3$. Two balls are drawn at random from the box, without replacement. Given that P(both balls drawn are red) = 1/15.

(a) Show that $n^2 - n - 90 = 0$. **(4)**

(b) Hence show that $n = 10$, explaining why the other solution of the equation is rejected. **(2)**

(c) Given that $n = 10$, find the probability that exactly one of the two balls drawn is red. **(3)**

(d) A third ball is now drawn at random from the box, without replacement, given that the first two balls drawn were both red. Find the probability that the third ball is blue. **(3)**

(Total for Question 21 is 12 marks)

Mark scheme · 2.3 Statistics: Probability

Question 1

(a) P(M) + P(P) - P(M and P)	**M1**
(a) cao 0.80	**A1**
(a) Answer: P(M or P) = 0.80	
(b) P(M) - P(M and P)	**M1**
(b) cao 0.40	**A1**
(b) Answer: P(M and not P) = 0.40	
(c) compute P(M) x P(P) = 0.65 x 0.40	**M1**
(c) P(M) x P(P) = 0.26	**A1**
(c) cso: 0.26 does not equal 0.25 (=P(M and P)), so M and P are not independent	**A1**
(c) Answer: Not independent, since P(M)P(P) = 0.26 not equal to P(M and P) = 0.25	
(d) 1 - P(M or P), following through from part (a)	**M1ft**
(d) cao 0.20	**A1ft**
(d) Answer: P(not M and not P) = 0.20	

Question 2

(a) first branches 2/3 (red) and 1/3 (blue)	**B1**
(a) second branches 5/8, 3/8 after red; 3/4, 1/4 after blue	**B1**
(a) P(RR) = 2/3 x 5/8	**M1**
(a) 5/12	**A1**
(a) Answer: 5/12	
(b) P(RR) + P(BB)	**M1**
(b) 1/2	**A1**
(b) Answer: 1/2	
(c) conditional probability: P(RB)/P(different)	**M1**
(c) 1/2	**A1**
(c) Answer: 1/2	

Question 3

(a) tennis total = 38 + 34	**M1**
(a) 72/136 = 9/17	**A1**
(a) Answer: 9/17	
(b) conditional probability: left-handed tennis players / all tennis players	**M1**
(b) 38/72 = 19/36	**A1**
(b) Answer: 19/36	
(c) left-handed swimmers / all left-handed members	**M1**
(c) 19/67 = 19/67	**A1**
(c) Answer: 19/67	

Question 4

(a) F only = 33 - 15	**M1**
(a) F only 18, S only 26, both 15	**A1**
(a) neither = 100 - 18 - 26 - 15 = 41	**A1**
(a) Answer: F only 18, S only 26, both 15, neither 41	
(b) P(F union S) = P(F) + P(S) - P(F and S)	**M1**
(b) 59/100	**A1**
(b) Answer: 59/100	
(c) P(F | S) = P(F and S)/P(S)	**M1**
(c) 15/41	**A1**
(c) Answer: 15/41	
(d) compares P(F) x P(S) = 1353/10000 with P(F and S) = 3/20	**M1**
(d) not independent	**A1**
(d) Answer: not independent	

Question 5

(a) first branches 1/2 (red) and 1/2 (blue)	**B1**
(a) second branches 5/11, 6/11 after red; 6/11, 5/11 after blue	**B1**
(a) P(RR) = 1/2 x 5/11	**M1**
(a) 5/22	**A1**
(a) Answer: 5/22	
(b) P(RR) + P(BB)	**M1**
(b) 5/11	**A1**
(b) Answer: 5/11	
(c) conditional probability: P(RB)/P(different)	**M1**
(c) 1/2	**A1**
(c) Answer: 1/2	

Question 6

(a) tennis total = 9 + 8	**M1**
(a) 17/112 = 17/112	**A1**
(a) Answer: 17/112	
(b) conditional probability: left-handed tennis players / all tennis players	**M1**
(b) 9/17 = 9/17	**A1**
(b) Answer: 9/17	
(c) left-handed swimmers / all left-handed members	**M1**
(c) 22/65 = 22/65	**A1**
(c) Answer: 22/65	

Question 7

(a) F only = 10 - 6 **M1**
(a) F only 4, S only 7, both 6 **A1**
(a) neither = 40 - 4 - 7 - 6 = 23 **A1**
(a) Answer: F only 4, S only 7, both 6, neither 23
(b) P(F union S) = P(F) + P(S) - P(F and S) **M1**
(b) 17/40 **A1**
(b) Answer: 17/40
(c) P(F | S) = P(F and S)/P(S) **M1**
(c) 6/13 **A1**
(c) Answer: 6/13
(d) compares P(F) x P(S) = 13/160 with P(F and S) = 3/20 **M1**
(d) not independent **A1**
(d) Answer: not independent

Question 8

(a) first branches 6/13 (red) and 7/13 (blue) **B1**
(a) second branches 5/12, 7/12 after red; 1/2, 1/2 after blue **B1**
(a) P(RR) = 6/13 x 5/12 **M1**
(a) 5/26 **A1**
(a) Answer: 5/26
(b) P(RR) + P(BB) **M1**
(b) 6/13 **A1**
(b) Answer: 6/13
(c) conditional probability: P(RB)/P(different) **M1**
(c) 1/2 **A1**
(c) Answer: 1/2

Question 9

(a) tennis total = 15 + 29 **M1**
(a) 44/138 = 22/69 **A1**
(a) Answer: 22/69
(b) conditional probability: left-handed tennis players / all tennis players **M1**
(b) 15/44 = 15/44 **A1**
(b) Answer: 15/44
(c) left-handed swimmers / all left-handed members **M1**
(c) 38/78 = 19/39 **A1**
(c) Answer: 19/39

Question 10

(a) independent so P(A and B) = P(A) x P(B) **M1**
(a) 2/35 **A1**
(a) P(A or B) = P(A) + P(B) - P(A and B) **M1**
(a) 3/7 **A1**
(a) Answer: P(A and B) = 2/35, P(A or B) = 3/7
(b) forms p + 2/7 - 2/7p = 3/7 **M1**
(b) rearranges: p(1 - 2/7) = 1/7 **M1**
(b) P(C) = 1/5 **A1**
(b) Answer: P(C) = 1/5
(c) P(A | B') = P(A and B')/P(B') = P(A)P(B')/P(B') **M1**
(c) 1/5 **A1**
(c) it equals P(A), confirming that A and B are independent **B1**
(c) Answer: 1/5

Question 11

(a) tennis total = 15 + 13 **M1**
(a) 28/150 = 14/75 **A1**
(a) Answer: 14/75
(b) conditional probability: left-handed tennis players / all tennis players **M1**
(b) 15/28 = 15/28 **A1**
(b) Answer: 15/28
(c) left-handed swimmers / all left-handed members **M1**
(c) 20/71 = 20/71 **A1**
(c) Answer: 20/71

Question 12

(a) independent so P(A and B) = P(A) x P(B) **M1**
(a) 9/40 **A1**
(a) P(A or B) = P(A) + P(B) - P(A and B) **M1**
(a) 33/40 **A1**
(a) Answer: P(A and B) = 9/40, P(A or B) = 33/40
(b) forms p + 3/4 - 3/4p = 33/40 **M1**
(b) rearranges: p(1 - 3/4) = 3/40 **M1**
(b) P(C) = 3/10 **A1**
(b) Answer: P(C) = 3/10
(c) P(A | B') = P(A and B')/P(B') = P(A)P(B')/P(B') **M1**
(c) 3/10 **A1**
(c) it equals P(A), confirming that A and B are independent **B1**
(c) Answer: 3/10

Question 13

(a) F only = 7 - 3 **M1**
(a) F only 4, S only 3, both 3 **A1**
(a) neither = 20 - 4 - 3 - 3 = 10 **A1**
(a) Answer: F only 4, S only 3, both 3, neither 10
(b) P(F union S) = P(F) + P(S) - P(F and S) **M1**
(b) 1/2 **A1**
(b) Answer: 1/2
(c) P(F | S) = P(F and S)/P(S) **M1**
(c) 1/2 **A1**
(c) Answer: 1/2
(d) compares P(F) x P(S) = 21/200 with P(F and S) = 3/20 **M1**
(d) not independent **A1**
(d) Answer: not independent

Question 14

(a) F only = 15 - 5 **M1**
(a) F only 10, S only 5, both 5 **A1**
(a) neither = 40 - 10 - 5 - 5 = 20 **A1**
(a) Answer: F only 10, S only 5, both 5, neither 20
(b) P(F union S) = P(F) + P(S) - P(F and S) **M1**
(b) 1/2 **A1**
(b) Answer: 1/2
(c) P(F | S) = P(F and S)/P(S) **M1**
(c) 1/2 **A1**
(c) Answer: 1/2
(d) compares P(F) x P(S) = 3/32 with P(F and S) = 1/8 **M1**
(d) not independent **A1**
(d) Answer: not independent

Question 15

(a) P(RR) = 5/12 x 4/11, oe **M1**

(a) P(BB) = 7/12 x 6/11, oe **M1**

(a) cao 31/66 (awrt 0.470) **A1**

(a) Answer: P(same colour) = 31/66 (awrt 0.470)

(b) P(BB) = 7/12 x 6/11 **M1**

(b) 1 - P(BB) **M1**

(b) cao 15/22 (awrt 0.682) **A1**

(b) Answer: P(at least one red) = 15/22 (awrt 0.682)

(c) identify P(RR) = 5/33 (ft from a) **M1**

(c) divide by P(same colour) = 31/66 (ft from a) **M1**

(c) cao 10/31 (awrt 0.323) **A1**

(c) Answer: P(both red | same colour) = 10/31 (awrt 0.323)

Question 16

(a) $10C3 \times 0.3^3 \times 0.7^7$ **M1**

(a) awrt 0.267 **A1**

(a) Answer: P(X = 3) = 0.267 (3 s.f.)

(b) P(X=0) + P(X=1) + P(X=2) **M1**

(b) correct individual terms: $0.7^{10} + 10x0.3x0.7^9 + 45x0.3^2x0.7^8$ **M1**

(b) awrt 0.383 **A1**

(b) Answer: P(X ≤ 2) = 0.383 (3 s.f.)

(c) 1 - P(X ≤ 3) **M1**

(c) P(X ≤ 3) = P(X ≤ 2) (ft from b) + P(X = 3) (ft from a) **M1**

(c) awrt 0.350 **A1**

(c) Answer: P(X ≥ 4) = 0.350 (3 s.f.)

Question 17

(a) sum of probabilities = 1: k(1+2+3+4) = 1, i.e. 10k = 1 **M1**

(a) cso: k = 0.1 **A1**

(a) Answer: k = 0.1

(b) P(X=3) + P(X=4) = 3k + 4k, using k = 0.1 (ft) **M1**

(b) cao 0.7 **A1**

(b) Answer: P(X ≥ 3) = 0.7

(c) sum of x P(x) with at least three correct terms **M1**

(c) all four terms correct: 1(0.1)+2(0.2)+3(0.3)+4(0.4) **M1**

(c) cao 3.0 **A1**

(c) Answer: E(X) = 3.0

(d) attempt $E(X^2)$ = sum x^2 P(x) **M1**

(d) correct value $E(X^2) = 10.0$ **M1**

(d) $Var(X) = E(X^2) - [E(X)]^2$, following through from c **M1ft**

(d) cao 1.0 **A1**

(d) Answer: Var(X) = 1.0

Question 18

(a) np = 6 and np(1-p) = 4.2 **M1**

(a) divide to eliminate n: (1-p) = 4.2/6 = 0.7 **M1**

(a) cso p = 0.3 **A1**

(a) Answer: p = 0.3

(b) n = 6 / 0.3, following through from part (a) **M1ft**

(b) cao n = 20 **A1**

(b) Answer: n = 20

(c) $20C8 \times 0.3^8 \times 0.7^{12}$, using n and p ft from a, b **M1ft**

(c) correct substitution of values **M1**

(c) awrt 0.114 **A1**

(c) Answer: P(X = 8) = 0.114 (3 s.f.)

Question 19

(a) P(C or Bi) = 1 - P(neither) = 0.80 **M1**

(a) use P(C or Bi) = P(C) + P(Bi) - P(C and Bi), rearranged for P(C and Bi) **M1**

(a) cso: 0.55 + 0.30 - 0.80 = 0.05 **A1**

(a) Answer: P(C and Bi) = 0.05

(b) P(Bi and not C) = P(Bi) - P(C and Bi) = 0.30 - 0.05, then x 200 (ft from a) **M1**

(b) cao 50 **A1**

(b) Answer: 50 people

(c) number owning a car = 0.55 x 200 = 110 **M1**

(c) number owning both a car and a bicycle = 0.05 x 200 = 10 (ft from a) **M1**

(c) (10/110) x (9/109), oe **M1**

(c) cao 9/1199 (awrt 0.00751) **A1**

(c) Answer: P(both own a bicycle) = 9/1199 (awrt 0.00751)

Question 20

(a) use P(A or B) = P(A) + P(B) - P(A)P(B), using independence **M1**

(a) correct substitution: p + (p+0.2) - p(p+0.2) = 0.75 **M1**

(a) expand and rearrange into a three-term quadratic set to zero **dM1**

(a) cso: $p^2 - 1.8p + 0.55 = 0$, fully shown **A1**

(a) Answer: $p^2 - 1.8p + 0.55 = 0$ (shown)

(b) solve $p^2 - 1.8p + 0.55 = 0$ using the quadratic formula or a calculator **M1**

(b) awrt p = 0.390 **A1**

(b) reason: the other root (awrt 1.41) is rejected as it exceeds 1 and probabilities cannot be greater than 1, oe **B1**

(b) Answer: p = 0.390 (3 s.f.); other root p = 1.41 rejected as impossible for a probability

(c) reason: since A and B are independent, A and B' are also independent, so P(A | B') = P(A) **B1**

(c) awrt 0.390, following through from part (b) **B1ft**

(c) Answer: P(A | B') = P(A) = 0.390 (3 s.f.)

Question 21

(a) P(both red) = (3/n) x (2/(n-1)), oe	**M1**
(a) set 6/(n(n-1)) = 1/15 and cross-multiply: 6 x 15 = n(n-1)	**M1**
(a) rearrange to form a three-term quadratic set to zero	**dM1**
(a) cso: n^2 - n - 90 = 0, fully shown	**A1**

(a) Answer: n^2 - n - 90 = 0 (shown)

(b) solve using the quadratic formula or factorisation: (n-10)(n+9) = 0	**M1**
(b) cso: n = 10 (n = -9 rejected since the number of balls cannot be negative)	**A1**

(b) Answer: n = 10 (n = -9 rejected)

(c) 2 x (3/10) x (7/9), oe	**M1**
(c) correct arithmetic simplification	**M1**
(c) cao 7/15 (awrt 0.467)	**A1**

(c) Answer: P(exactly one red) = 7/15 (awrt 0.467)

(d) identify the remaining balls after removing 2 red: 1 red and 7 blue (8 balls left)	**M1**
(d) correct ratio: 7/8	**M1**
(d) cao 7/8 (= 0.875)	**A1**

(d) Answer: P(third ball is blue | first two red) = 7/8 (= 0.875)