2.4 Statistics: Statistical Distributions (Binomial/Normal): 1-hour Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Tom works at a call centre and makes 25 sales calls each day. The probability that any call results in a sale is 0.24, independently of other calls. Let X be the number of sales Tom makes in a day, so X ~ B(25, 0.24). Tom's daily bonus, in pounds, is L = 8X + 15.

(a) Find E(X) and Var(X). **(2)**

(b) Find E(L) and Var(L). **(4)**

(c) Find the standard deviation of Tom's daily bonus, giving your answer to 3 significant figures. **(2)**

(Total for Question 1 is 8 marks)

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2 Mr Hussain, a maths tutor, asks his class to investigate the shape of the binomial distribution X ~ B(25, 0.4).

(a) Show that, for r = 1, 2, ..., 25, P(X=r) / P(X=r-1) = [(26-r)/r] x (2/3). **(3)**

(b) Hence determine the mode of X, justifying your answer. **(3)**

(c) Using your answer to part (a), explain, without further calculation, why P(X=25) is much smaller than P(X=10). **(2)**

(Total for Question 2 is 8 marks)

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3 A fair-looking spinner is spun 16 times and a success is a spin landing on blue. The probability of success on each trial is 0.45. Let X be the number of successes, so X ~ B(16, 0.45).

(a) Find P(X = 5). **(2)**

(b) Find P(X ≥ 5). **(2)**

(c) Find E(X) and Var(X), and hence the standard deviation of X to 3 significant figures. **(3)**

(d) Find P(4 ≤ X ≤ 6). **(2)**

(Total for Question 3 is 9 marks)

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4 The random variable X has distribution B(50, 0.4).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find P(X ≥ 20), to 3 significant figures. **(3)**

(c) Calculate the exact value of P(X ≥ 20) and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 4 is 8 marks)

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5 Priya is a marketing analyst. A company claims that the probability a customer makes a purchase within a week of receiving a promotional email is 0.3. After a new campaign, Priya suspects this probability has increased. She takes a random sample of 20 customers who receive the email and finds that 10 make a purchase within a week. Let X be the number of customers, out of 20, who make a purchase, and let p be the true probability of a purchase under the new campaign.

(a) State suitable null and alternative hypotheses for Priya's test. **(2)**

(b) Find the critical region for a test at the 5% significance level, in the form $X \geq c$, showing that your value of c is correct. **(4)**

(c) Using the observed value $X = 10$, state the conclusion of the test in context. **(2)**

(d) State the actual significance level of this test. **(1)**

(Total for Question 5 is 9 marks)

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6 Mrs Chen grows pea plants on her allotment in Bristol. The probability that a planted seed germinates is 0.85, independently of other seeds. She plants 16 seeds. Let X be the number of seeds that germinate, and let L be the pod length, in cm, of a germinated plant, modelled by $L \sim N(9.2, 1.1^2)$.

(a) State the distribution of X, including its parameters. **(1)**

(b) Calculate the probability that all 16 seeds germinate. **(2)**

(c) Calculate the probability that at least 14 seeds germinate. **(3)**

(d) For a randomly selected germinated plant, calculate $P(L > 10.5)$. **(3)**

(e) State one assumption made in modelling L using a normal distribution that may not hold in practice. **(1)**

(Total for Question 6 is 10 marks)

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Mark scheme · 2.4 Statistics: Statistical Distributions (Binomial/Normal): 1-hour Pack

Question 1

(a) E(X) = 25 x 0.24 = 6, cao **B1**

(a) Var(X) = 25 x 0.24 x 0.76 = 4.56, cao **B1**

(a) Answer: E(X) = 6, Var(X) = 4.56

(b) uses E(L) = 8E(X) + 15 **M1**

(b) E(L) = 63, cao **A1**

(b) uses Var(L) = 8^2 Var(X) **M1**

(b) Var(L) = 291.84, cao **A1**

(b) Answer: E(L) = 63, Var(L) = 291.84

(c) uses SD(L) = $\sqrt{\text{Var(L)}}$ **M1**

(c) SD(L) = awrt 17.1 (pounds) **A1**

(c) Answer: SD(L) = awrt £17.1

Question 2

(a) writes the ratio of binomial coefficients C(25,r)/C(25,r-1) = (25-r+1)/r = (26-r)/r **M1**

(a) writes the ratio of probability terms as (p/(1-p)) = 0.4/0.6 = 2/3 **M1**

(a) combines to give P(X=r)/P(X=r-1) = [(26-r)/r] x (2/3), cso **A1**

(a) Answer: P(X=r)/P(X=r-1) = [(26-r)/r] x (2/3)

(b) sets [(26-r)/r] x (2/3) ≥ 1 and solves to find r ≤ 10.4 **M1**

(b) deduces P(X=r) > P(X=r-1) for r ≤ 10, and P(X=r) < P(X=r-1) for r ≥ 11 **A1**

(b) mode of X = 10, cao **A1**

(b) Answer: Mode = 10

(c) for r>10.4 the ratio [(26-r)/r]x(2/3) is less than 1, so probabilities decrease steadily as r increases from 11 to 25 **B1**

(c) at r=25 the ratio [(26-25)/25]x(2/3) = 2/75, a very small factor, so after many successive decreasing multiplications P(X=25) is far smaller than the peak value P(X=10), oe **B1**

(c) Answer: The ratio [(26-r)/r]x(2/3) falls well below 1 for large r (e.g. 2/75 at r=25), so repeated multiplication by small factors makes P(X=25) far smaller than P(X=10)

Question 3

(a) 16C5 x 0.45^5 x 0.55^{11} **M1**

(a) awrt 0.112 **A1**

(a) Answer: awrt 0.112

(b) 1 - P(X ≤ 4) **M1**

(b) awrt 0.915 **A1**

(b) Answer: awrt 0.915

(c) E(X) = 16 x 0.45 = 7.2 **B1**

(c) Var(X) = 16 x 0.45 x 0.55 = 3.960 **B1**

(c) standard deviation = 1.99 **B1**

(c) Answer: E(X) = 7.2, Var(X) = 3.960, sd 1.99

(d) P(X ≤ 6) - P(X ≤ 3) **M1**

(d) awrt 0.338 **A1**

(d) Answer: awrt 0.338

Question 4

(a) n is large and p is close enough to 0.5 (np = 20 and n(1 - p) = 30 both large) **B1**

(a) mean 20, variance 12.00 **B1**

(a) Answer: N(20, 12.00)

(b) continuity correction: P(Y > 19.5) **M1**

(b) z = (19.5 - 20)/3.4641 = -0.1443 **M1**

(b) 0.557 **A1**

(b) Answer: 0.557

(c) 1 - P(X ≤ 19) with the binomial distribution **M1**

(c) 0.554 **A1**

(c) difference 0.0039 **A1**

(c) Answer: exact 0.554, difference 0.0039

Question 5

(a) H0: p = 0.3 **B1**

(a) H1: p > 0.3 **B1**

(a) Answer: H0: p=0.3, H1: p>0.3

(b) under H0, X ~ B(20, 0.3), and attempts P(X≥k) for values of k near the upper tail **M1**

(b) P(X≥10) = awrt 0.0479 (< 0.05) **A1**

(b) P(X≥9) = awrt 0.1133 (> 0.05) **A1**

(b) critical region is X ≥ 10, since this is the smallest region with probability ≤ 0.05, cso **A1**

(b) Answer: Critical region: X ≥ 10

(c) X=10 lies in the critical region (X≥10), so H0 is rejected **B1**

(c) there is sufficient evidence, at the 5% level, that the proportion of customers making a purchase has increased since the new campaign, cao **B1**

(c) Answer: Reject H0; evidence that the purchase proportion has increased

(d) actual significance level = P(X≥10) = awrt 0.0479 (4.79%) **B1**

(d) Answer: 0.0479 (4.79%)

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Question 6

(a) X ~ B(16, 0.85) **B1**

(a) Answer: X ~ B(16, 0.85)

(b) uses $P(X=16) = 0.85^{16}$ **M1**

(b) P(X=16) = awrt 0.0743 **A1**

(b) Answer: P(X=16) = awrt 0.0743

(c) identifies $P(X \geq 14) = P(X=14)+P(X=15)+P(X=16)$ **M1**

(c) computes each term correctly using the binomial formula **M1**

(c) $P(X \geq 14)$ = awrt 0.561 **A1**

(c) Answer: P(X≥14) = awrt 0.561

(d) standardises: z = (10.5-9.2)/1.1 **M1**

(d) z = 1.18 **A1**

(d) P(L>10.5) = awrt 0.119 **A1**

(d) Answer: P(L>10.5) = awrt 0.119

(e) assumes pod length is symmetric about the mean and can theoretically take any value (including implausibly large or negative lengths), which may not be realistic, oe **B1**

(e) Answer: Pod length cannot realistically be negative or unbounded, but the normal model allows this

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