2.4 Statistics: Statistical Distributions (Binomial/Normal): 2-hour Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A fair-looking spinner is spun 27 times and a success is a spin landing on blue. The probability of success on each trial is 0.45. Let X be the number of successes, so X ~ B(27, 0.45).

(a) Find P(X = 12). **(2)**

(b) Find P(X ≥ 12). **(2)**

(c) Find E(X) and Var(X), and hence the standard deviation of X to 3 significant figures. **(3)**

(d) Find P(11 ≤ X ≤ 13). **(2)**

(Total for Question 1 is 9 marks)

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2 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that P(X < 93) = 0.1587 and P(X < 108) = 0.6915.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 2 is 7 marks)

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3 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that P(X < 47) = 0.1587 and P(X < 63) = 0.8413.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 3 is 7 marks)

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4 The times, in minutes, taken by customers to be served at a bank are modelled by $X \sim N(164, 2^2)$.

(a) Find $P(X > 166)$. **(3)**

(b) Find $P(161 < X < 166)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.95$, to 3 significant figures. **(3)**

(Total for Question 4 is 9 marks)

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5 The random variable X has distribution B(50, 0.6).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find $P(X \geq 25)$, to 3 significant figures. **(3)**

(c) Calculate the exact value of $P(X \geq 25)$ and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 5 is 8 marks)

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6 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $P(X < 40) = 0.3085$ and $P(X < 46) = 0.8413$.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 6 is 7 marks)

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7 The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $P(X < 60) = 0.0668$ and $P(X < 66) = 0.3085$.

(a) Find the z values corresponding to the two probabilities, using the inverse normal distribution function, to 2 decimal places. **(3)**

(b) Hence write two equations in μ and σ and solve them to find μ and σ, each to 1 decimal place. **(4)**

(Total for Question 7 is 7 marks)

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8 The masses, in grams, of apples from an orchard are modelled by $X \sim N(81, 5^2)$.

(a) Find $P(X > 91)$. **(3)**

(b) Find $P(78.5 < X < 91)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.8$, to 3 significant figures. **(3)**

(Total for Question 8 is 9 marks)

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9 The random variable X has distribution B(80, 0.3).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find $P(X \geq 26)$, to 3 significant figures. **(3)**

(c) Calculate the exact value of $P(X \geq 26)$ and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 9 is 8 marks)

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10 A machine produces components, and each is defective with a fixed probability, independently of the others. A sample of 15 is taken. The probability of success on each trial is 0.25. Let X be the number of successes, so $X \sim B(15, 0.25)$.

(a) Find $P(X = 5)$. **(2)**

(b) Find $P(X \geq 5)$. **(2)**

(c) Find E(X) and Var(X), and hence the standard deviation of X to 3 significant figures. **(3)**

(d) Find $P(4 \leq X \leq 6)$. **(2)**

(Total for Question 10 is 9 marks)

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11 The times, in minutes, taken by customers to be served at a bank are modelled by $X \sim N(56, 8^2)$.

(a) Find $P(X > 60)$. **(3)**

(b) Find $P(52 < X < 60)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.05$, to 3 significant figures. **(3)**

(Total for Question 11 is 9 marks)

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12 The daily rainfall, in mm, at a weather station in the wet season is modelled by $X \sim N(117, 8^2)$.

(a) Find $P(X > 129)$. **(3)**

(b) Find $P(105 < X < 129)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.9$, to 3 significant figures. **(3)**

(Total for Question 12 is 9 marks)

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Mark scheme · 2.4 Statistics: Statistical Distributions (Binomial/Normal): 2-hour Pack

Question 1

(a) $27C12 \times 0.45^{12} \times 0.55^{15}$	**M1**
(a) awrt 0.153	**A1**
(a) Answer: awrt 0.153	
(b) 1 - P(X ≤ 11)	**M1**
(b) awrt 0.597	**A1**
(b) Answer: awrt 0.597	
(c) E(X) = 27 x 0.45 = 12.15	**B1**
(c) Var(X) = 27 x 0.45 x 0.55 = 6.6825	**B1**
(c) standard deviation = 2.59	**B1**
(c) Answer: E(X) = 12.15, Var(X) = 6.6825, sd 2.59	
(d) P(X ≤ 13) - P(X ≤ 10)	**M1**
(d) awrt 0.437	**A1**
(d) Answer: awrt 0.437	

Question 2

(a) inverse normal of 0.1587	**M1**
(a) z = -1.00	**A1**
(a) z = 0.50	**A1**
(a) Answer: z = -1.00 and 0.50	
(b) (93 - μ)/σ = -1.00 and (108 - μ)/σ = 0.50	**M1**
(b) subtracts to eliminate μ	**M1**
(b) σ = 10.0	**A1**
(b) μ = 103.0	**A1**
(b) Answer: μ = 103.0, σ = 10.0	

Question 3

(a) inverse normal of 0.1587	**M1**
(a) z = -1.00	**A1**
(a) z = 1.00	**A1**
(a) Answer: z = -1.00 and 1.00	
(b) (47 - μ)/σ = -1.00 and (63 - μ)/σ = 1.00	**M1**
(b) subtracts to eliminate μ	**M1**
(b) σ = 8.0	**A1**
(b) μ = 55.0	**A1**
(b) Answer: μ = 55.0, σ = 8.0	

Question 4

(a) standardises: z = (166 - 164)/2 = 1	**M1**
(a) P(Z > 1) = 1 - 0.8413	**A1**
(a) awrt 0.159	**A1**
(a) Answer: awrt 0.159	
(b) finds both z values: -1.5 and 1	**M1**
(b) P(Z < upper) - P(Z < lower)	**M1**
(b) awrt 0.775	**A1**
(b) Answer: awrt 0.775	
(c) z = inverse normal of 0.95 = 1.6449	**M1**
(c) a = 164 + z x 2	**M1**
(c) a = 167	**A1**
(c) Answer: a = 167	

Question 5

(a) n is large and p is close enough to 0.5 (np = 30 and n(1 - p) = 20 both large)	**B1**
(a) mean 30, variance 12.00	**B1**
(a) Answer: N(30, 12.00)	
(b) continuity correction: P(Y > 24.5)	**M1**
(b) z = (24.5 - 30)/3.4641 = -1.5877	**M1**
(b) 0.944	**A1**
(b) Answer: 0.944	
(c) 1 - P(X ≤ 24) with the binomial distribution	**M1**
(c) 0.943	**A1**
(c) difference 0.0012	**A1**
(c) Answer: exact 0.943, difference 0.0012	

Question 6

(a) inverse normal of 0.3085	**M1**
(a) z = -0.50	**A1**
(a) z = 1.00	**A1**
(a) Answer: z = -0.50 and 1.00	
(b) (40 - μ)/σ = -0.50 and (46 - μ)/σ = 1.00	**M1**
(b) subtracts to eliminate μ	**M1**
(b) σ = 4.0	**A1**
(b) μ = 42.0	**A1**
(b) Answer: μ = 42.0, σ = 4.0	

Question 7

(a) inverse normal of 0.0668	**M1**
(a) z = -1.50	**A1**
(a) z = -0.50	**A1**
(a) Answer: z = -1.50 and -0.50	
(b) (60 - μ)/σ = -1.50 and (66 - μ)/σ = -0.50	**M1**
(b) subtracts to eliminate μ	**M1**
(b) σ = 6.0	**A1**
(b) μ = 69.0	**A1**
(b) Answer: μ = 69.0, σ = 6.0	

Question 8

(a) standardises: z = (91 - 81)/5 = 2	**M1**
(a) P(Z > 2) = 1 - 0.9772	**A1**
(a) awrt 0.0228	**A1**
(a) Answer: awrt 0.0228	
(b) finds both z values: -0.5 and 2	**M1**
(b) P(Z < upper) - P(Z < lower)	**M1**
(b) awrt 0.669	**A1**
(b) Answer: awrt 0.669	
(c) z = inverse normal of 0.8 = 0.8416	**M1**
(c) a = 81 + z x 5	**M1**
(c) a = 85.2	**A1**
(c) Answer: a = 85.2	

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Question 9

(a) n is large and p is close enough to 0.5 (np = 24 and n(1 - p) = 56 both large) **B1**

(a) mean 24, variance 16.80 **B1**

(a) Answer: N(24, 16.80)

(b) continuity correction: P(Y > 25.5) **M1**

(b) z = (25.5 - 24)/4.0988 = 0.3660 **M1**

(b) 0.357 **A1**

(b) Answer: 0.357

(c) 1 - P(X ≤ 25) with the binomial distribution **M1**

(c) 0.352 **A1**

(c) difference 0.0051 **A1**

(c) Answer: exact 0.352, difference 0.0051

Question 10

(a) $15C5 \times 0.25^5 \times 0.75^{10}$ **M1**

(a) awrt 0.165 **A1**

(a) Answer: awrt 0.165

(b) 1 - P(X ≤ 4) **M1**

(b) awrt 0.314 **A1**

(b) Answer: awrt 0.314

(c) E(X) = 15 x 0.25 = 3.75 **B1**

(c) Var(X) = 15 x 0.25 x 0.75 = 2.8125 **B1**

(c) standard deviation = 1.68 **B1**

(c) Answer: E(X) = 3.75, Var(X) = 2.8125, sd 1.68

(d) P(X ≤ 6) - P(X ≤ 3) **M1**

(d) awrt 0.482 **A1**

(d) Answer: awrt 0.482

Question 11

(a) standardises: z = (60 - 56)/8 = 0.5 **M1**

(a) P(Z > 0.5) = 1 - 0.6915 **A1**

(a) awrt 0.309 **A1**

(a) Answer: awrt 0.309

(b) finds both z values: -0.5 and 0.5 **M1**

(b) P(Z < upper) - P(Z < lower) **M1**

(b) awrt 0.383 **A1**

(b) Answer: awrt 0.383

(c) z = inverse normal of 0.05 = -1.6449 **M1**

(c) a = 56 + z x 8 **M1**

(c) a = 42.8 **A1**

(c) Answer: a = 42.8

Question 12

(a) standardises: z = (129 - 117)/8 = 1.5 **M1**

(a) P(Z > 1.5) = 1 - 0.9332 **A1**

(a) awrt 0.0668 **A1**

(a) Answer: awrt 0.0668

(b) finds both z values: -1.5 and 1.5 **M1**

(b) P(Z < upper) - P(Z < lower) **M1**

(b) awrt 0.866 **A1**

(b) Answer: awrt 0.866

(c) z = inverse normal of 0.9 = 1.2816 **M1**

(c) a = 117 + z x 8 **M1**

(c) a = 127 **A1**

(c) Answer: a = 127

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