2.4 Statistics: Statistical Distributions (Binomial/Normal): 30-minute Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 30 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Aaliyah is analysing height data. Heights of adult women in the UK, X cm, are modelled by $X \sim N(163, 6.5^2)$.

(a) Calculate $P(X > 170)$. **(3)**

(b) Calculate $P(155 < X < 170)$. **(3)**

(c) Give one criticism of using a normal distribution to model adult female height. **(1)**

(Total for Question 1 is 7 marks)

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2 Grace is the quality control manager at a factory in Leeds that fills bags of sugar. The mass of sugar in a bag, W grams, is modelled by $W \sim N(1010, \sigma^2)$. It is known that 10% of bags weigh less than 1000 g.

(a) State the value of z such that $P(Z < z) = 0.10$. **(1)**

(b) Find the value of σ, giving your answer to 3 significant figures. **(3)**

(c) Using this value of σ, calculate the probability that a randomly selected bag weighs more than 1020 g. **(2)**

(d) Calculate $P(995 < W < 1020)$. **(3)**

(Total for Question 2 is 9 marks)

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3 Zara, a statistics student, is investigating a binomial model $X \sim B(n, p)$ where $n = 48$. She is given that $\text{Var}(X) = 0.75 \times E(X)$.

(a) Show that $p = 0.25$. **(4)**

(b) State the values of np and $n(1-p)$, and explain why a normal approximation to X would be appropriate. **(2)**

(c) Using the normal approximation to X, with a continuity correction, estimate $P(X \leq 8)$. **(4)**

(Total for Question 3 is 10 marks)

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Mark scheme · 2.4 Statistics: Statistical Distributions (Binomial/Normal): 30-minute Pack

Question 1

(a) standardises: z = (170-163)/6.5, oe **M1**

(a) z = 1.08 (or better) and uses the normal distribution correctly **A1**

(a) P(X>170) = awrt 0.141 (allow 0.140 if z is rounded to 2 d.p. and tables are used) **A1**

(a) Answer: P(X>170) = awrt 0.141 (0.140 if z is rounded to 1.08)

(b) finds both z-values: z1=(155-163)/6.5=-1.23, z2=(170-163)/6.5=1.08 **M1**

(b) uses P(155<X<170) = Phi(1.08) - Phi(-1.23), oe **A1**

(b) P(155<X<170) = awrt 0.750 (allow 0.751 if z-values are rounded to 2 d.p. and tables are used) **A1**

(b) Answer: P(155<X<170) = awrt 0.750 (0.751 if z-values are rounded to 2 d.p.)

(c) the normal distribution is defined for all real values (including negative values), but height cannot be negative, oe; or the true distribution may have a natural upper/lower limit not reflected by the normal model **B1**

(c) Answer: The normal model allows impossible negative heights, so it can only be an approximation

Question 2

(a) z = -1.2816 (or -1.28), using the fact that P(Z<1.2816)=0.90 **B1**

(a) Answer: z = -1.2816

(b) forms the equation (1000-1010)/σ = -1.2816, oe **M1**

(b) rearranges to σ = 10/1.2816 **M1**

(b) σ = awrt 7.80 **A1**

(b) Answer: σ = awrt 7.80

(c) standardises: z = (1020-1010)/7.80 = 1.28 (ft their σ) **M1ft**

(c) P(W>1020) = awrt 0.100 **A1ft**

(c) Answer: P(W>1020) = awrt 0.100

(d) finds both z-values: z1=(995-1010)/7.80=-1.92, z2=(1020-1010)/7.80=1.28 **M1ft**

(d) uses P(995<W<1020) = Phi(1.28) - Phi(-1.92), oe **A1ft**

(d) P(995<W<1020) = awrt 0.873 (allow 0.872 if z-values are rounded to 2 d.p. and tables are used) **A1ft**

(d) Answer: P(995<W<1020) = awrt 0.873 (0.872 if z-values are rounded to 2 d.p.)

Question 3

(a) states E(X) = 48p **M1**

(a) states Var(X) = 48p(1-p) **M1**

(a) forms 48p(1-p) = 0.75 x 48p and divides both sides by 48p (p not 0) to get 1-p = 0.75 **M1**

(a) p = 0.25, cso **A1**

(a) Answer: p = 0.25

(b) np = 12 and n(1-p) = 36 **B1**

(b) both values are greater than 5, so X can be well approximated by a normal distribution, oe **B1**

(b) Answer: np=12, n(1-p)=36; both exceed 5, so normal approximation is appropriate

(c) states Y ~ N(12, 9), since Var(X)=9 from part (a) **B1**

(c) applies continuity correction: P(X≤8) approx P(Y<8.5) **M1**

(c) standardises: z = (8.5-12)/3 = -1.17 **M1**

(c) P(X≤8) = awrt 0.122 (allow 0.121 if z is rounded to -1.17 and tables are used) **A1**

(c) Answer: P(X≤8) = awrt 0.122 (0.121 if z is rounded to -1.17)

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