2.4 Statistics: Statistical Distributions (Binomial/Normal): Test Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Daniel is a quality inspector at a factory that produces steel bolts. From long-term records, 8% of bolts produced are defective. Daniel selects a random sample of 10 bolts for inspection. Let X be the number of defective bolts in the sample.

(a) State two conditions needed for X to be modelled by a binomial distribution. **(2)**

(b) Given that $X \sim B(10, 0.08)$, calculate $P(X = 2)$. **(2)**

(c) Calculate $P(X \leq 1)$. **(2)**

(Total for Question 1 is 6 marks)

---

2 A manufacturing engineer, Mr Whitfield, times how long a robot arm takes to complete an assembly task. The time, X seconds, is modelled by $X \sim N(72, 5^2)$.

(a) Find the value of a such that $P(X > a) = 0.2005$. **(4)**

(b) Find the value of b such that $P(X < b) = 0.2005$. **(2)**

(c) Hence, or otherwise, calculate $P(b < X < a)$. **(2)**

(Total for Question 2 is 8 marks)

---

3 The random variable X has distribution B(100, 0.5).

(a) Explain why X may be approximated by a normal distribution, and state the parameters of this approximating distribution. **(2)**

(b) Use a normal approximation to find $P(X \geq 46)$, to 3 significant figures. **(3)**

(c) Calculate the exact value of $P(X \geq 46)$ and the difference from the approximation, to 3 significant figures. **(3)**

(Total for Question 3 is 8 marks)

© Revision Library | revisionlibrary.co.uk

4 The times, in minutes, taken by customers to be served at a bank are modelled by $X \sim N(92, 6^2)$.

(a) Find $P(X > 95)$. **(3)**

(b) Find $P(86 < X < 95)$. **(3)**

(c) Find the value of a such that $P(X < a) = 0.8$, to 3 significant figures. **(3)**

(Total for Question 4 is 9 marks)

---

5 Dr Okafor, a lecturer, models the scores on a Sociology module test as $X \sim N(\mu, \sigma^2)$. It is known that $P(X < 60) = 0.2$ and $P(X > 85) = 0.1$.

(a) State the two z-values used, giving each to 4 decimal places. **(2)**

(b) Form two equations in μ and σ. **(2)**

(c) Solve the equations to find the values of μ and σ, giving each to 3 significant figures. **(4)**

(Total for Question 5 is 8 marks)

---

6 Liam works for a polling company, Meridian Polling. National figures suggest 55% of voters support a proposed policy. Liam surveys a random sample of 200 voters. Let X be the number, out of 200, who support the policy, so $X \sim B(200, 0.55)$.

(a) Give a reason why a normal approximation to X is appropriate here. **(1)**

(b) Using a suitable approximation with a continuity correction, calculate $P(X > 110)$. **(4)**

(c) Using a suitable approximation with a continuity correction, calculate $P(100 \leq X \leq 120)$. **(4)**

(Total for Question 6 is 9 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 2.4 Statistics: Statistical Distributions (Binomial/Normal): Test Pack

Question 1

(a) there is a fixed number of trials (n=10, known in advance), oe **B1**

(a) each bolt is independent with only two outcomes (defective/not defective) and a constant probability of being defective (p=0.08), oe **B1**

(a) Answer: Fixed number of trials; independent trials with constant probability of defect

(b) uses $P(X=2) = C(10,2) \times 0.08^2 \times 0.92^8$ **M1**

(b) P(X=2) = awrt 0.148 **A1**

(b) Answer: P(X=2) = awrt 0.148

(c) finds $P(X=0) + P(X=1) = 0.92^{10} + 10x0.08x0.92^9$ **M1**

(c) P(X≤1) = awrt 0.812 **A1**

(c) Answer: P(X≤1) = awrt 0.812

Question 2

(a) recognises P(Z<z) = 1-0.2005 = 0.7995 **M1**

(a) z = 0.84 **A1**

(a) forms a = 72 + 0.84x5 **M1**

(a) a = 76.2 **A1**

(a) Answer: a = 76.2

(b) uses symmetry: z = -0.84 **B1**

(b) b = 72 - 0.84x5 = 67.8 **A1**

(b) Answer: b = 67.8

(c) uses P(b<X<a) = 1 - 2(0.2005), oe **M1**

(c) P(67.8<X<76.2) = 0.599, cao **A1**

(c) Answer: P(67.8<X<76.2) = 0.599

Question 3

(a) n is large and p is close enough to 0.5 (np = 50 and n(1 - p) = 50 both large) **B1**

(a) mean 50, variance 25.00 **B1**

(a) Answer: N(50, 25.00)

(b) continuity correction: P(Y > 45.5) **M1**

(b) z = (45.5 - 50)/5.0000 = -0.9000 **M1**

(b) 0.816 **A1**

(b) Answer: 0.816

(c) 1 - P(X ≤ 45) with the binomial distribution **M1**

(c) 0.816 **A1**

(c) difference 0.000041 **A1**

(c) Answer: exact 0.816, difference 0.000041

Question 4

(a) standardises: z = (95 - 92)/6 = 0.5 **M1**

(a) P(Z > 0.5) = 1 - 0.6915 **A1**

(a) awrt 0.309 **A1**

(a) Answer: awrt 0.309

(b) finds both z values: -1 and 0.5 **M1**

(b) P(Z < upper) - P(Z < lower) **M1**

(b) awrt 0.533 **A1**

(b) Answer: awrt 0.533

(c) z = inverse normal of 0.8 = 0.8416 **M1**

(c) a = 92 + z x 6 **M1**

(c) a = 97.0 **A1**

(c) Answer: a = 97.0

Question 5

(a) z1 = -0.8416, since P(Z< 0.8416)=0.80 **B1**

(a) z2 = 1.2816, since P(Z<1.2816)=0.90 **B1**

(a) Answer: z1 = -0.8416, z2 = 1.2816

(b) 60 = μ - 0.8416 σ, oe **M1**

(b) 85 = μ + 1.2816 σ, oe **M1**

(b) Answer: μ - 0.8416sigma = 60 and μ + 1.2816sigma = 85

(c) subtracts the equations to eliminate μ: 25 = 2.1232 σ **M1**

(c) solves for σ = 25/2.1232 **dM1**

(c) σ = awrt 11.8 **A1**

(c) μ = awrt 69.9 (ft their σ substituted into either equation) **A1ft**

(c) Answer: μ = awrt 69.9, σ = awrt 11.8

Question 6

(a) n is large and both np=110>5 and n(1-p)=90>5, so a normal approximation is valid, oe **B1**

(a) Answer: np=110>5 and n(1-p)=90>5, so normal approximation is valid

(b) states Y ~ N(110, 49.5), since np=110, npq=200x0.55x0.45=49.5 **B1**

(b) applies continuity correction: P(X>110) approx P(Y>110.5) **M1**

(b) standardises: $z = (110.5-110)/\sqrt{49.5} = 0.07$ **M1**

(b) P(X>110) = awrt 0.472 **A1**

(b) Answer: P(X>110) = awrt 0.472

(c) applies continuity correction at both ends: P(99.5 < Y < 120.5) **M1**

(c) standardises both values: z1=(99.5-110)/7.036=-1.49, z2=(120.5-110)/7.036=1.49 **M1**

(c) uses P(100≤X≤120) approx 2Phi(1.49)-1, oe **A1**

(c) P(100≤X≤120) = awrt 0.864 **A1**

(c) Answer: P(100≤X≤120) = awrt 0.864

© Revision Library | revisionlibrary.co.uk